AMATH 483/583
High Performance Scientific Computing

Lecture 20:
Advanced Message Passing, Cannon's Algorithm

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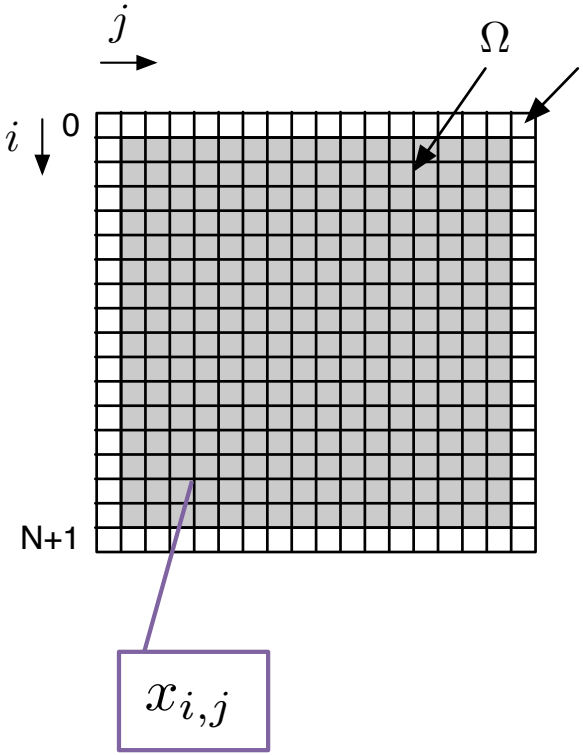
Administrative

- Fill out course evaluations!

Laplace's Equation on a Regular Grid



$$\begin{aligned} \nabla^2\phi &= 0 \quad \text{on } \Omega \\ \phi &= f \quad \text{on } \partial\Omega \end{aligned}$$

$$\frac{1}{h^2}\begin{bmatrix} 4 & -1 & \cdots & -1 & & \\ -1 & \ddots & \ddots & \ddots & \ddots & \\ \vdots & \ddots & \ddots & \ddots & \ddots & -1 \\ -1 & \ddots & \ddots & \ddots & \ddots & \vdots \\ & \ddots & \ddots & \ddots & \ddots & -1 \\ & & -1 & \cdots & -1 & 4 \end{bmatrix} \begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ \vdots \\ \\ \\ \end{bmatrix} = \begin{bmatrix} b_0 \\ b_1 \\ b_2 \\ \vdots \\ \\ \\ \end{bmatrix}$$

Discretization

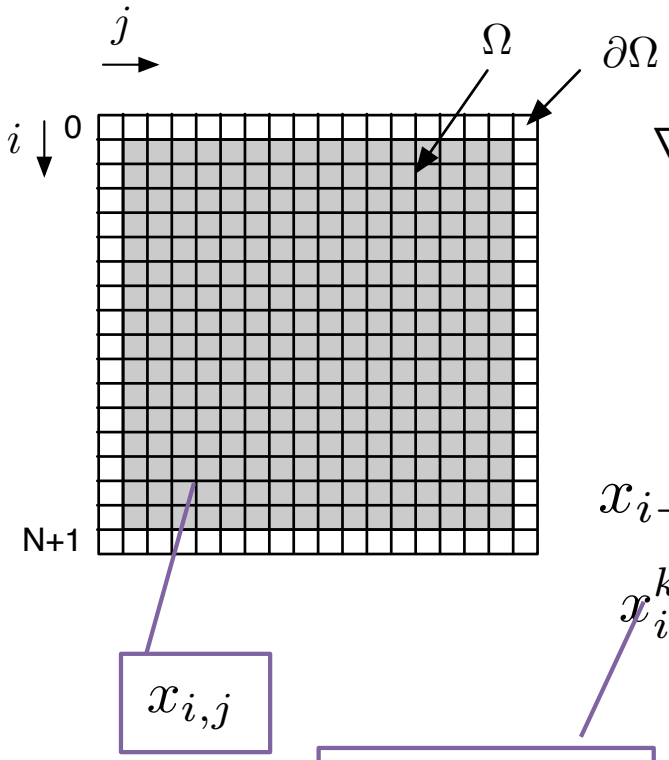
$$x_{i-1,j} + x_{i+1,j} + x_{i,j-1} + x_{i,j+1} - 4x_{i,j} = 0$$

$$x_{i,j} = (x_{i-1,j} + x_{i+1,j} + x_{i,j-1} + x_{i,j+1})/4$$

The value of each point on the grid

The average of its neighbors

Jacobi Iteration



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Discretization

$$x_{i-1,j} + x_{i+1,j} + x_{i,j-1} + x_{i,j+1} - 4x_{i,j} = 0$$

$$x_{i,j}^{k+1} = (x_{i-1,j}^{k} + x_{i+1,j}^{k} + x_{i,j-1}^{k} + x_{i,j+1}^{k})/4$$

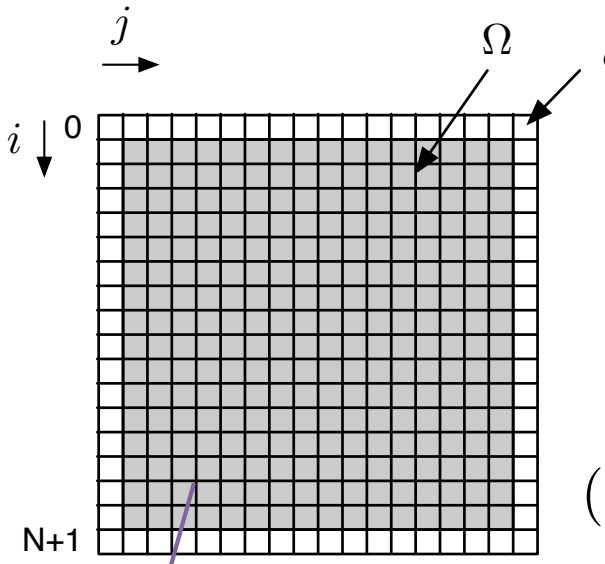
Iteration k+1

The value of each point on the grid

The average of its neighbors

Iteration k

Jacobi Iteration



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$$\frac{1}{h^2}\begin{bmatrix} 4 & -1 & \cdots & -1 & & \\ -1 & \ddots & \ddots & \ddots & \ddots & \\ \vdots & \ddots & \ddots & \ddots & \ddots & -1 \\ -1 & \ddots & \ddots & \ddots & \ddots & \vdots \\ & \ddots & \ddots & \ddots & \ddots & -1 \\ & & -1 & \cdots & -1 & 4 \end{bmatrix}\begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ \vdots \\ \\ \\ \end{bmatrix} = \begin{bmatrix} b_0 \\ b_1 \\ b_2 \\ \vdots \\ \\ \\ \end{bmatrix}$$

Discretization

$$(x^k_{i-1,j} + x^k_{i+1,j} + x^k_{i,j-1} + x^k_{i,j+1}) - 4x^{k+1}_{i,j} = 0$$

$$x^{k+1}_{i,j} = (x^k_{i-1,j} + x^k_{i+1,j} + x^k_{i,j-1} + x^k_{i,j+1})/4$$

$x_{i,j}$

Iteration k+1

The value of each point on the grid

The average of its neighbors

Iteration k

Jacobi Iteration

$$Ax = b$$

$$4x_{i,j} - (x_{i-1,j} + x_{i+1,j} + x_{i,j-1} + x_{i,j+1}) = 0$$

$$\frac{1}{h^2}\begin{bmatrix} 4 & -1 & \cdots & -1 & & \\ -1 & \ddots & \ddots & \ddots & \ddots & \\ \vdots & \ddots & \ddots & \ddots & \ddots & -1 \\ -1 & \ddots & \ddots & \ddots & \ddots & \vdots \\ & \ddots & \ddots & \ddots & \ddots & -1 \\ & & -1 & \cdots & -1 & 4 \end{bmatrix} \begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ \vdots \\ \end{bmatrix} = \begin{bmatrix} b_0 \\ b_1 \\ b_2 \\ \vdots \\ \end{bmatrix}$$

A

$$4x_{i,j}^{k+1} - (x_{i-1,j}^{k} + x_{i+1,j}^{k} + x_{i,j-1}^{k} + x_{i,j+1}^{k}) = 0$$

$$A = M - N$$

$$\frac{1}{h^2}\begin{bmatrix} 4 & 0 & \cdots & 0 & & \\ 0 & \ddots & \ddots & \ddots & \ddots & \\ \vdots & \ddots & \ddots & \ddots & \ddots & 0 \\ 0 & \ddots & \ddots & \ddots & \ddots & \vdots \\ & \ddots & \ddots & \ddots & \ddots & 0 \\ & & 0 & \cdots & 0 & 4 \end{bmatrix} \begin{bmatrix} x_0^{k+1} \\ x_1^{k+1} \\ x_2^{k+1} \\ \vdots \\ \end{bmatrix} - \frac{1}{h^2}\begin{bmatrix} 0 & -1 & \cdots & -1 & & \\ -1 & \ddots & \ddots & \ddots & \ddots & \\ \vdots & \ddots & \ddots & \ddots & \ddots & -1 \\ -1 & \ddots & \ddots & \ddots & \ddots & \vdots \\ & \ddots & \ddots & \ddots & \ddots & -1 \\ & & -1 & \cdots & -1 & 0 \end{bmatrix} \begin{bmatrix} x_0^{k} \\ x_1^{k} \\ x_2^{k} \\ \vdots \\ \end{bmatrix} = \begin{bmatrix} b_0 \\ b_1 \\ b_2 \\ \vdots \\ \end{bmatrix}$$

M

N

Jacobi Iteration

$Ax = b$

A

$$\frac{1}{h^2}\begin{bmatrix} 4 & -1 & \cdots & -1 & & \\ -1 & \ddots & \ddots & \ddots & \ddots & \\ \vdots & \ddots & \ddots & \ddots & \ddots & -1 \\ -1 & \ddots & \ddots & \ddots & \ddots & \vdots \\ & \ddots & \ddots & \ddots & \ddots & -1 \\ & & -1 & \cdots & -1 & 4 \end{bmatrix} \begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ \vdots \\ \end{bmatrix} = \begin{bmatrix} b_0 \\ b_1 \\ b_2 \\ \vdots \\ \end{bmatrix}$$

$$4x_{i,j} - (x_{i-1,j} + x_{i+1,j} + x_{i,j-1} + x_{i,j+1}) = 0$$

$$4x^{k+1}_{i,j} - (x^k_{i-1,j} + x^k_{i+1,j} + x^k_{i,j-1} + x^k_{i,j+1}) = 0$$

$A = M - N$

$$Mx^{k+1} - Nx^k = b$$

$$x^{k+1}_{i,j} = \frac{1}{4}(x^k_{i-1,j} + x^k_{i+1,j} + x^k_{i,j-1} + x^k_{i,j+1})$$

$$x^{k+1} = M^{-1}(Nx^k + b)$$

Average of neighbors

Still a stencil application

class Grid

```
class Grid {

public:
  explicit Grid(size_t x, size_t y)                    :
      xPoints(x+2), yPoints(y+2), arrayData(xPoints*yPoints) {}

        double &operator()(size_t i, size_t j)
            { return arrayData[i*yPoints + j]; }
  const double &operator()(size_t i, size_t j) const
            { return arrayData[i*yPoints + j]; }

  size_t numX() const { return xPoints; }
  size_t numY() const { return yPoints; }

private:
  size_t xPoints, yPoints;
  std::vector<double> arrayData;
};
```

Grid is a 2D array

Constructor

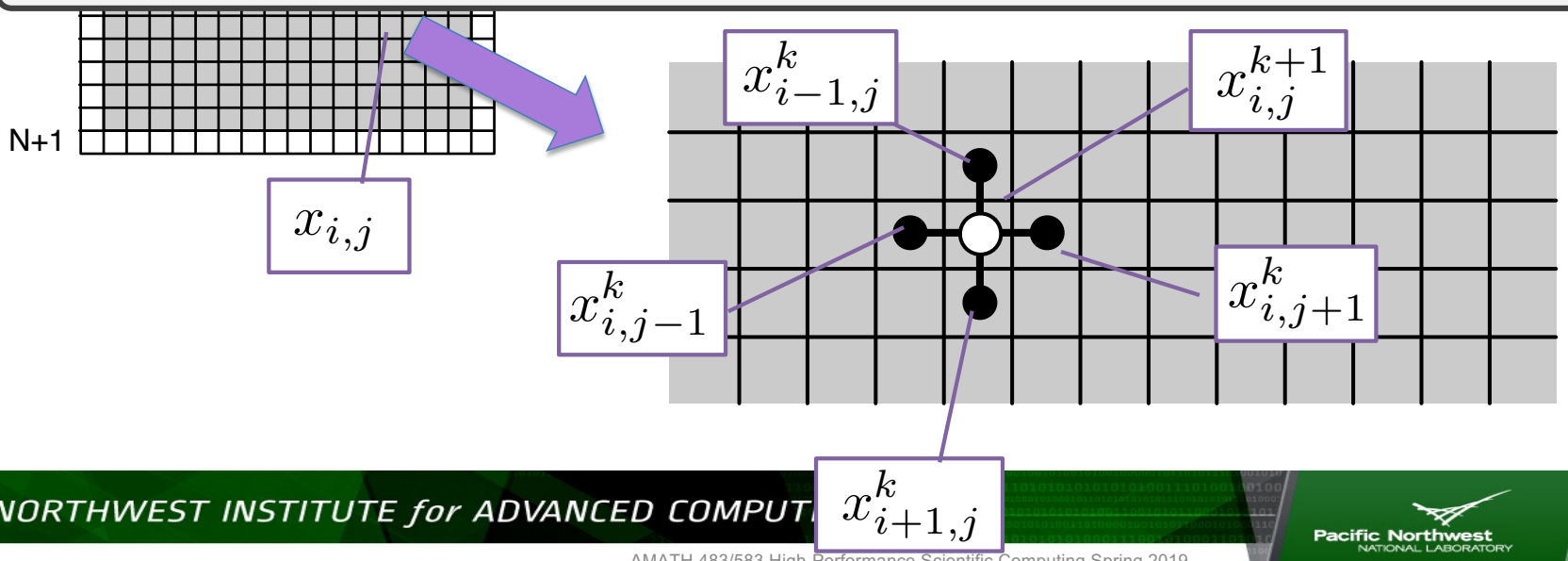
Accessor

Storage

Iterating for a solution

```
while (! converged()) {
  for (size_t k = 0; k < max_k; ++k) {
    for (size_t i = 1; i < N+1; ++i)
      for (size_t j = 1; j < N+1; ++j)
        x[k+1](i,j) = (x[k](i-1,j) + x[k](i+1,j) + x[k](i,j-1) + x[k](i,j+1))/4.0;
}
```

Claim: We only ever need two grids



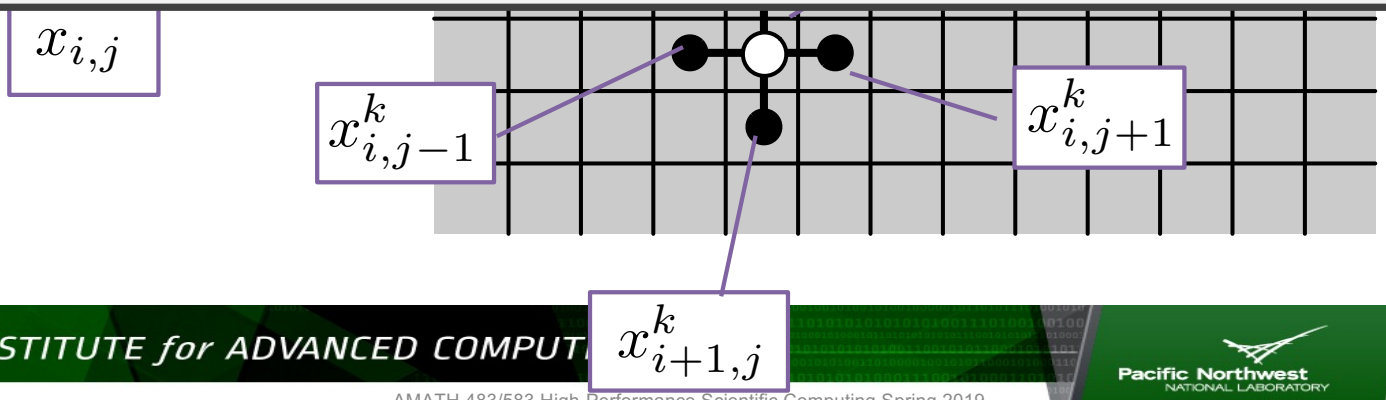
Iterating for a solution

```
while (! converged()) {
  for (size_t i = 1; i < N+1; ++i) {
    for (size_t j = 1; j < N+1; ++j) {
      xp(i,j) = (x(i-1,j) + x(i+1,j) + x(i,j-1) + x(i,j+1))/4.0;
    }
  }
  swap(xp, x);
}
```

Claim: We only ever need two grids

Make current the previous

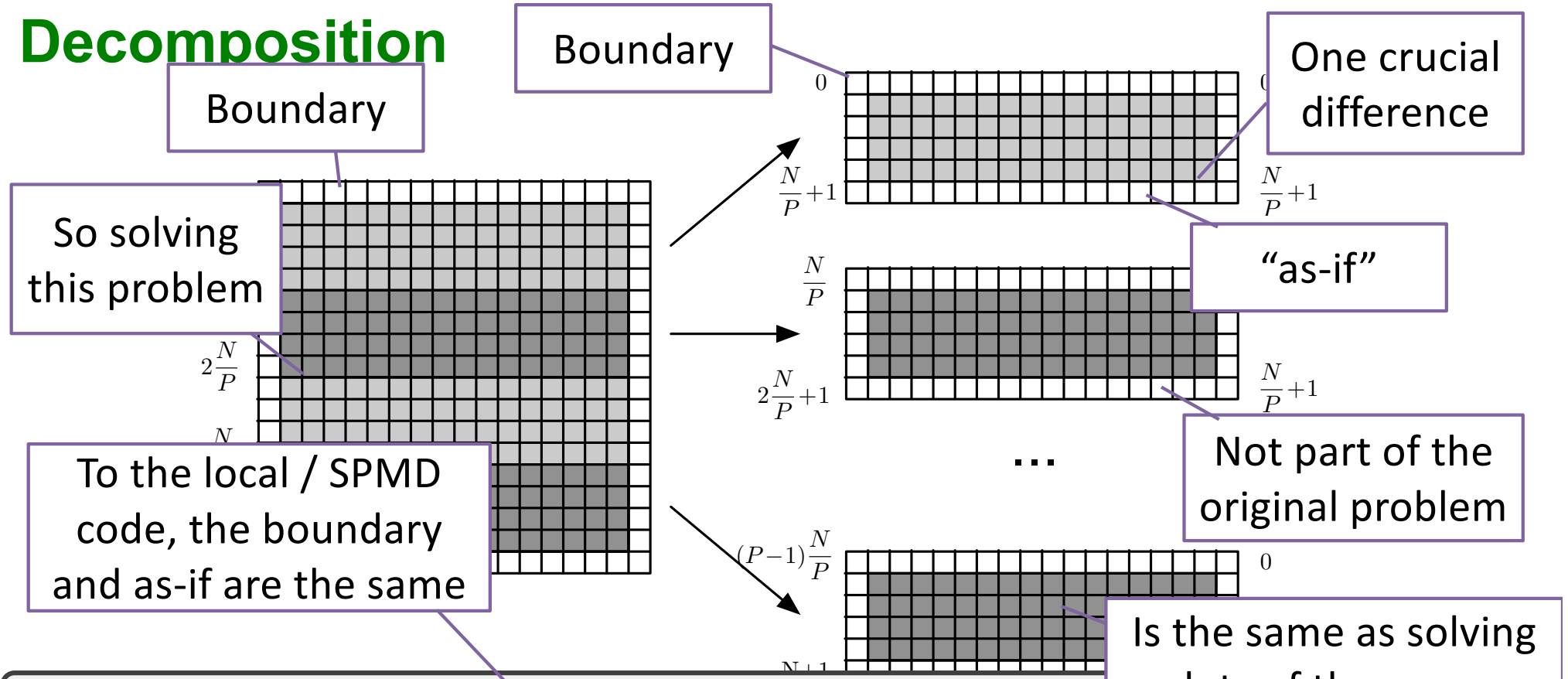
Could copy instead, but…



Sequential

```
void jacobi(Grid& x, Grid& xp) {
  while (! converged()) {
    for (size_t i = 1; i < x.num_x()-1; ++i) {
      for (size_t j = 1; j < x.num_y()-1; ++j) {
        xp(i,j) = (x(i-1,j) + x(i+1,j) + x(i,j-1) + x(i,j+1))/4.0;
      }
    }
    swap(xp, x);
  }
}
```

Decomposition

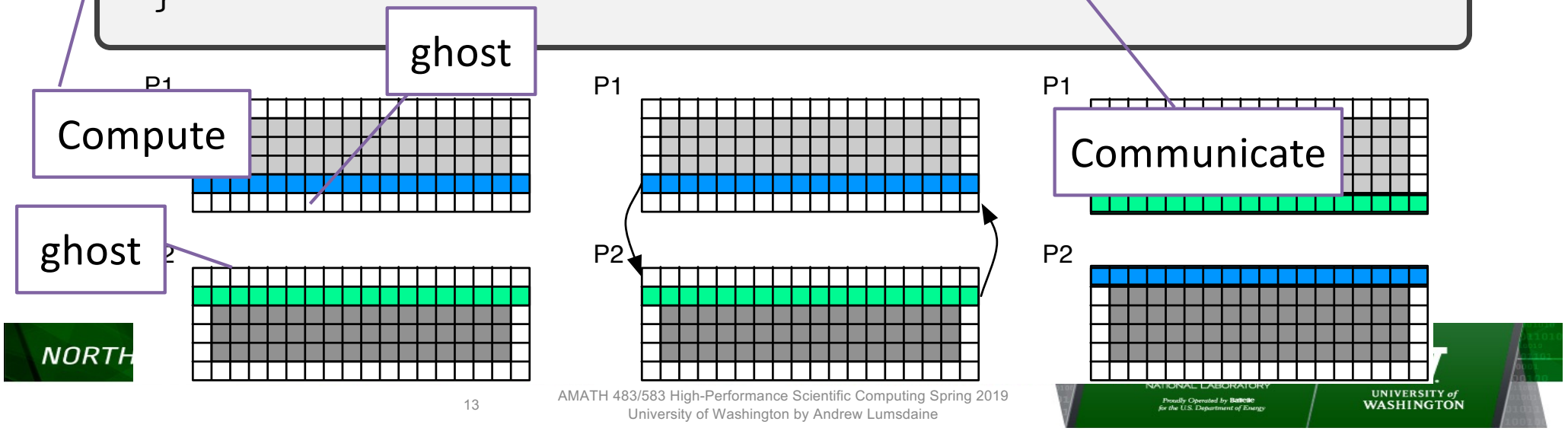



```
for (size_t i = 1; i < N/P+1; ++i)
  for (size_t j = 1; j < N+1; ++j)
    y(i,j) = (x(i-1,j) + x(i+1,j) + x(i,j-1) + x(i,j+1))/4.0;
```

Compute / Communicate

```
while (! converged()) {
  for (size_t i = 1; i < N+1; ++i)
    for (size_t j = 1; j < N+1; ++j)
      y(i,j) = (x(i-1,j) + x(i+1,j) + x(i,j-1) + x(i,j+1))/4.0;
  swap(x,y);
  make_as_if(x); // Communicate ghost cells
}
```

Standard terminology for as-if boundary is "ghost cell" or "halo"



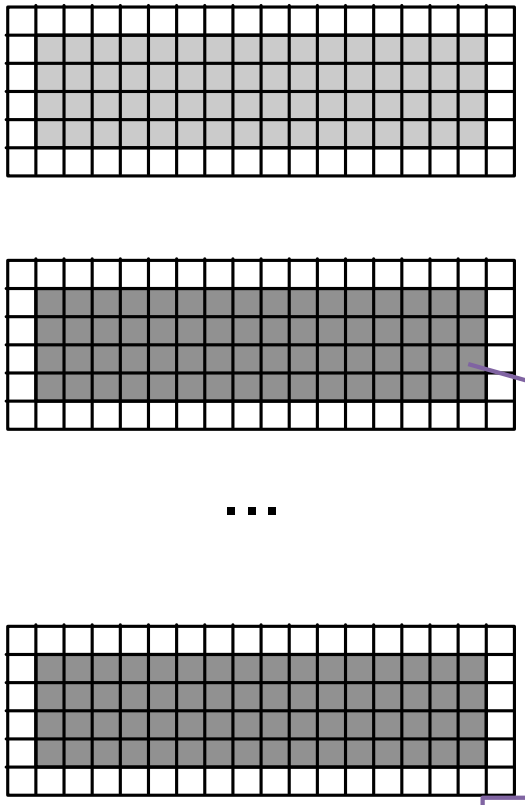
SPMD

```
void jacobi(Grid& x, Grid& xp) {
  while (! converged()) {
    for (size_t i = 1; i < x.num_x()-1; ++i) {
      for (size_t j = 1; j < x.num_y()-1; ++j) {
        xp(i,j) = (x(i-1,j) + x(i+1,j) + x(i,j-1) + x(i,j+1))/4.0;
      }
    }
    swap(xp, x);
  }
}
```

Or here

Communicate here

Decomposition



```
MPI::COMM_WORLD.Send(to myrank + 1)
MPI::COMM_WORLD.Send(to myrank - 1)
MPI::COMM_WORLD.Recv(from myrank - 1)
MPI::COMM_WORLD.Recv(from myrank + 1)
```

"myrank"

Which match?

Message sent "up"

Received from below

...

```
MPI::COMM_WORLD.Send(to myrank + 1, uptag)
MPI::COMM_WORLD.Send(to myrank - 1, downtag)
MPI::COMM_WORLD.Recv(from myrank - 1, uptag)
MPI::COMM_WORLD.Recv(from myrank + 1, downtag)
```

Tags really Necessary?

Message sent "up"

Received from below

Details

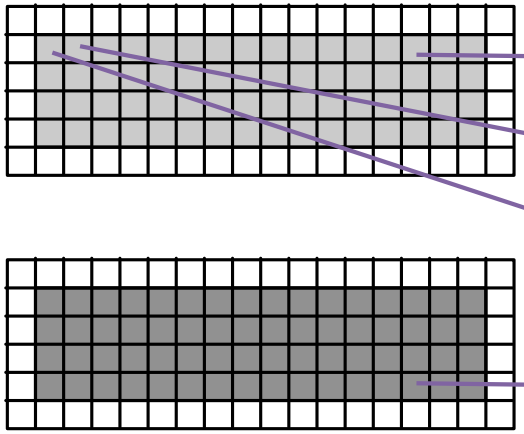
```
MPI::COMM_WORLD.Send(to myrank + 1)
MPI::COMM_WORLD.Send(to myrank - 1)
MPI::COMM_WORLD.Recv(from myrank - 1)
MPI::COMM_WORLD.Recv(from myrank + 1)
```

```
void Comm::Send(const void* buf, int count, const Datatype&
    datatype, int dest, int tag);
```

What are these actually?

Details

```
void Comm::Send(const void* buf, int count, const Datatype&
    datatype, int dest, int tag);
```



We want to send this row “up”

Address in memory of the data we want to send

First element is here

Next element is here

Why?

Important!

We want to send this row “down”

Details

```
void Comm::Send(const void* buf, int count, const Datatype&
    datatype, int dest, int tag);
```

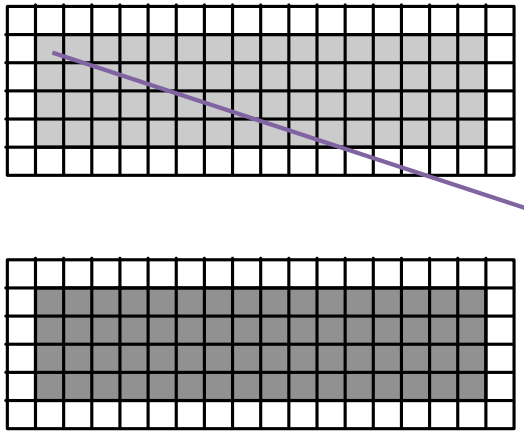
How many?

What type?

`x.num_y()`

`MPI::DOUBLE`

Address in memory of the data we want to send



First element is here

What is its address?

How do we access it?

`&x(1,1)`

`x(1,1)`

Address

Details

```
void Comm::Send(const void* buf, int count, const Datatype&
    datatype, int dest, int tag);
```

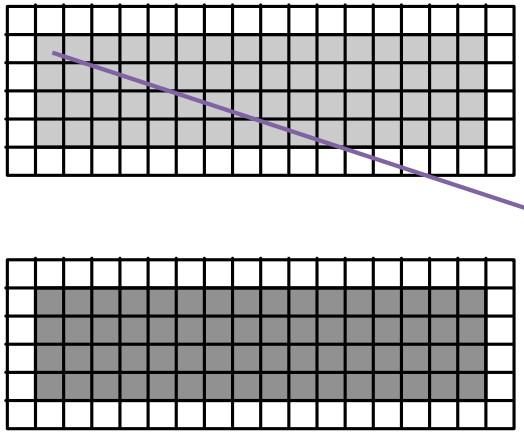
How many?

What type?

`x.num_y()-2`

`MPI::DOUBLE`

Address in memory of the data we want to send



First element is here

What is its address?

How do we access it?

`&x(1,1)`

`x(1,1)`

Address

Alternatively

```
void Comm::Send(const void* buf, int count, const Datatype&
    datatype, int dest, int tag);
```

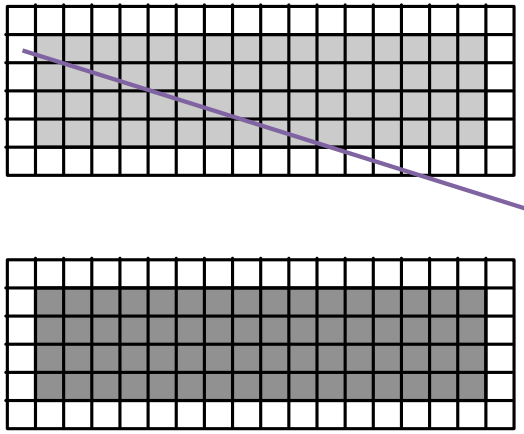
How many?

What type?

x.num_y()

MPI::DOUBLE

Address in memory of the data we want to send



First element is here

What is its address?

How do we access it?

&x(1,0)

x(1,0)

Address

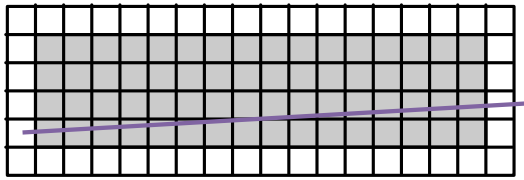
Sending "up"

```
MPI::COMM_WORLD.Send(&x(1, 0)), x.num_y(), MPI::DOUBLE, myrank+1, uptag);
```

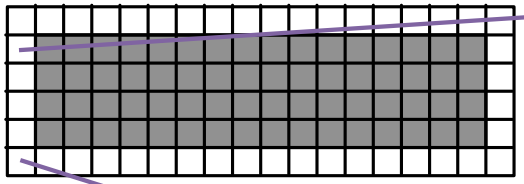
May need const cast

What is corresponding receive?

```
MPI::COMM_WORLD.Recv(&x(x.num_x()-1, 0)), x.num_y(), MPI::DOUBLE, myrank-1, uptag);
```



Send "down": First element is here



Receive "down": First element is here

Yes?

Need to handle top and bottom correctly

And not deadlock

First element is here

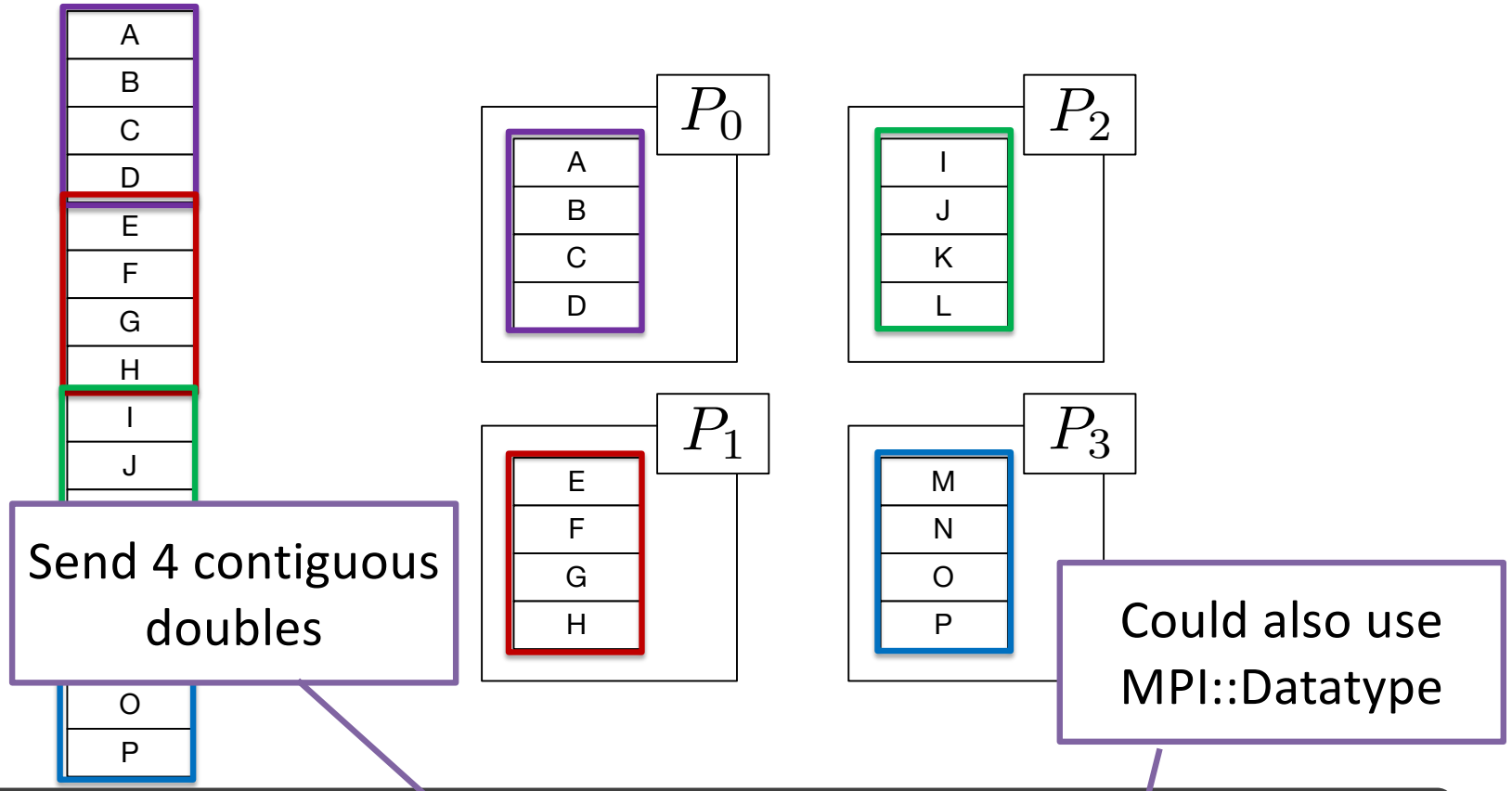
Same size and type

Same tag

Parallel Matrix-Matrix Multiply

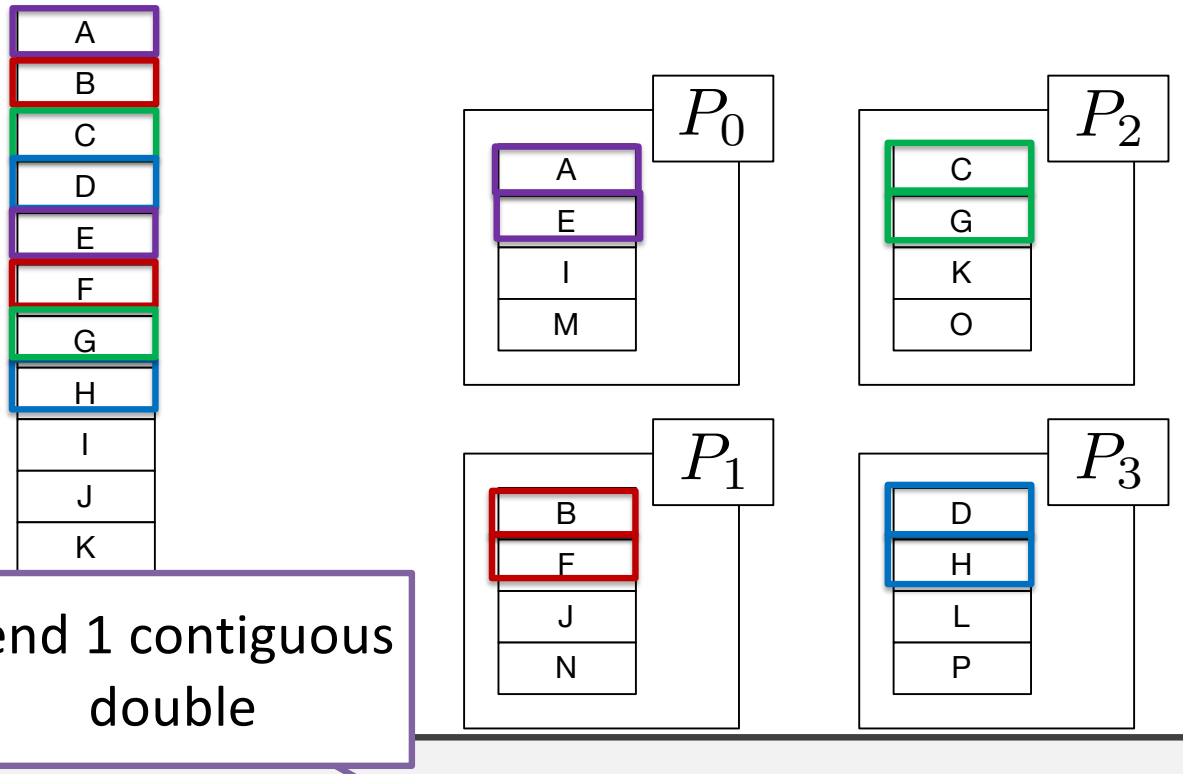
- Use block algorithm
- Partition matrix into blocks
- Assign blocks to processors
- Orchestrate communication and computation
- ***Owner computes***

Block Partitioning




```
MPI::COMM_WORLD.Scatter(&x(0), 4, MPI::DOUBLE, &x(0), 4, MPI::DOUBLE, 0);
```

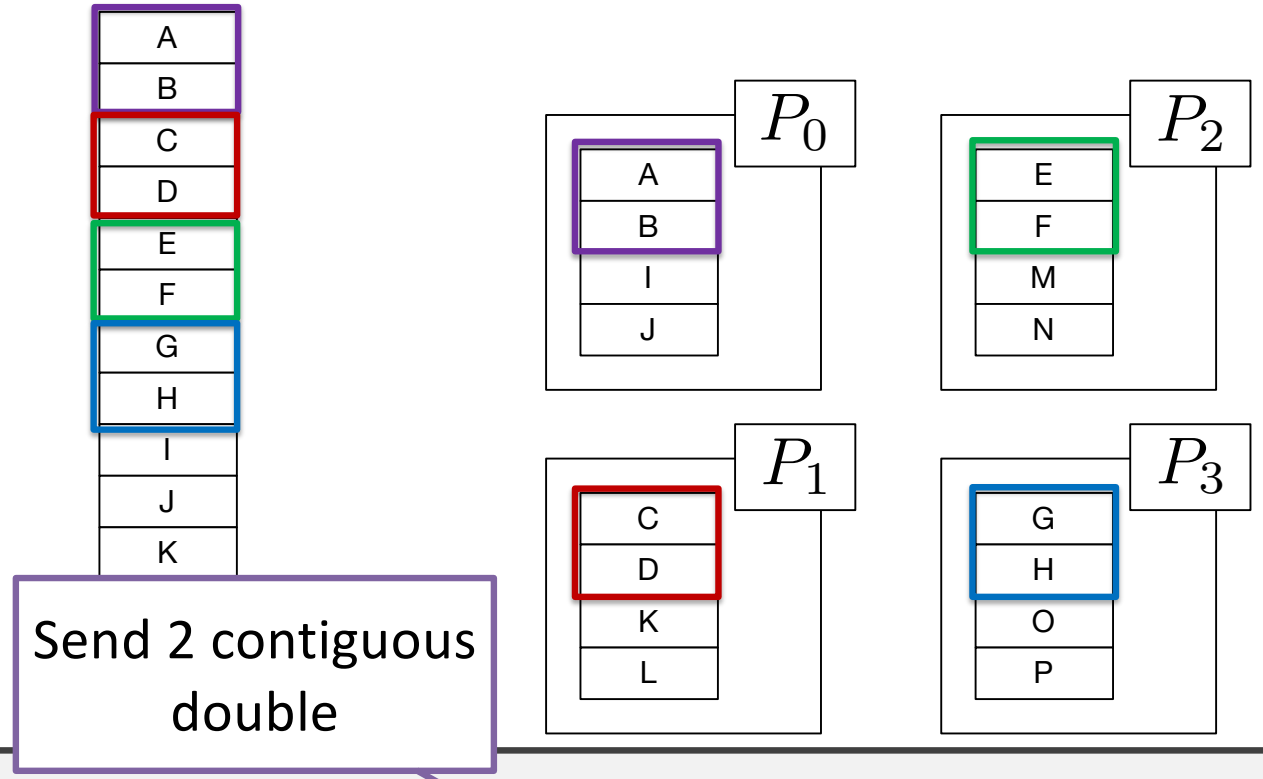
Cyclic Partitioning



Send 1 contiguous double

```
for (size_t i = 0; i < 4; ++i) {
  MPI::COMM_WORLD.Scatter(&x(i*4), 1, MPI::DOUBLE, &x(i*4), 1, MPI::DOUBLE, 0);
}
```

Block Cyclic Partitioning

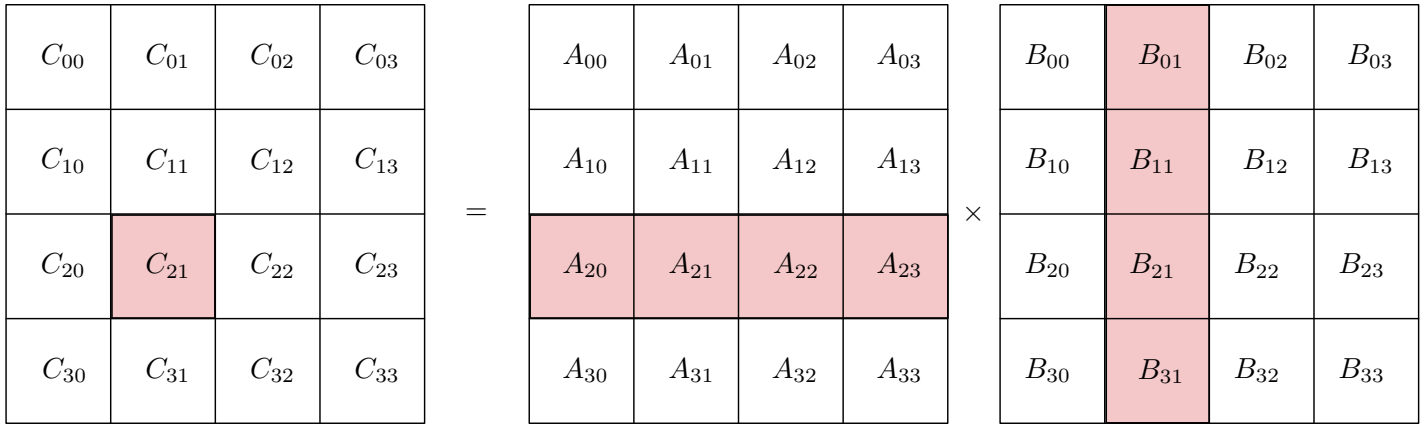


Send 2 contiguous double

```
for (size_t i = 0; i < 2; ++i) {
  MPI::COMM_WORLD.Scatter(&x(i*8), 2, MPI::DOUBLE, &x(i*8), 2, MPI::DOUBLE, 0);
}
```

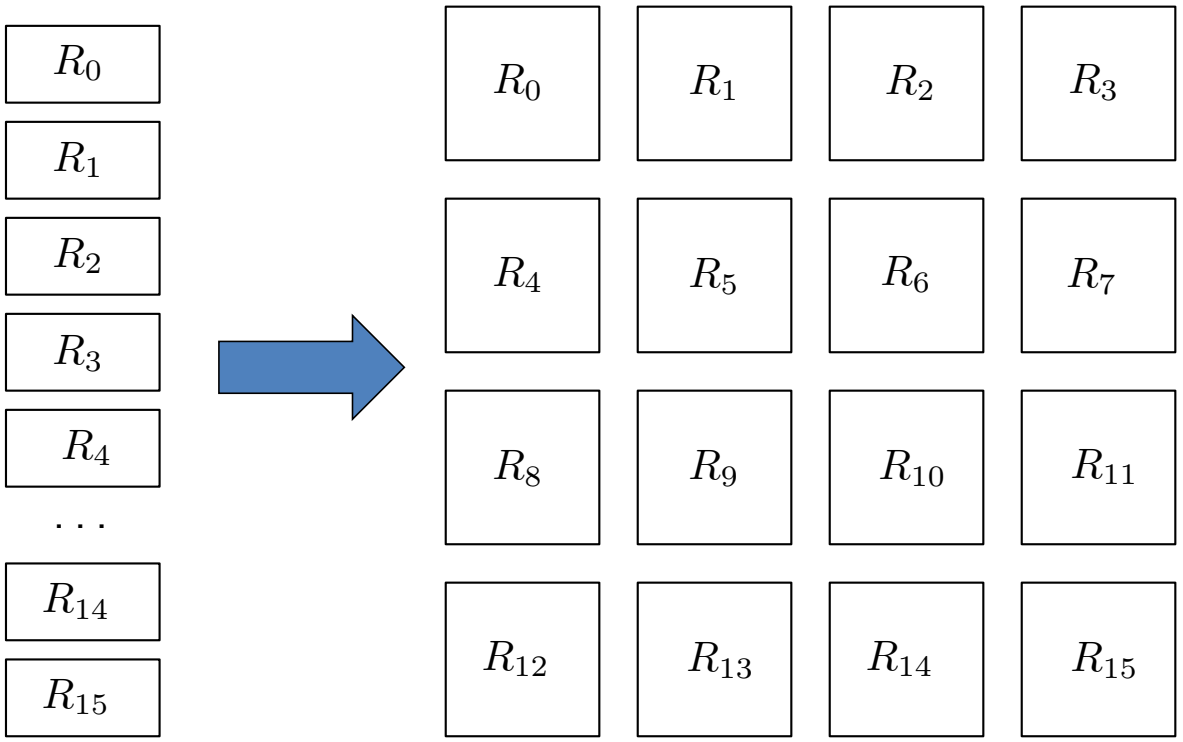
Block Matrix-Matrix Product

$$C_{IJ} = \sum_K A_{IK} B_{KJ}$$

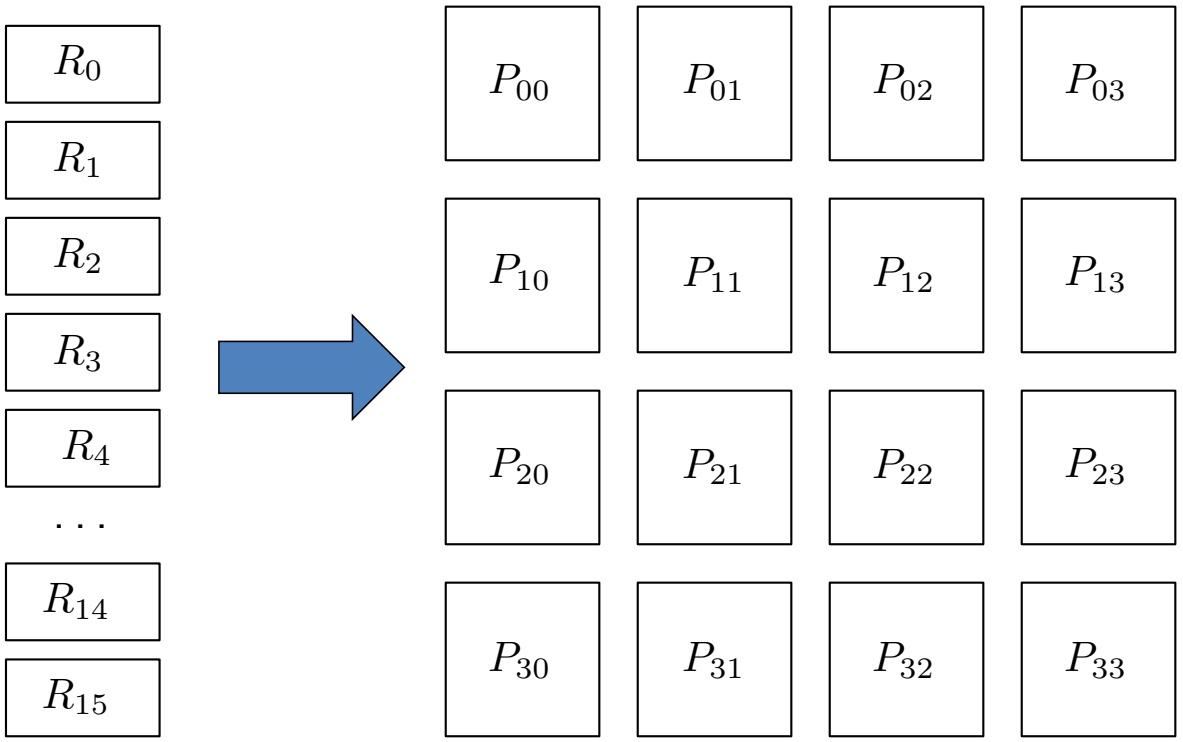


$$C_{21} = A_{20}B_{01} + A_{21}B_{11} + A_{22}B_{21} + A_{23}B_{31}$$

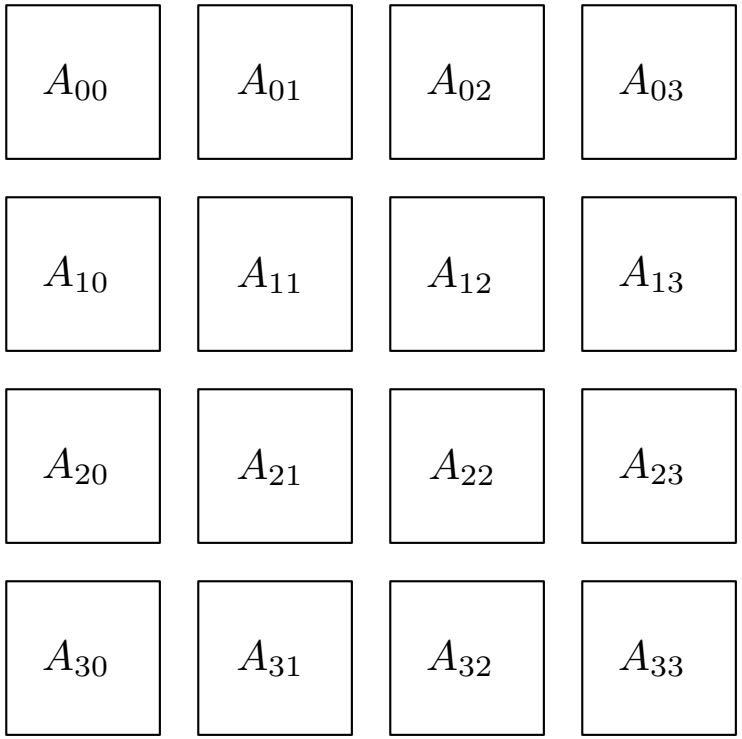
Processor Grid



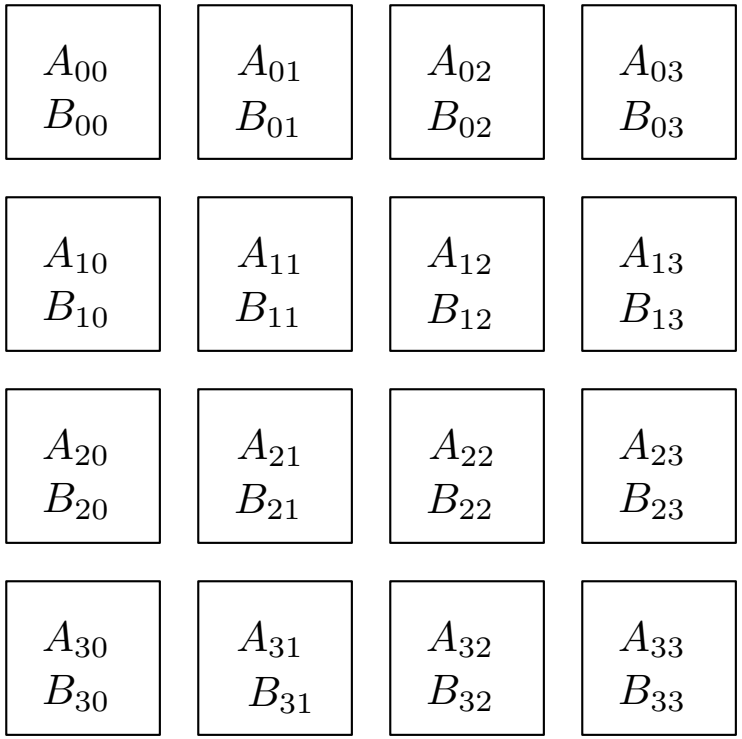
Processor Grid



Matrix Block Partitioning



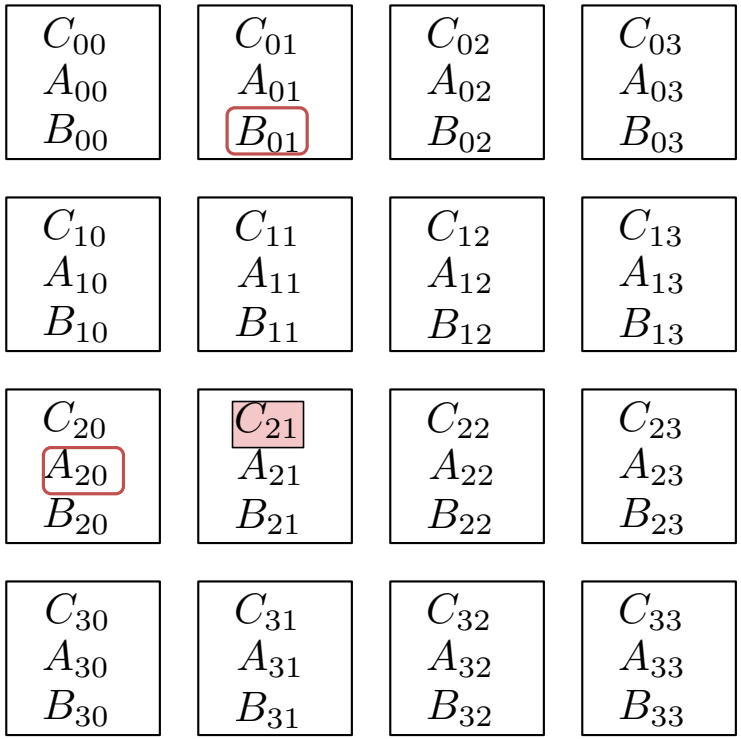
Matrix Block Partitioning



Matrix Block Partitioning

C_{00} A_{00} B_{00}	C_{01} A_{01} B_{01}	C_{02} A_{02} B_{02}	C_{03} A_{03} B_{03}
C_{10} A_{10} B_{10}	C_{11} A_{11} B_{11}	C_{12} A_{12} B_{12}	C_{13} A_{13} B_{13}
C_{20} A_{20} B_{20}	C_{21} A_{21} B_{21}	C_{22} A_{22} B_{22}	C_{23} A_{23} B_{23}
C_{30} A_{30} B_{30}	C_{31} A_{31} B_{31}	C_{32} A_{32} B_{32}	C_{33} A_{33} B_{33}

Matrix Block Partitioning



$$C_{IJ} = \sum_K A_{IK} B_{KJ}$$ (Owner computes)

$$C_{21} = A_{20} B_{01} + A_{21} B_{11} + A_{22} B_{21} + A_{23} B_{31}$$

Matrix Block Partitioning

C_{00} A_{00} B_{00}	C_{01} A_{01} B_{01}	C_{02} A_{02} B_{02}	C_{03} A_{03} B_{03}
C_{10} A_{10} B_{10}	C_{11} A_{11} B_{11}	C_{12} A_{12} B_{12}	C_{13} A_{13} B_{13}
C_{20} A_{20} B_{20}	C_{21} A_{21} B_{21}	C_{22} A_{22} B_{22}	C_{23} A_{23} B_{23}
C_{30} A_{30} B_{30}	C_{31} A_{31} B_{31}	C_{32} A_{32} B_{32}	C_{33} A_{33} B_{33}

$$C_{IJ} = \sum_K A_{IK} B_{KJ}$$ (Owner computes)

$$C_{21} = A_{20}B_{01} + A_{21}B_{11} + A_{22}B_{21} + A_{23}B_{31}$$

Matrix Block Partitioning

C_{00} A_{00} B_{00}	C_{01} A_{01} B_{01}	C_{02} A_{02} B_{02}	C_{03} A_{03} B_{03}
C_{10} A_{10} B_{10}	C_{11} A_{11} B_{11}	C_{12} A_{12} B_{12}	C_{13} A_{13} B_{13}
C_{20} A_{20} B_{20}	C_{21} A_{21} B_{21}	C_{22} A_{22} B_{22}	C_{23} A_{23} B_{23}
C_{30} A_{30} B_{30}	C_{31} A_{31} B_{31}	C_{32} A_{32} B_{32}	C_{33} A_{33} B_{33}

$$C_{IJ} = \sum_K A_{IK} B_{KJ}$$ (Owner computes)

$$C_{21} = A_{20}B_{01} + A_{21}B_{11} + A_{22}B_{21} + A_{23}B_{31}$$

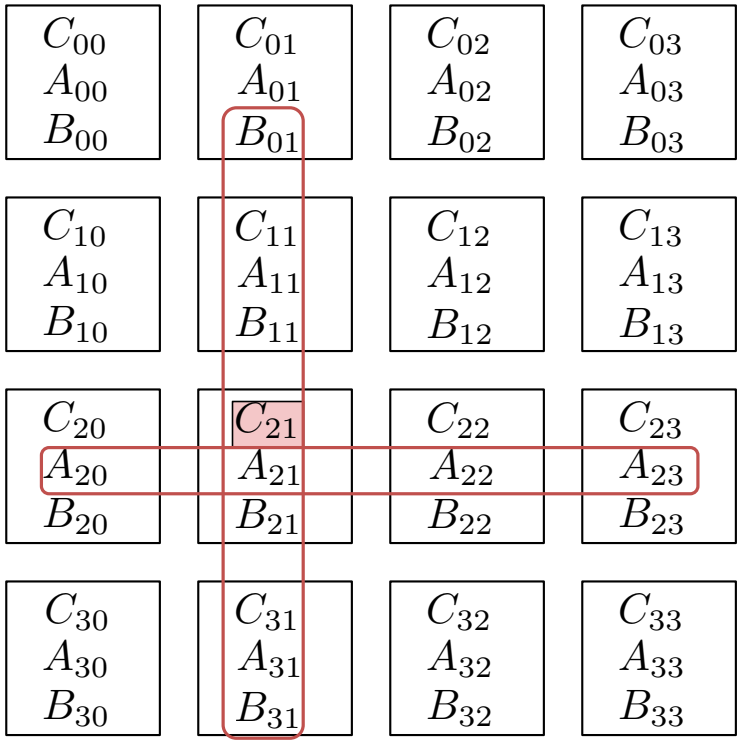
Matrix Block Partitioning

C_{00} A_{00} B_{00}	C_{01} A_{01} B_{01}	C_{02} A_{02} B_{02}	C_{03} A_{03} B_{03}
C_{10} A_{10} B_{10}	C_{11} A_{11} B_{11}	C_{12} A_{12} B_{12}	C_{13} A_{13} B_{13}
C_{20} A_{20} B_{20}	C_{21} A_{21} B_{21}	C_{22} A_{22} B_{22}	C_{23} A_{23} B_{23}
C_{30} A_{30} B_{30}	C_{31} A_{31} B_{31}	C_{32} A_{32} B_{32}	C_{33} A_{33} B_{33}

$$C_{IJ} = \sum_K A_{IK} B_{KJ}$$ (Owner computes)

$$C_{21} = A_{20}B_{01} + A_{21}B_{11} + A_{22}B_{21} + A_{23}B_{31}$$

Matrix Block Partitioning



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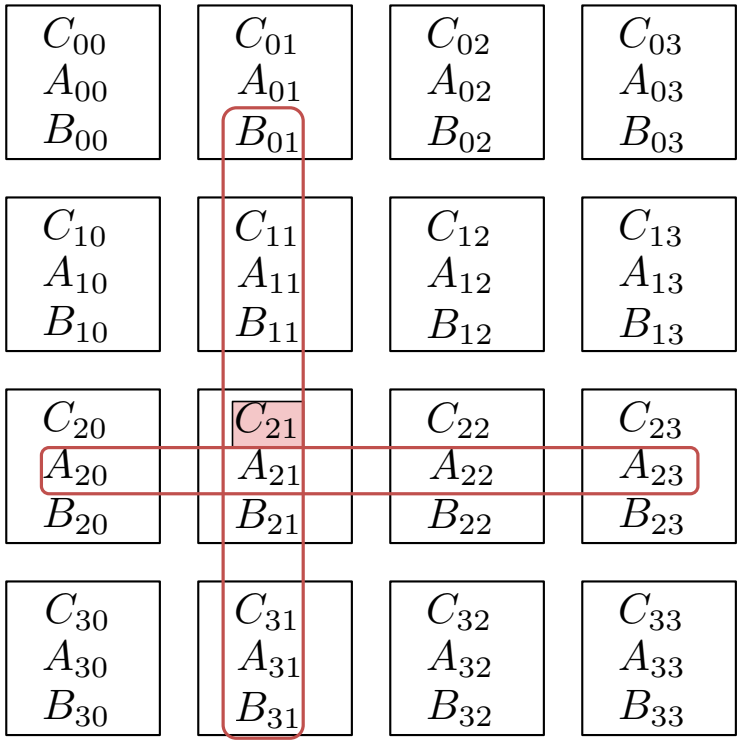
$$C_{21} = A_{20}B_{01} + A_{21}B_{11} + A_{22}B_{21} + A_{23}B_{31}$$

Matrix Block Partitioning



$$C_{IJ} = \sum_K A_{IK} B_{KJ}$$ (Owner computes)

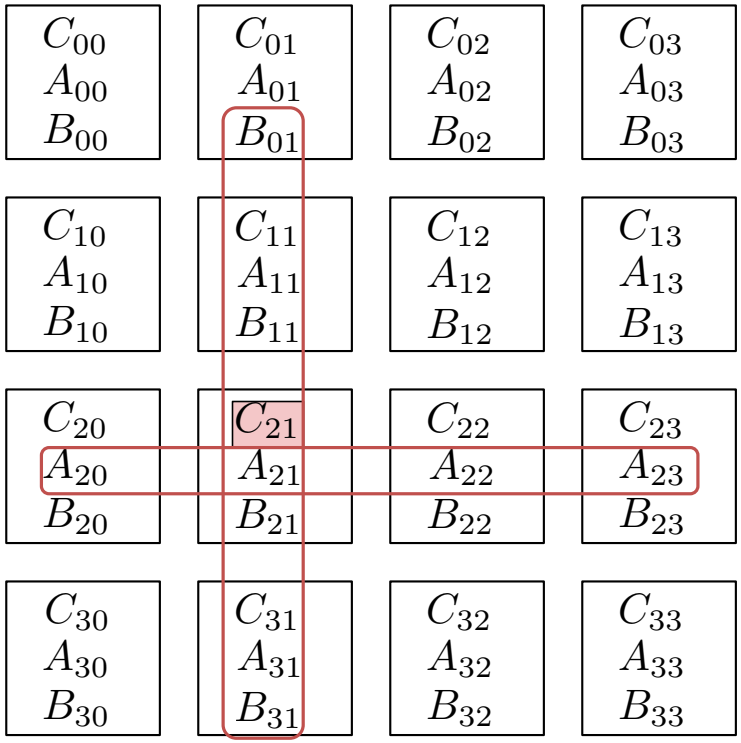
- At each step K, arrange for

  $A_{I,(I+J+K)}$ $B_{(I+J+K),J}$

  to be on processor I,J

$$C_{21} = A_{20}B_{01} + A_{21}B_{11} + A_{22}B_{21} + A_{23}B_{31}$$

Cannon's Algorithm



$$C_{IJ} = \sum_K A_{IK} B_{KJ}$$

- At each step K, arrange for

  $A_{I,(I+J+K)}$ $B_{(I+J+K),J}$

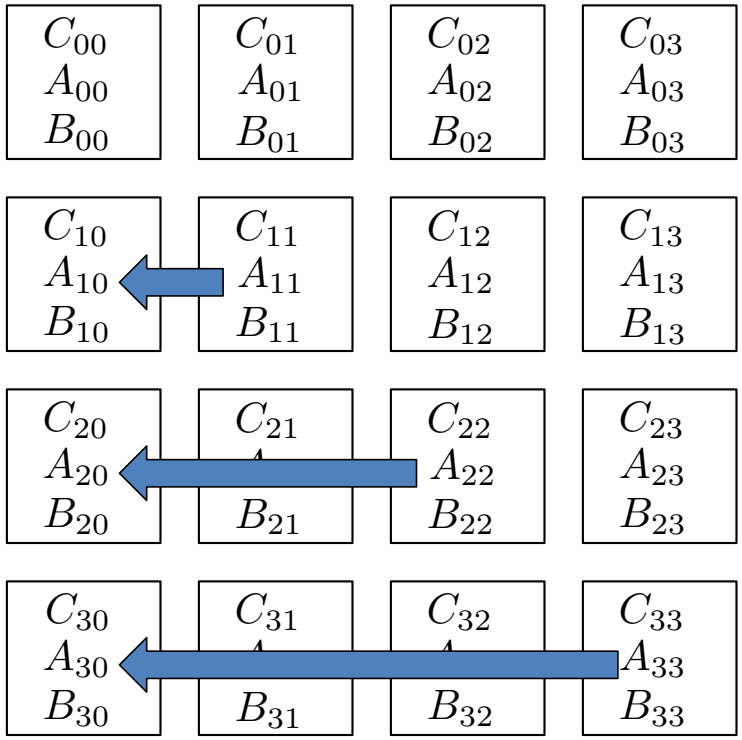
  to be on processor I,J

- Compute

  $C_{IJ} \mathrel{+}= A_{I,(I+J+K)} \times B_{(I+J+K),J}$

$$C_{21} = A_{20}B_{01} + A_{21}B_{11} + A_{22}B_{21} + A_{23}B_{31}$$

Cannon's Algorithm: Setup (K = 0)



$$C_{IJ} = \sum_K A_{IK} B_{KJ}$$

- At each step K, arrange for

  $A_{I,(I+J+K)}$ $B_{(I+J+K),J}$

  to be on processor I,J

- Compute

  $C_{IJ} \mathrel{+}= A_{I,(I+J+K)} \times B_{(I+J+K),J}$

Cannon's Algorithm: Setup (K = 0)

C_{00} A_{00} B_{00}	C_{01} A_{01} B_{01}	C_{02} A_{02} B_{02}	C_{03} A_{03} B_{03}
C_{10} A_{11} B_{10}	C_{11} A_{12} B_{11}	C_{12} A_{13} B_{12}	C_{13} A_{10} B_{13}
C_{20} A_{22} B_{20}	C_{21} A_{23} B_{21}	C_{22} A_{20} B_{22}	C_{23} A_{21} B_{23}
C_{30} A_{33} B_{30}	C_{31} A_{30} B_{31}	C_{32} A_{31} B_{32}	C_{33} A_{32} B_{33}

$$C_{IJ} = \sum_K A_{IK} B_{KJ}$$

- At each step K, arrange for

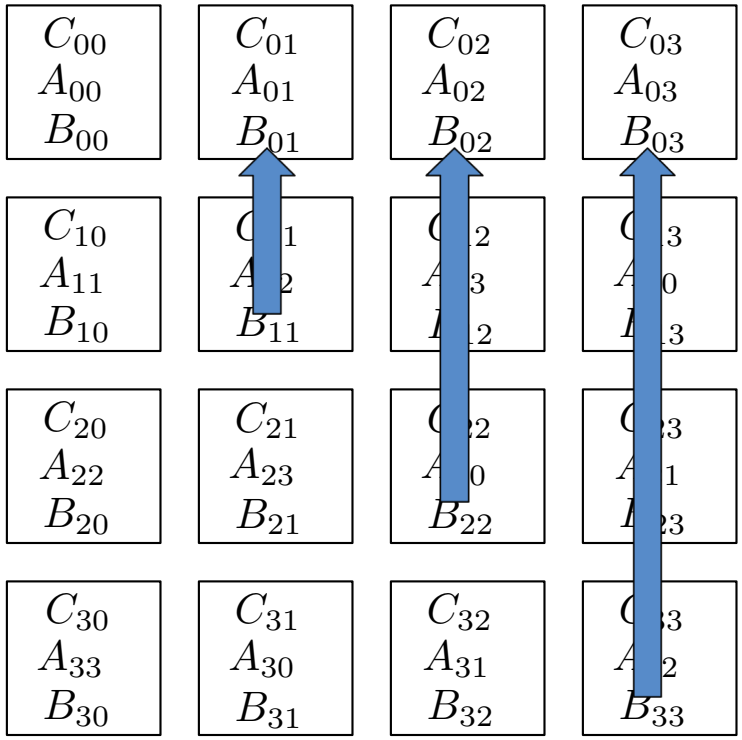
  $A_{I,(I+J+K)}$ $B_{(I+J+K),J}$

  to be on processor I,J

- Compute

  $C_{IJ} \mathrel{+}= A_{I,(I+J+K)} \times B_{(I+J+K),J}$

Cannon's Algorithm: Setup (K = 0)



$$C_{IJ} = \sum_K A_{IK} B_{KJ}$$

- At each step K, arrange for

$A_{I,(I+J+K)}$ $B_{(I+J+K),J}$

to be on processor I,J

- Compute

$$C_{IJ} \mathrel{+}= A_{I,(I+J+K)} \times B_{(I+J+K),J}$$

Cannon's Algorithm: Setup

C_{00} A_{00} B_{00}	C_{01} A_{01} B_{11}	C_{02} A_{02} B_{22}	C_{03} A_{03} B_{33}
C_{10} A_{11} B_{10}	C_{11} A_{12} B_{21}	C_{12} A_{13} B_{32}	C_{13} A_{10} B_{03}
C_{20} A_{22} B_{20}	C_{21} A_{23} B_{31}	C_{22} A_{20} B_{02}	C_{23} A_{21} B_{13}
C_{30} A_{33} B_{30}	C_{31} A_{30} B_{01}	C_{32} A_{31} B_{12}	C_{33} A_{32} B_{23}

$$C_{IJ} = \sum_K A_{IK} B_{KJ}$$

- At each step K, arrange for

  $A_{I,(I+J+K)}$ $B_{(I+J+K),J}$

  to be on processor I,J
- Compute

  $C_{IJ} \mathrel{+}= A_{I,(I+J+K)} \times B_{(I+J+K),J}$

Cannon's Algorithm: K = 0

C_{00} A_{00} B_{00}	C_{01} A_{01} B_{11}	C_{02} A_{02} B_{22}	C_{03} A_{03} B_{33}
C_{10} A_{11} B_{10}	C_{11} A_{12} B_{21}	C_{12} A_{13} B_{32}	C_{13} A_{10} B_{03}
C_{20} A_{22} B_{20}	C_{21} A_{23} B_{31}	C_{22} A_{20} B_{02}	C_{23} A_{21} B_{13}
C_{30} A_{33} B_{30}	C_{31} A_{30} B_{01}	C_{32} A_{31} B_{12}	C_{33} A_{32} B_{23}

$$C_{IJ} = \sum_K A_{IK} B_{KJ}$$

- At each step K, arrange for

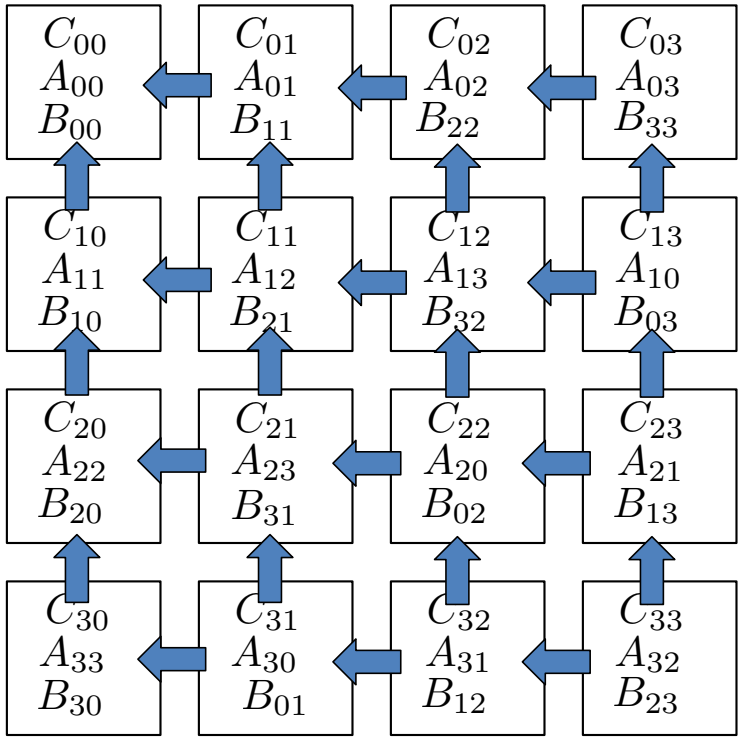
  $A_{I,(I+J+K)}$ $B_{(I+J+K),J}$

  to be on processor I,J

- Compute

  $C_{IJ} \mathrel{+}= A_{I,(I+J+K)} \times B_{(I+J+K),J}$

Cannon's Algorithm: K = 1



$$C_{IJ} = \sum_K A_{IK} B_{KJ}$$

- At each step K, arrange for

  $A_{I,(I+J+K)}$ $B_{(I+J+K),J}$

  to be on processor I,J

- Compute

  $C_{IJ} \mathrel{+}= A_{I,(I+J+K)} \times B_{(I+J+K),J}$

Cannon's Algorithm: K = 1

C_{00} A_{01} B_{10}	C_{01} A_{02} B_{21}	C_{02} A_{03} B_{32}	C_{03} A_{00} B_{03}
C_{10} A_{12} B_{20}	C_{11} A_{13} B_{31}	C_{12} A_{10} B_{02}	C_{13} A_{11} B_{13}
C_{20} A_{23} B_{30}	C_{21} A_{20} B_{01}	C_{22} A_{21} B_{12}	C_{23} A_{22} B_{23}
C_{30} A_{30} B_{00}	C_{31} A_{31} B_{11}	C_{32} A_{32} B_{22}	C_{33} A_{33} B_{33}

$$C_{IJ} = \sum_K A_{IK} B_{KJ}$$

- At each step K, arrange for

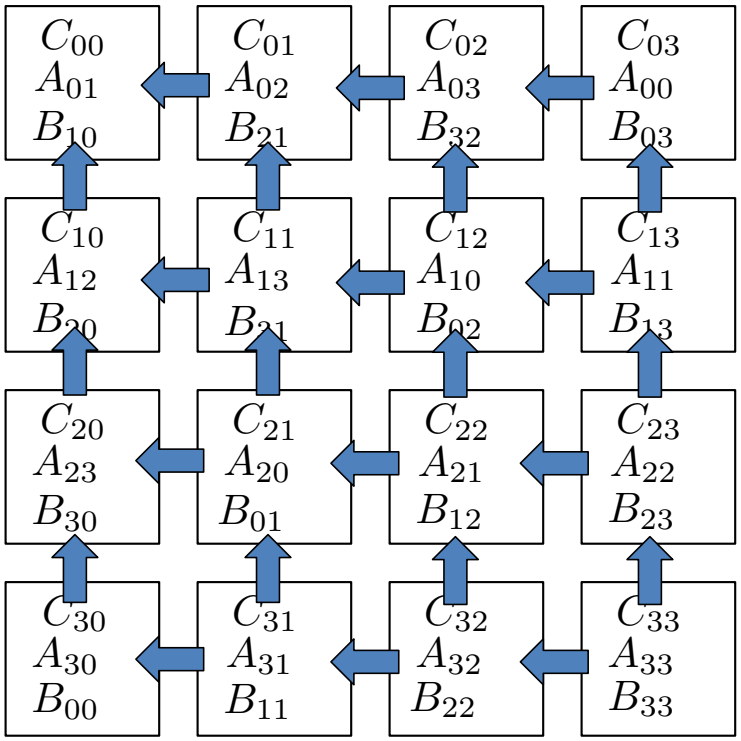
  $A_{I,(I+J+K)}$ $B_{(I+J+K),J}$

  to be on processor I,J

- Compute

  $C_{IJ} \mathrel{+}= A_{I,(I+J+K)} \times B_{(I+J+K),J}$

Cannon's Algorithm: K = 2



$$C_{IJ} = \sum_K A_{IK} B_{KJ}$$

- At each step K, arrange for

  $A_{I,(I+J+K)}$ $B_{(I+J+K),J}$

  to be on processor I,J

- Compute

  $C_{IJ} \mathrel{+}= A_{I,(I+J+K)} \times B_{(I+J+K),J}$

Cannon's Algorithm: K = 2

C_{00} A_{02} B_{20}	C_{01} A_{03} B_{31}	C_{02} A_{00} B_{02}	C_{03} A_{01} B_{13}
C_{10} A_{13} B_{30}	C_{11} A_{10} B_{01}	C_{12} A_{11} B_{12}	C_{13} A_{12} B_{23}
C_{20} A_{20} B_{00}	C_{21} A_{21} B_{11}	C_{22} A_{22} B_{22}	C_{23} A_{23} B_{33}
C_{30} A_{31} B_{10}	C_{31} A_{32} B_{21}	C_{32} A_{33} B_{32}	C_{33} A_{30} B_{03}

$$C_{IJ} = \sum_K A_{IK} B_{KJ}$$

- At each step K, arrange for

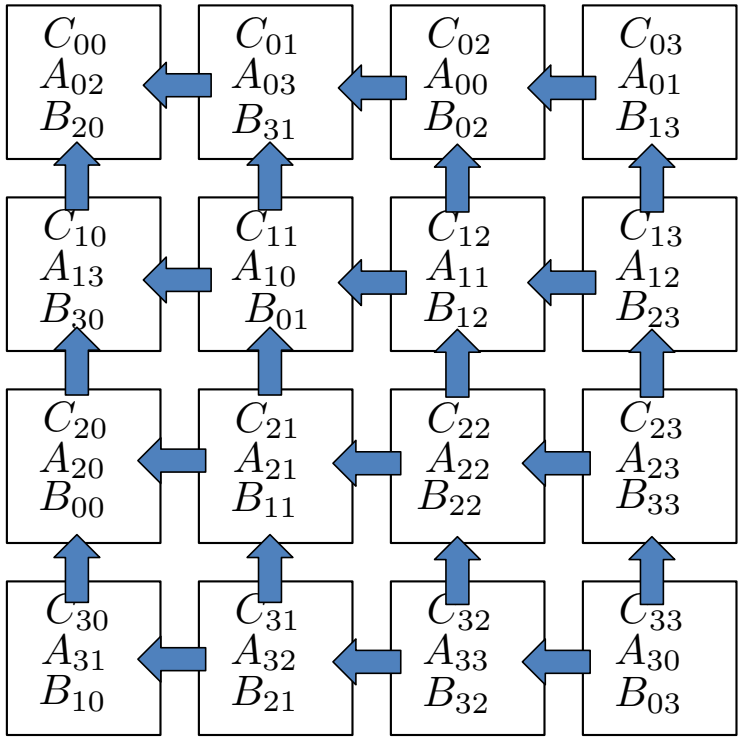
  $A_{I,(I+J+K)}$ $B_{(I+J+K),J}$

  to be on processor I,J

- Compute

  $C_{IJ} \mathrel{+}= A_{I,(I+J+K)} \times B_{(I+J+K),J}$

Cannon's Algorithm: K = 3



$$C_{IJ} = \sum_K A_{IK} B_{KJ}$$

- At each step K, arrange for

  $A_{I,(I+J+K)}$ $B_{(I+J+K),J}$

  to be on processor I,J

- Compute

  $C_{IJ} \mathrel{+}= A_{I,(I+J+K)} \times B_{(I+J+K),J}$

Cannon's Algorithm: K = 3

C_{00} A_{03} B_{30}	C_{01} A_{00} B_{01}	C_{02} A_{01} B_{12}	C_{03} A_{02} B_{23}
C_{10} A_{10} B_{00}	C_{11} A_{11} B_{11}	C_{12} A_{12} B_{22}	C_{13} A_{13} B_{33}
C_{20} A_{21} B_{10}	C_{21} A_{22} B_{21}	C_{22} A_{23} B_{32}	C_{23} A_{20} B_{03}
C_{30} A_{32} B_{20}	C_{31} A_{33} B_{31}	C_{32} A_{30} B_{02}	C_{33} A_{31} B_{13}

$$C_{IJ} = \sum_K A_{IK} B_{KJ}$$

- At each step K, arrange for

  $A_{I,(I+J+K)}$ $B_{(I+J+K),J}$

  to be on processor I,J

- Compute

  $C_{IJ} \mathrel{+}= A_{I,(I+J+K)} \times B_{(I+J+K),J}$

Implementation

- Two-D decomposition of matrices A, B, C
- Move A and B to starting positions
- Local matrix-matrix product
- Shift left
- Shift up
- Move A and B back to initial distributions

MPI Mental Model

All MPI communication takes place in the context of an ***MPI Communicator***

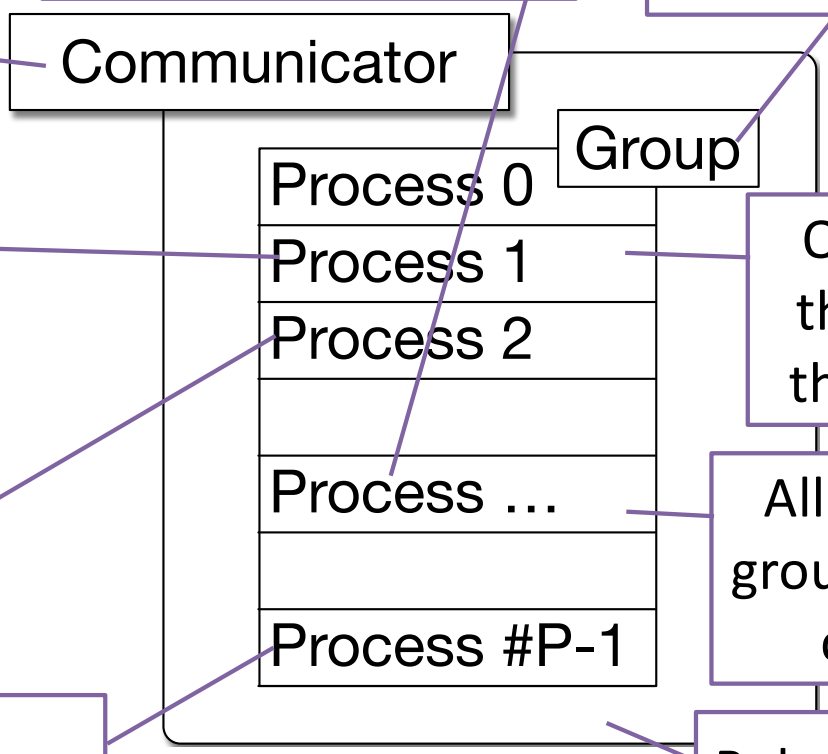
An MPI Group translates from ***rank*** in the group to actual process

We us the index (***rank***) of a process in the group to identify other processes

The ***size*** of a communicator is the size of the group

Processes can query for size and for their own rank in group

An MPI Communicator contains an ***MPI Group***

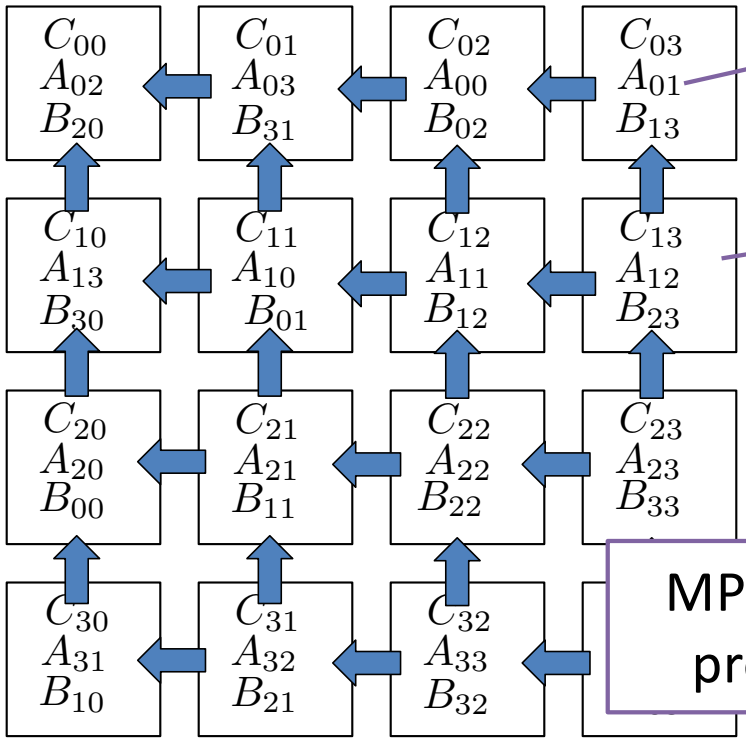


Only processes in the group can use the communicator

All processes in the group see an identical communicator

Behavior is ***as if*** it were global and shared

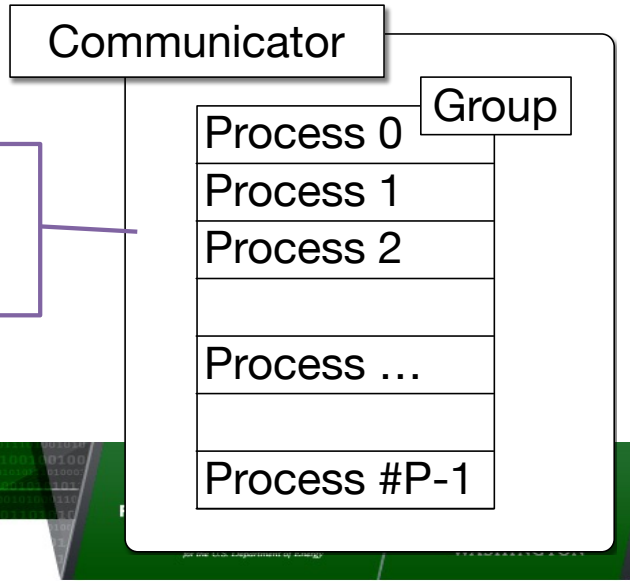
Shifting North, East, West, South



This is a useful way to reason about the algorithm

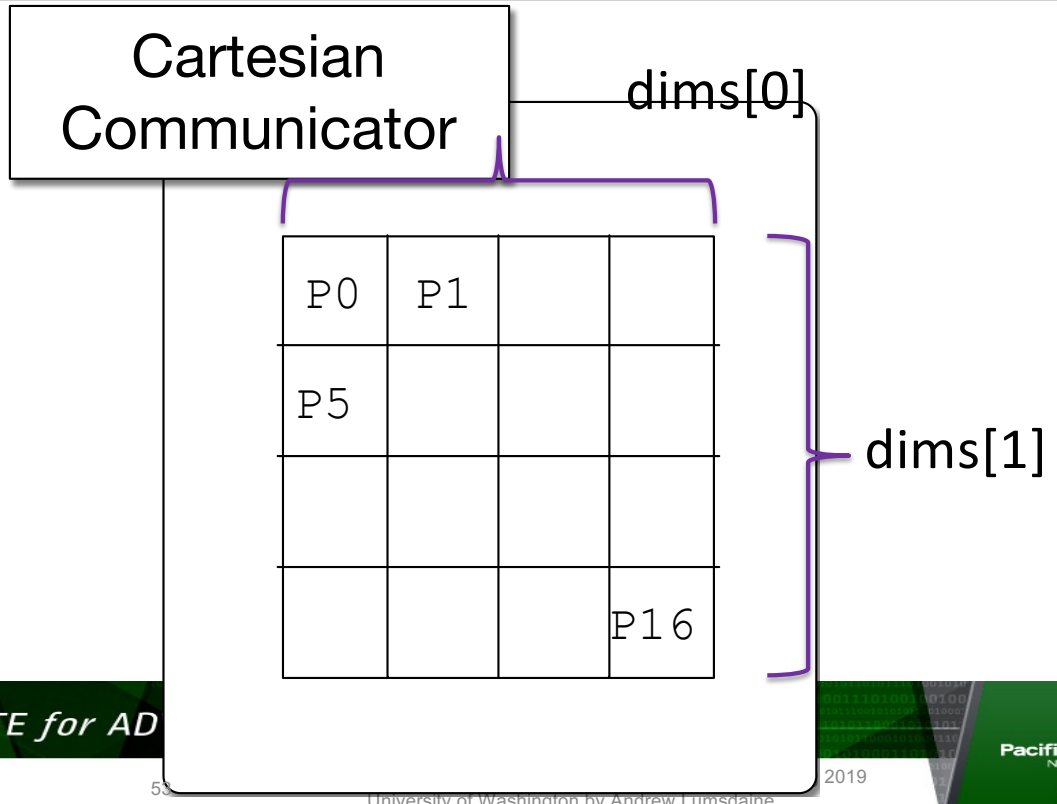
Also turns out to be efficient

MPI communicator has processes in an array



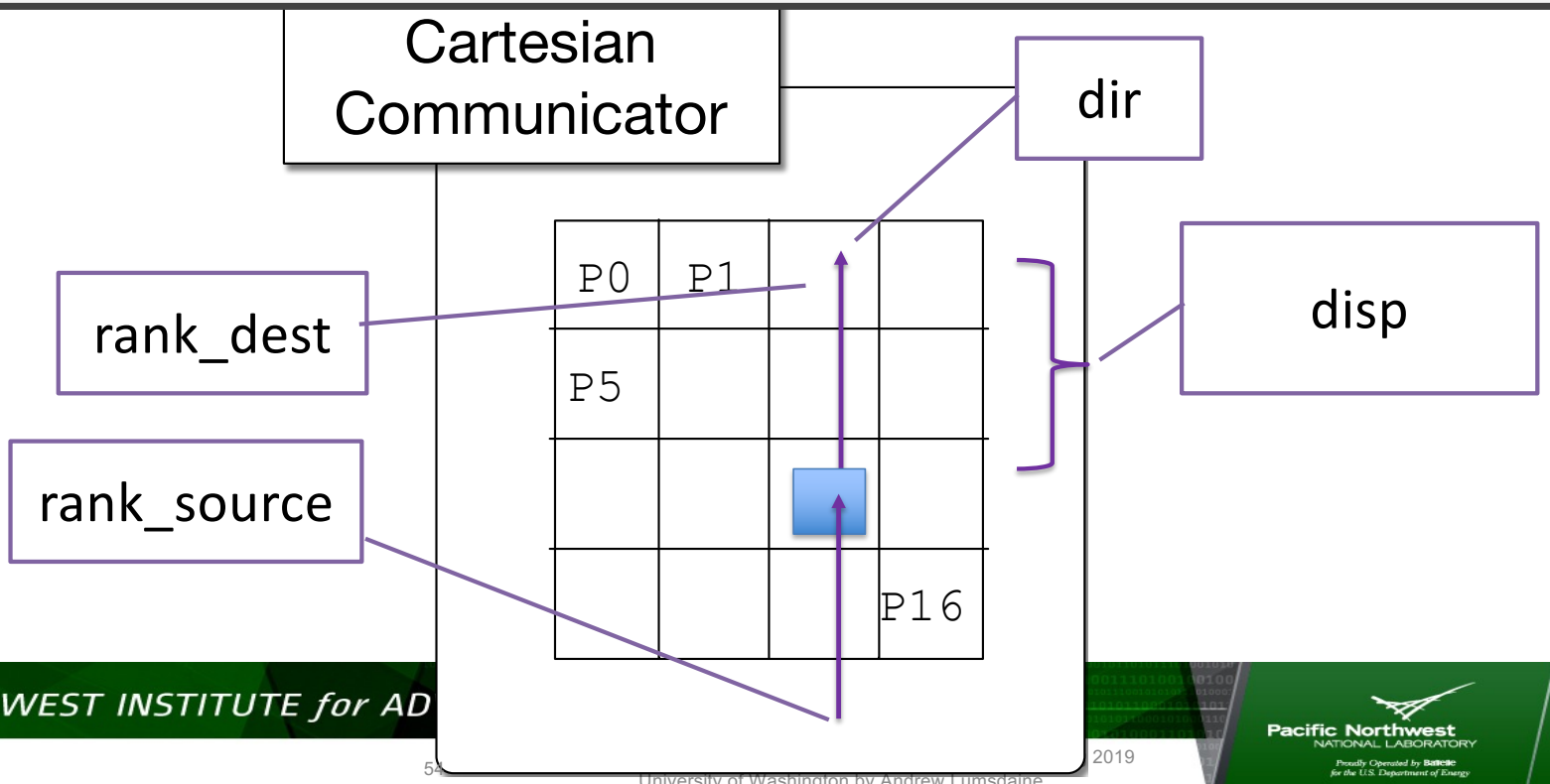
Cartesian Communicator

```
Cartcomm Intracomm.Create_cart(int ndims, int dims[], const bool periods[],
↪  bool reorder) const
```



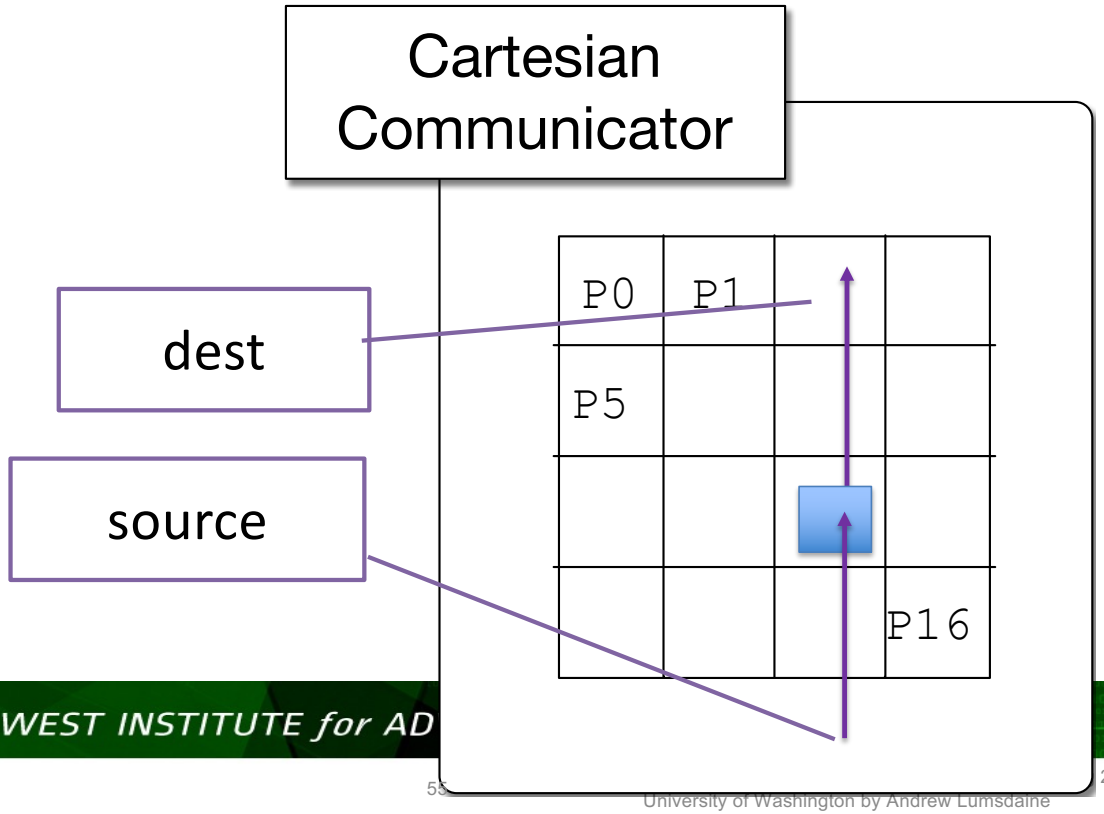
Cartesian Communicator

```
void Cartcomm::Shift(int direction, int disp, int& rank_source,
↪  int& rank_dest) const
```



Cartesian Communicator

```
void Comm::Sendrecv_replace(void* buf, int count, const Datatype& datatype,
↪  int dest, int sendtag, int source, int recvtag) const
```



Implementation

```
1  void cannonMultiplyMV(const Matrix& A, const Matrix& B, Matrix& C) {
2    size_t mysize = MPI::COMM_WORLD.Get_size();
3
4    // Set up grid topology and a grid (Cartesian) communicator
5    int dims[2] = { (int) std::sqrt(mysize), (int) std::sqrt(mysize) };
6    bool periods[2] = { true, true };
7
8    MPI::Cartcomm gridComm = MPI::COMM_WORLD.Create_cart(2, dims, periods, true);
9    size_t myrank = gridComm.Get_rank();
10
11   int mycoords[2];
12   gridComm.Get_coords(myrank, 2, mycoords);
13
14   int northRank, eastRank, westRank, southRank;
15   gridComm.Shift(0, -1, westRank,  eastRank);
16   gridComm.Shift(1, -1, southRank, northRank);
17
18   // Move A and B where they need to be to start
19   int shiftSource, shiftDest;
20   gridComm.Shift(0, -mycoords[0], shiftSource, shiftDest);
21   gridComm.Sendrecv_replace(const_cast<double*>(&A(0,0)), A.numRows()*A.numCols(),
22                             MPI::DOUBLE, shiftDest, 314, shiftSource, 314);
23
24   gridComm.Shift(1, -mycoords[1], shiftSource, shiftDest);
25   gridComm.Sendrecv_replace(const_cast<double*>(&B(0,0)), B.numRows()*B.numCols(),
26                             MPI::DOUBLE, shiftDest, 314, shiftSource, 315);
27
28
29   // Main loop
30   for (int k = 0; k < dims[0]; ++k) {
31     hoistedCopyBlockedTiledMultiply2x2(A, B, C); // Local block matmat
32
33     gridComm.Sendrecv_replace(const_cast<double*>(&A(0,0)), A.numRows()*A.numCols(),
34                               MPI::DOUBLE, westRank,  316, eastRank,  316);
35     gridComm.Sendrecv_replace(const_cast<double*>(&B(0,0)), B.numRows()*A.numCols(),
36                               MPI::DOUBLE, northRank, 317, southRank, 317);
37   }
38
39   // Restore A and B to initial distribution
40   gridComm.Shift(0, +mycoords[0], shiftSource, shiftDest);
41   gridComm.Sendrecv_replace(const_cast<double*>(&A(0,0)), A.numRows()*A.numCols(),
42                             MPI::DOUBLE, shiftDest, 318, shiftSource, 318);
43
44   gridComm.Shift(1, +mycoords[1], shiftSource, shiftDest);
45   gridComm.Sendrecv_replace(const_cast<double*>(&B(0,0)), B.numRows()*B.numCols(),
46                             MPI::DOUBLE, shiftDest, 319, shiftSource, 319);
47
48   gridComm.Free();
49 }
```

Implementation

```
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13
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17
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19   int shiftSource, shiftDest;
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21   gridComm.Sendrecv_replace(const_cast<double*>(&A(0,0)), A.numRows()*A.numCols(),
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23
24   gridComm.Shift(1, -mycoords[1], shiftSource, shiftDest);
25   gridComm.Sendrecv_replace(const_cast<double*>(&B(0,0)), B.numRows()*B.numCols(),
26                             MPI::DOUBLE, shiftDest, 314, shiftSource, 315);
27
28
29   // Main loop
30   for (int k = 0; k < dims[0]; ++k) {
31     hoistedCopyBlockedTiledMultiply2x2(A, B, C); // Local block matmat
32
33     gridComm.Sendrecv_replace(const_cast<double*>(&A(0,0)), A.numRows()*A.numCols(),
34                               MPI::DOUBLE, westRank,  316, eastRank,  316);
35     gridComm.Sendrecv_replace(const_cast<double*>(&B(0,0)), B.numRows()*A.numCols(),
36                               MPI::DOUBLE, northRank, 317, southRank, 317);
37   }
38
39   // Restore A and B to initial distribution
40   gridComm.Shift(0, +mycoords[0], shiftSource, shiftDest);
41   gridComm.Sendrecv_replace(const_cast<double*>(&A(0,0)), A.numRows()*A.numCols(),
42                             MPI::DOUBLE, shiftDest, 318, shiftSource, 318);
43
44   gridComm.Shift(1, +mycoords[1], shiftSource, shiftDest);
45   gridComm.Sendrecv_replace(const_cast<double*>(&B(0,0)), B.numRows()*B.numCols(),
46                             MPI::DOUBLE, shiftDest, 319, shiftSource, 319);
47
48   gridComm.Free();
49 }
```

Implementation

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12   gridComm.Get_coords(myrank, 2, mycoords);
13
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15   gridComm.Shift(0, -1, westRank,  eastRank);
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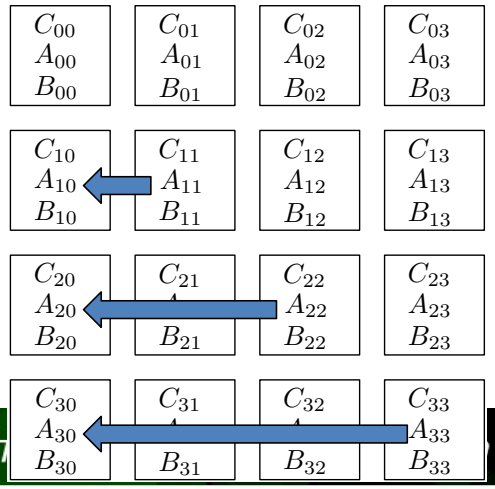
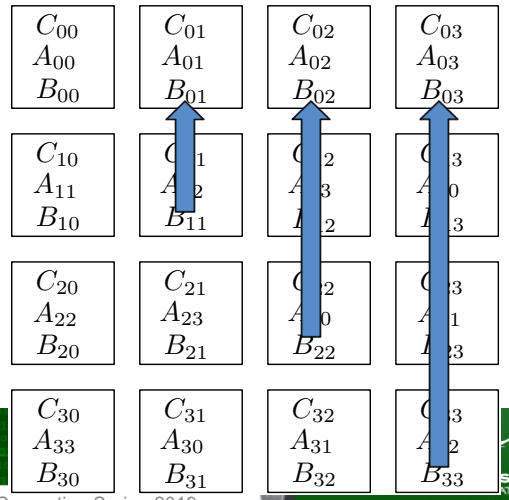
```

Implementation

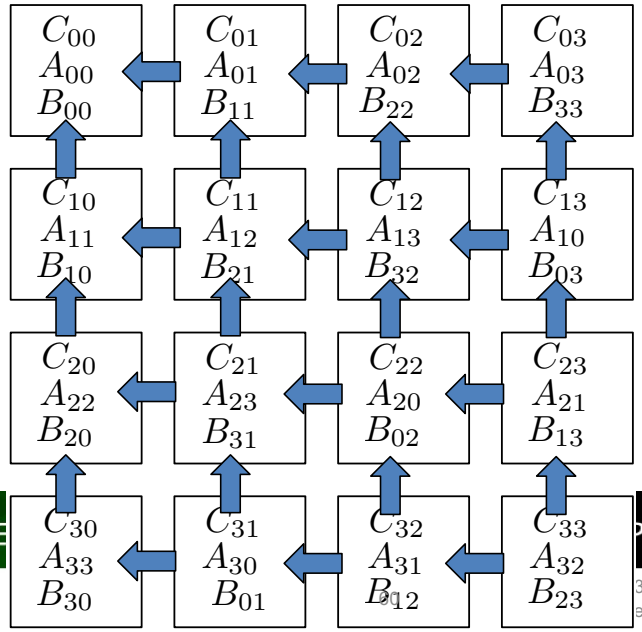
```
17
18      // Move A and B where they need to be to start
19      int shiftSource, shiftDest;
20      gridComm.Shift(0, -mycoords[0], shiftSource, shiftDest);
21      gridComm.Sendrecv_replace(const_cast<double*>(&A(0,0)), A.numRows()*A.numCols(),
22                                MPI::DOUBLE, shiftDest, 314, shiftSource, 314);
23
24      gridComm.Shift(1, -mycoords[1], shiftSource, shiftDest);
25      gridComm.Sendrecv_replace(const_cast<double*>(&B(0,0)), B.numRows()*B.numCols(),
26                                MPI::DOUBLE, shiftDest, 314, shiftSource, 315);
27
```





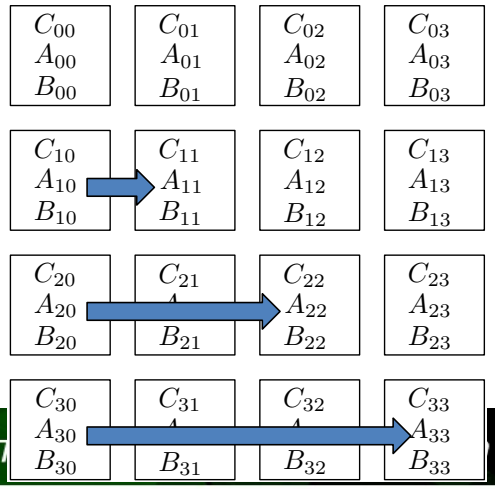
Implementation

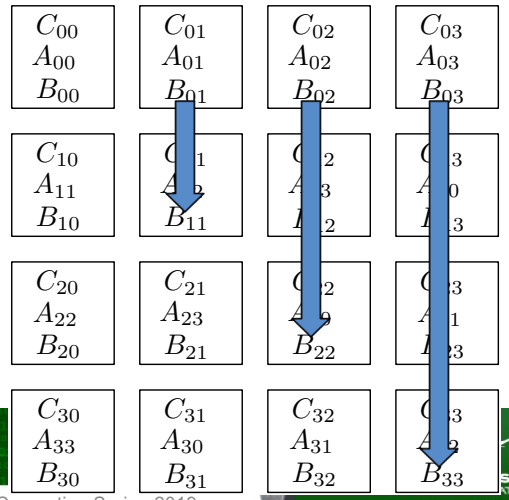
```
28
29      // Main loop
30      for (int k = 0; k < dims[0]; ++k) {
31        hoistedCopyBlockedTiledMultiply2x2(A, B, C); // Local block matmat
32
33        gridComm.Sendrecv_replace(const_cast<double*>(&A(0,0)), A.numRows()*A.numCols(),
34                                  MPI::DOUBLE, westRank,  316, eastRank,  316);
35        gridComm.Sendrecv_replace(const_cast<double*>(&B(0,0)), B.numRows()*A.numCols(),
36                                  MPI::DOUBLE, northRank, 317, southRank, 317);
37      }
```



Implementation

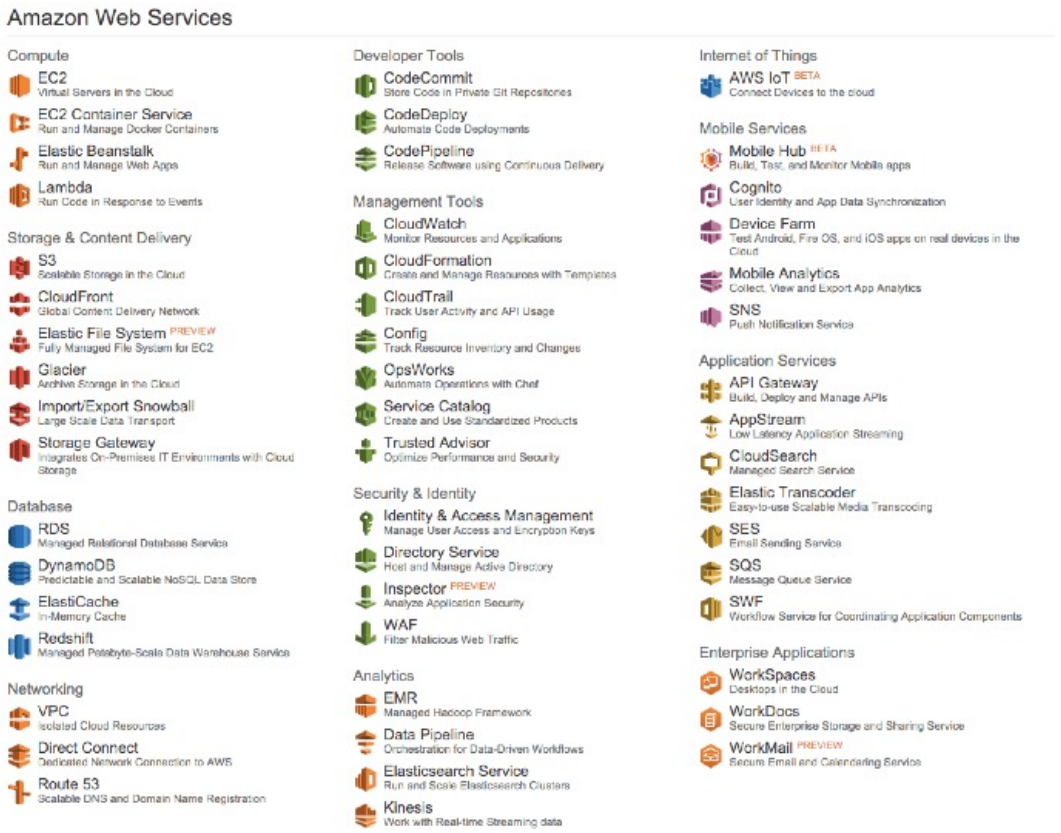
```
38
39      // Restore A and B to initial distribution
40      gridComm.Shift(0, +mycoords[0], shiftSource, shiftDest);
41      gridComm.Sendrecv_replace(const_cast<double*>(&A(0,0)), A.numRows()*A.numCols(),
42                                MPI::DOUBLE, shiftDest, 318, shiftSource, 318);
43
44      gridComm.Shift(1, +mycoords[1], shiftSource, shiftDest);
45      gridComm.Sendrecv_replace(const_cast<double*>(&B(0,0)), B.numRows()*B.numCols(),
46                                MPI::DOUBLE, shiftDest, 319, shiftSource, 319);
47
48      gridComm.Free();
49  }
```





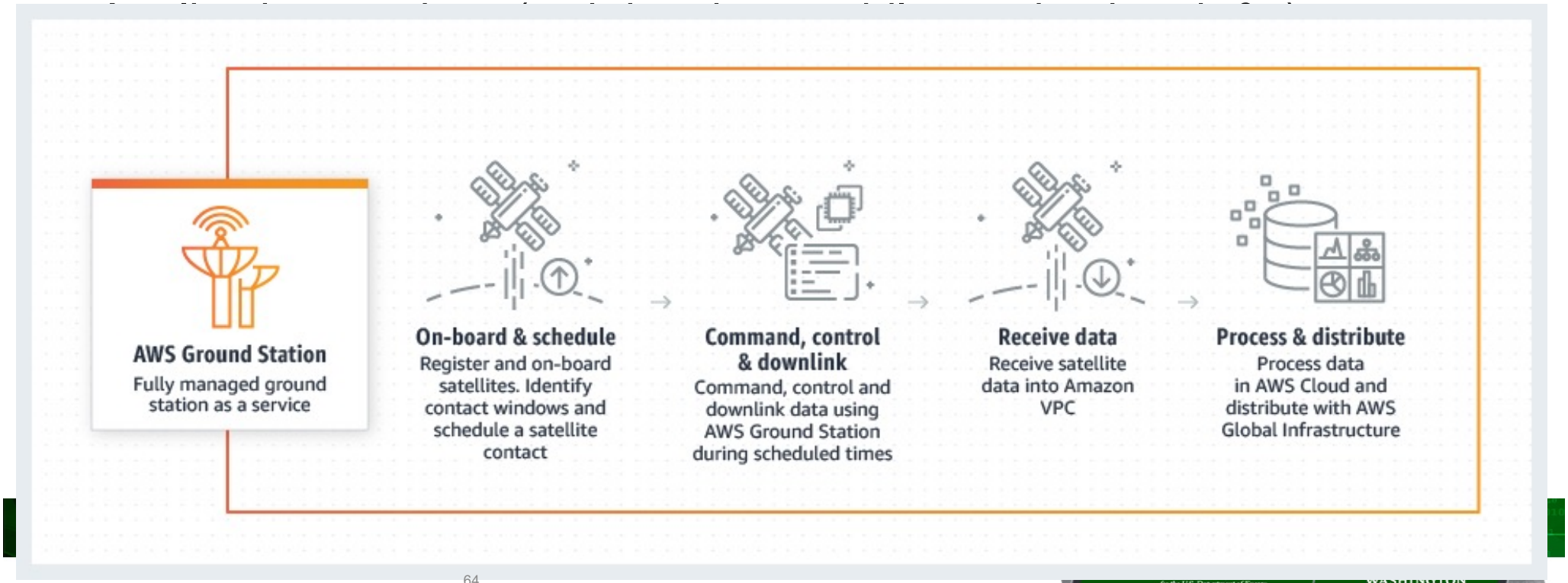
Where do we go from here?

Clouds = Services



Services: On Demand Access

- Data Storage (blob, file, unstructured, SQL, &c)
- Computing (VM, cluster, GPU)



What's Next

- Machine learning
- Quantum computing
- 5G
- IoT / edge computing

Thank You!

- Be well
- Do good work
- Stay in touch

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