

# AMATH 483/583

## High Performance Scientific Computing

### STL, Parallel STL

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## A programming problem

- “I’ve assigned this problem in courses at Bell Labs and IBM. Professional programmers had a couple of hours to convert the description into a programming language of their choice; a high-level pseudo code was fine... Ninety percent of the programmers found bugs in their programs (and I wasn’t always convinced of the correctness of the code in which no bugs were found).”
  - Jon Bentley, Programming Pearls, 1986

This must be a  
complicated  
algorithm!

## Binary search solution

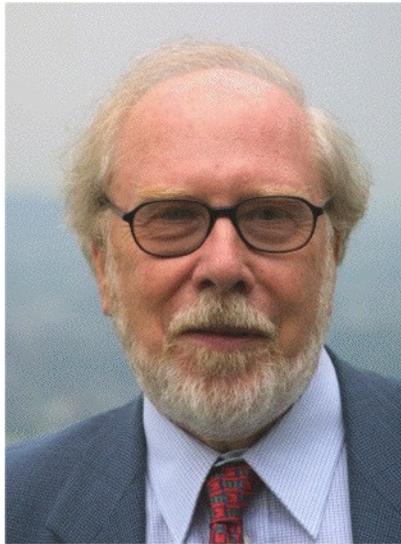
```
int* lower_bound(int* first, int* last, int x)
{
    while (first != last)
    {
        int* middle = first + (last - first) / 2;

        if (*middle < x) first = middle + 1;
        else last = middle;
    }

    return first;
}
```

# Let an expert write binary search

- Once and for all



# Matrix-Matrix Product

```
template <typename MatrixType>
MatrixType operator*(const MatrixType& A, const MatrixType& B) {
    MatrixType C(A.num_rows(), B.num_cols());
    for (size_t i = 0; i < A.num_rows(); ++i) {
        for (size_t j = 0; j < B.num_cols(); ++j) {
            for (size_t k = 0; k < A.num_cols(); ++k) {
                C(i, j) += A(i, k) * B(k, j);
            }
        }
    }
    return C;
}
```

This will work for any type that meets requirements for MatrixType

Constructor  
Accessor

```
NewMatrix A(32, 32);
NewMatrix B(32, 32);
NewMatrix C = A * B;
```

```
RowMatrix A(32, 32);
RowMatrix B(32, 32);
RowMatrix C = A * B;
```

```
ColMatrix A(32, 32);
ColMatrix B(32, 32);
ColMatrix C = A * B;
```

```
Matrix A(32, 32);
Matrix B(32, 32);
Matrix C = A * B;
```

# Generic Programming Methodology

1. Study the concrete implementations of an algorithm
2. **Lift** away unnecessary requirements to produce a more abstract algorithm
  - a) Catalog these requirements.
  - b) Bundle requirements into **concepts**.
3. Repeat the lifting process until we have obtained a generic algorithm that:
  - a) Instantiates to efficient concrete implementations.
  - b) Captures the essence of the “higher truth” of that algorithm.

# Before

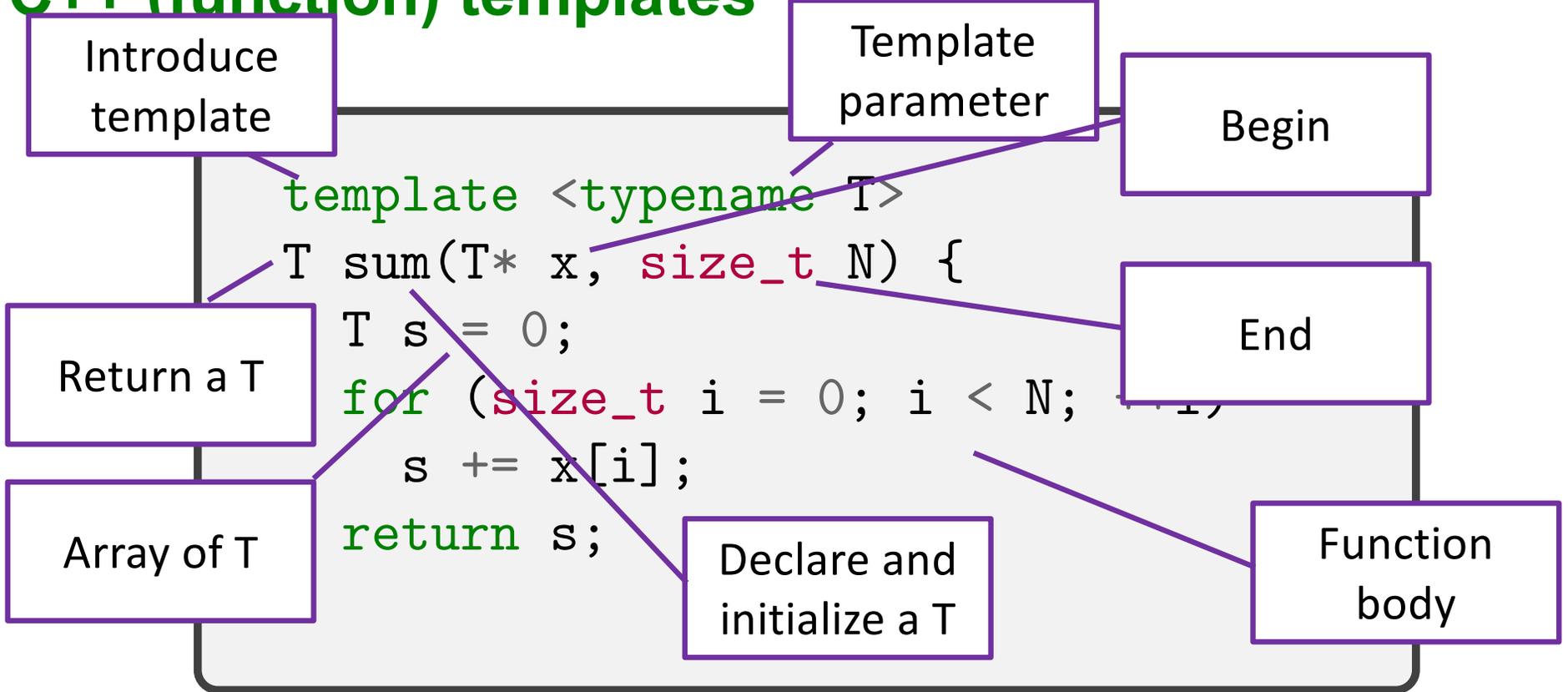
```
double sum(double* x, size_t N) {  
    double s = 0;  
    for (size_t i = 0; i < N; ++i)  
        s += x[i];  
    return s;  
}
```

# Before

```
float sum(float* x, size_t N) {  
    float s = 0;  
    for (size_t i = 0; i < N; ++i)  
        s += x[i];  
    return s;  
}
```

Exactly the  
same loop

# C++ (function) templates



# C++ (function) templates

```
template <typename T>
T sum(T* x, size_t N)
{
    T s = 0;
    for (size_t i = 0; i < N; ++i)
        s += x[i];
    return s;
}
```

At this level

```
double* dx;
size_t dN;
double dz = sum(dx, dN);
```

Bind T to be a double

```
float* dx;
size_t dN;
float dz = sum(dx, dN);
```

Bind T to be a float

# Template for sum so far

We have a thing we want to sum over

Beginning of the thing

End of the thing

```
template <typename T>
T sum(T* x, size_t N) {
    T s = 0;
    for (size_t i = 0; i < N; ++i)
        s += x[i];
    return s;
}
```

The type of s can be set to zero

Go through the thing

Get a value out of the thing

The value is not the same as the thing

The value of the thing can be added to s

The value is not the same as the thing

# Lifting

```
template <typename T>
T sum (T* begin, T* end) {
    T s = 0;
    for (T* p = begin; p < end; ++p) {
        s += *p;
    }
    return s;
}
```

We don't know what kind of thing, so parameterize

From begin to end

Use pointer

Dereference pointer

Get a value

OK for linked list?

Almost, actually

# Lifting

```
template <typename T>
T sum (T* begin, T* end) {
    T s = 0;
    for (T* p = begin; p < end; ++p) {
        s += *p;
    }
    return s;
}
```

Get value

Go to next element

Linked list can do this

# Lifting

begin

```
template <typename T>
T sum (T* begin, T* end) {
    T s = 0;
    for (T* p = begin; p < end; ++p) {
        s += *p;
    }
    return s;
}
```

```
double sum(element* x) {
    double s = 0;
    while (x != nullptr) {
        s += x->val;
        x = x->next;
    }
    return s;
}
```

Get value

Go to next  
element

end

We can do all the  
things needed for sum

But we can't use  
it with our sum

Because of syntax

# For your consideration (Element son)

Element  
"thing"

```
struct element_ptr {  
    element_ptr(element* x) : x(x) {}  
    element_ptr operator++() { x = x->next; return x; }  
    element_ptr operator++(int) { element* y = x; x = x->next; return y; }  
    double operator*() { return x->val; }  
    bool operator==(element_ptr y) { return x == y.x; }  
    bool operator!=(element_ptr y) { return x != y.x; }  
  
    element* x;  
};
```

We also need to  
compare

Get value

Go to next

# Lifting

Get value

```
template <typename T>
T sum (T begin, T end) {
    T s = 0;
    for (T p = begin; p != end; ++p) {
        s += *p;
    }
    return s;
}
```

Compare for equality

Check

Wrong type for s

Move to next

Will s be compatible with "0"

```
struct element_ptr {
    element_ptr(element* x) :
    element_ptr operator++() { return x; }
    element_ptr operator++(int) { element* y = x; x = x->next; return y; }
    double operator*() { return x->val; }
    bool operator==(element_ptr y) { return x == y.x; }
    bool operator!=(element_ptr y) { return x != y.x; }

    element* x;
};
```

Check

Check

# Lifting

Rename  
some things

```
template <typename Iter, typename T>
T sum (Iter begin, Iter end, T init) {
    for (T p = begin; p != end; ++p) {
        init += *p;
    }
    return init;
}
```

And pass in  
initial value  
of s

Parameterize  
the type of s

```
struct element_ptr {
    element_ptr(element* x) : x(x) {}
    element_ptr operator++() { x = x->next; return x; }
    element_ptr operator++(int) { element* y = x; x = x->next; return y; }
    double operator*() { return x->val; }
    bool operator==(element_ptr y) { return x == y.x; }
    bool operator!=(element_ptr y) { return x != y.x; }

    element* x;
};
```

# Final

Lets us iterate through our thing

The thing is holding values of type T

```
template <typename ForwardIterator, typename T>
T sum(ForwardIterator begin, ForwardIterator end, T init) {
    while (begin != end)
        init += *begin++;
    return init;
}
```

Use iterators to mark begin and end of what we want to sum

Compare

Get value

Move to next

Add to init

# Requirements

```
template <typename ForwardIterator, typename T>
T sum(ForwardIterator begin, ForwardIterator end, T init) {
    while (begin != end)
        init += *begin++;
    return init;
}
```

If the type we bind to ForwardIterator has these expressions, we can use sum

- Need dereference – `*i`
- Need increment – `i++`
- Need equality comparison – `i == j` (equiv `i != j`)

# Lifting

```
template <typename ForwardIterator, typename T>
T sum(ForwardIterator begin, ForwardIterator end, T init) {
    while (begin != end)
        init += *begin++;
    return init;
}
```

```
double dd = sum(dx, dx + dN, 0.0);
double ff = sum(fx, fx + fN, 0.0);
double ll = sum(lx, nullptr, 0.0);
```

# Specialization

```
double dd = sum(dx, dx + dN, 0.0);  
double ff = sum(fx, fx + fN, 0.0);  
double ll = sum(lx, nullptr, 0.0);
```

```
double sum(double* x, size_t N) {  
    double s = 0;  
    for (size_t i = 0; i < N; ++i)  
        s += x[i];  
    return s;  
}
```

```
float sum(float* x, size_t N) {  
    float s = 0;  
    for (size_t i = 0; i < N; ++i)  
        s += x[i];  
    return s;  
}
```

```
double sum(element* x) {  
    double s = 0;  
    while (x != nullptr) {  
        s += x->val;  
        x = x->next;  
    }  
    return s;  
}
```

# Interface Specification

- Lets formalize what we just did
- What does the interface really consist of?
- Operations supported by the parameterized type
- Other types associated with the parameterized type
- Semantics, complexity guarantees
- This set of requirements is called a *concept*
- A type meeting the requirements of a concept is said to *model* the concept

# Concepts in Generic Programming

- Generic programming is sometimes called “programming with concepts”
- Syntax
  - Valid expressions
  - Associated types
- Semantics
- Complexity

# Iterator Concepts

- In our example, the iterator required `*`, `++`, `!=`
- These are requirements for an `InputIterator`
- C++ SL has a number of other iterator concepts
- The name of the required concept is usually indicated by the template name

# C++ SL Iterator Concepts

- Trivial Iterator: \*
- Input Iterator: \*, ++
- Output Iterator: \*, ++
- Forward Iterator: \*, ++
- Bidirectional Iterator: \*, ++, --
- Random Access Iterator: \*, ++, --, []

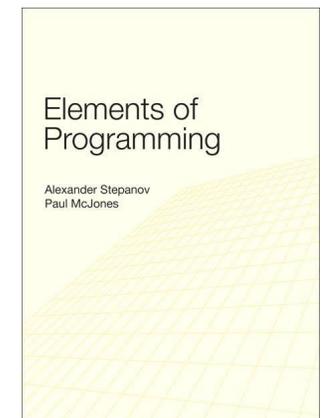
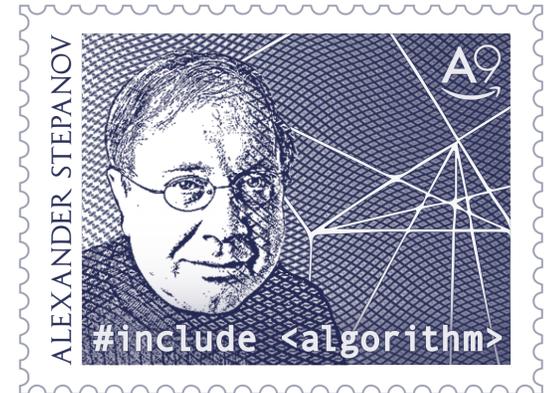
# The Standard Template Library

- In early-mid 90s Stepanov, Musser, Lee applied principles of **generic programming** to C++
- Leveraged templates / parametric polymorphism

```
std::set  
std::list  
std::map  
std::vector  
...
```

```
ForwardIterator  
ReverseIterator  
RandomAccessIterator
```

```
std::for_each  
std::sort  
std::accumulate  
std::min_element  
...
```



Alexander Stepanov and Paul McJones. 2009. *Elements of Programming* (1st ed.). Addison-Wesley Professional.

# Generic Programming

- Algorithms are **generic** (parametrically polymorphic)
- Algorithms can be used on **any** type that meets algorithmic reqts
  - Valid expressions, associated types
  - Not just std. ::types

Standard Library container

```
vector<double> array(N);
```

```
...
```

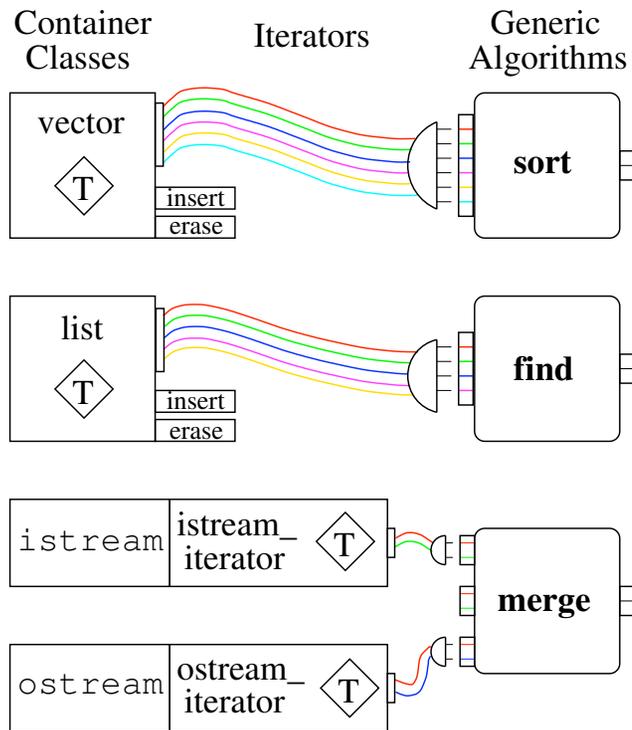
```
std::accumulate(array.begin(), array.end(), 0.0);
```

iterator

iterator

Initial value

# Algorithms and data structures connected by iterators



++ **Increment**    ==, &    **Compare, Reference**  
 = **Assign**        --        **Decrement**  
 \* **Dereference**    +, -, <    **Random Access**

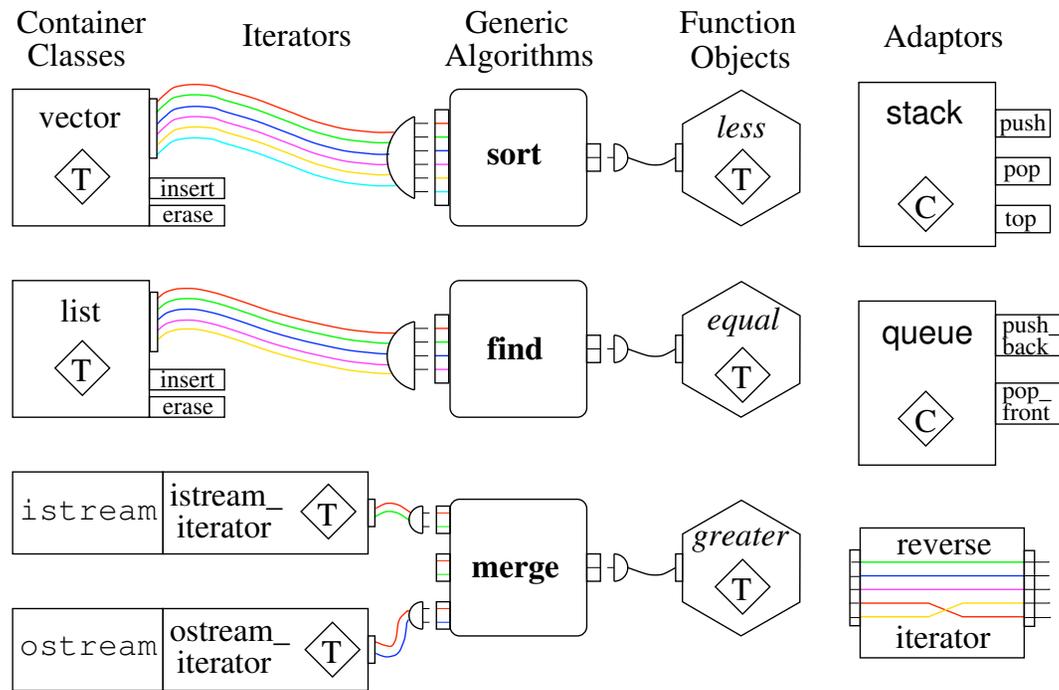
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# Five of the six major STL components



**++ Increment**      ==, &      Compare, Reference  
**= Assign**            --        Decrement  
**\* Dereference**      +, -, <      Random Access

◇ Generic Parameter

# std Containers

- Note that all containers have **same** interface
- (Actually a hierarchy, we'll come back to this)
- We will primarily be focusing on vector

Headers		<u>&lt;vector&gt;</u>	<u>&lt;deque&gt;</u>	<u>&lt;list&gt;</u>
Members		<u>vector</u>	<u>deque</u>	<u>list</u>
	constructor	<u>vector</u>	<u>deque</u>	<u>list</u>
	operator=	<u>operator=</u>	<u>operator=</u>	<u>operator=</u>
iterators	begin	<u>begin</u>	<u>begin</u>	<u>begin</u>
	end	<u>end</u>	<u>end</u>	<u>end</u>
capacity	size	<u>size</u>	<u>size</u>	<u>size</u>
	max_size	<u>max_size</u>	<u>max_size</u>	<u>max_size</u>
	empty	<u>empty</u>	<u>empty</u>	<u>empty</u>
	resize	<u>resize</u>	<u>resize</u>	<u>resize</u>
element access	front	<u>front</u>	<u>front</u>	<u>front</u>
	back	<u>back</u>	<u>back</u>	<u>back</u>
	operator[]	<u>operator[]</u>	<u>operator[]</u>	
modifiers	insert	<u>insert</u>	<u>insert</u>	<u>insert</u>
	erase	<u>erase</u>	<u>erase</u>	<u>erase</u>
	push_back	<u>push_back</u>	<u>push_back</u>	<u>push_back</u>
	pop_back	<u>pop_back</u>	<u>pop_back</u>	<u>pop_back</u>
	swap	<u>swap</u>	<u>swap</u>	<u>swap</u>

# std Containers

- std containers “contain” elements

```
vector<double> array(N);
```

vector of doubles

```
vector<int> array(N);
```

vector of ints

```
list<vector<complex<double> > > thing;
```

list of vectors of complex doubles

- Implementation of list, vector, complex is the same regardless of what is being contained

# Generic Programming

- Algorithms are **generic** (parametrically polymorphic)
- Algorithms can be used on **any** type that meets algorithmic reqts
  - Valid expressions, associated types
  - Not just std. ::types

Standard Library container

```
list<vector<complex<double>>> thing(N);
```

...

```
std::accumulate(thing.begin(), thing.end(), 0.0);
```

iterator

iterator

Initial value

# std Containers

- The std containers are **class templates** (not “template classes”)

```
template <typename T> class vector;  
template <typename T> class deque;  
template <typename T> class list;
```

What follows is  
a template

The template  
parameter is a  
type placeholder

A class  
template

- Don't need details for now

```
vector<double>
```

# Example

```
#include <iostream>
#include <algorithm>
#include <vector>
#include <numeric>

int main() {

    std::vector<int> x(10);
    std::iota(x.begin(), x.end(), 0);
    std::copy(x.begin(), x.end(),
              std::ostream_iterator<int>(std::cout, "\n"));

    return 0;
}
```

```
$ g++ copy_print_vector.cpp
$ ./a.out
0
1
2
3
4
5
6
7
8
9
```

# Class templates

What is the first rule?

```
template <typename T>
class Vector {
public:
    Vector(size_t M) : num_rows_(M), storage_(num_rows_) {}

    double& operator()(size_t i)      { return storage_[i]; }
    const double& operator()(size_t i) const { return storage_[i]; }

    size_t num_rows() const { return num_rows_; }

private:
    size_t      num_rows_;
    std::vector<T> storage_;
};
```

```
Vector<double> av(10);
```

# Instantiation model

- Instantiation model— For each template instance, a separate piece of code is generated, and compiled.
- Not directly required by the standard, but all language rules assume it. Used by every C++ compiler (except for Lunar).
- Different from Generic Java, Eiffel, ... which compile generic definitions to a single skeleton code.
- Speed/space trade-off: instantiation model often faster, but prone to code bloat.
- Some evidence suggest otherwise (Mark Jones, Haskell dictionary passing).

# Unconstrained genericity

```
template <typename ForwardIterator, type  
T sum(ForwardIterator begin, ForwardIterator end, T init) {  
    while (begin != end)  
        init += *begin++;  
    return init;  
}
```

Even though we have a spec for this

How much of this can we type check (and when)?

```
double dd = sum(dx, dx + dN, 0.0);  
double ff = sum(fx, fx + fN, 0.0);  
double ll = sum(lx, nullptr, 0.0);
```

For each template instance, a separate piece of code is generated, and compiled

# Unconstrained genericity

- Maximal reusability (structural conformance). Concise: no need to express constraints.
- No separate type checking, error diagnostics must be delayed until instantiation.
- Errors may occur deep inside a generic library. Errors difficult to interpret, difficult to assign blame.

# Error messages

```
#include <iostream>
#include <algorithm>
#include <iterator>
#include <vector>
class A {};

int main() {
    std::vector<A> a;
    // ...
    std::copy(a.begin(), a.end(),
        ↪ std::ostream_iterator<A>(std::cout,
        ↪ "\n"));
}
```

# Error messages

```
/usr/include/c++/3.2/bits/ostream.tcc:55: candidates are:
std::basic_ostream<CharT, _Traits>& std::basic_ostream<CharT,
_Traits>::operator<<(std::basic_ostream<CharT,
_Traits>&*) (std::basic_ostream<CharT, _Traits>&*) [with _CharT =
char, _Traits = std::char_traits<char>]
/usr/include/c++/3.2/bits/ostream.tcc:77:
std::basic_ostream<CharT, _Traits>& std::basic_ostream<CharT,
_Traits>::operator<<(std::basic_ios<CharT, _Traits>&*) [with _CharT =
char, _Traits = std::char_traits<char>]
/usr/include/c++/3.2/bits/ostream.tcc:99:
std::basic_ostream<CharT, _Traits>& std::basic_ostream<CharT,
_Traits>::operator<<(std::ios_base&*) (std::ios_base&*) [with
_CharT = char, _Traits = std::char_traits<char>]
/usr/include/c++/3.2/bits/ostream.tcc:171:
std::basic_ostream<CharT, _Traits>& std::basic_ostream<CharT,
_Traits>::operator<<(long int) [with _CharT = char, _Traits =
std::char_traits<char>] /usr/include/c++/3.2/bits/ostream.tcc:208:
std::basic_ostream<CharT, _Traits>& std::basic_ostream<CharT,
_Traits>::operator<<(long unsigned int) [with _CharT = char,
_Traits = std::char_traits<char>]
/usr/include/c++/3.2/bits/ostream.tcc:146:
std::basic_ostream<CharT, _Traits>& std::basic_ostream<CharT,
_Traits>::operator<<(bool) [with _CharT = char, _Traits =
std::char_traits<char>] /usr/include/c++/3.2/ostream:104:
std::basic_ostream<CharT, _Traits>& std::basic_ostream<CharT,
_Traits>::operator<<(short int) [with _CharT = char, _Traits =
std::char_traits<char>] /usr/include/c++/3.2/ostream:118:
std::basic_ostream<CharT, _Traits>& std::basic_ostream<CharT,
_Traits>::operator<<(short unsigned int) [with _CharT = char,
_Traits = std::char_traits<char>] /usr/include/c++/3.2/ostream:119:
std::basic_ostream<CharT, _Traits>& std::basic_ostream<CharT,
_Traits>::operator<<(int) [with _CharT = char, _Traits =
std::char_traits<char>] /usr/include/c++/3.2/ostream:130:
std::basic_ostream<CharT, _Traits>& std::basic_ostream<CharT,
_Traits>::operator<<(unsigned int) [with _CharT = char, _Traits =
std::char_traits<char>] /usr/include/c++/3.2/bits/ostream.tcc:234:
std::basic_ostream<CharT, _Traits>& std::basic_ostream<CharT,
_Traits>::operator<<(long long int) [with _CharT = char, _Traits =
std::char_traits<char>] /usr/include/c++/3.2/bits/ostream.tcc:272:
std::basic_ostream<CharT, _Traits>& std::basic_ostream<CharT,
_Traits>::operator<<(long long unsigned int) [with _CharT = char,
_Traits = std::char_traits<char>]
/usr/include/c++/3.2/bits/ostream.tcc:298:
std::basic_ostream<CharT, _Traits>& std::basic_ostream<CharT,
_Traits>::operator<<(double) [with _CharT = char, _Traits =
std::char_traits<char>] /usr/include/c++/3.2/ostream:145:
std::basic_ostream<CharT, _Traits>& std::basic_ostream<CharT,
_Traits>::operator<<(float) [with _CharT = char, _Traits =
std::char_traits<char>] /usr/include/c++/3.2/bits/ostream.tcc:323:
std::basic_ostream<CharT, _Traits>& std::basic_ostream<CharT,
_Traits>::operator<<(long double) [with _CharT = char, _Traits =
std::char_traits<char>] /usr/include/c++/3.2/bits/ostream.tcc:348:
std::basic_ostream<CharT, _Traits>& std::basic_ostream<CharT,
_Traits>::operator<<(const void*) [with _CharT = char, _Traits =
std::char_traits<char>] /usr/include/c++/3.2/bits/ostream.tcc:120:
std::basic_ostream<CharT, _Traits>& std::basic_ostream<CharT,
_Traits>::operator<<(std::basic_streambuf<CharT, _Traits>*) [with
_CharT = char, _Traits = std::char_traits<char>]
/usr/include/c++/3.2/ostream:211: std::basic_ostream<CharT,
_Traits>& std::operator<<(std::basic_ostream<CharT, _Traits>&,
char) [with _CharT = char, _Traits = std::char_traits<char>]
/usr/include/c++/3.2/bits/ostream.tcc:500: std::basic_ostream<char,
_Traits>& std::operator<<(std::basic_ostream<char, _Traits>&, char)
[with _Traits = std::char_traits<char>]
/usr/include/c++/3.2/ostream:222: std::basic_ostream<char,
_Traits>& std::operator<<(std::basic_ostream<char, _Traits>&,
signed char) [with _Traits = std::char_traits<char>]
/usr/include/c++/3.2/ostream:227: std::basic_ostream<char,
_Traits>& std::operator<<(std::basic_ostream<char, _Traits>&,
unsigned char) [with _Traits = std::char_traits<char>]
/usr/include/c++/3.2/bits/ostream.tcc:572:
std::basic_ostream<CharT, _Traits>&
std::operator<<(std::basic_ostream<CharT, _Traits>&, const char*)
[with _CharT = char, _Traits = std::char_traits<char>]
/usr/include/c++/3.2/bits/ostream.tcc:622: std::basic_ostream<char,
_Traits>& std::operator<<(std::basic_ostream<char, _Traits>&, const
char) [with _Traits = std::char_traits<char>]
/usr/include/c++/3.2/ostream:246: std::basic_ostream<char,
_Traits>& std::operator<<(std::basic_ostream<char, _Traits>&, const
signed char*) [with _Traits = std::char_traits<char>]
/usr/include/c++/3.2/ostream:251: std::basic_ostream<char,
_Traits>& std::operator<<(std::basic_ostream<char, _Traits>&, const
unsigned char*) [with _Traits = std::char_traits<char>]
```

```
#include <iostream>
#include <algorithm>
#include <iterator>
#include <vector>
class A {};
```

```
int main() {
    std::vector<A> a;
    // ...
    std::copy(a.begin(), a.end(),
        std::ostream_iterator<A>(std::cout,
        "\n"));
}
```

What is wrong here?

E for ADVANCED COMPUTING

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# Terminology

- A **parametrically polymorphic** function can accept **any** types to be bound to its parameters
  - Including ones that don't work
- A function that is **bounded polymorphic** only accepts a specific set of types to be bound to its parameters
- Two approaches for conformance
  - Structural conformance: a type satisfies every requirement listed in the concept/base class/type class/signature/...
  - Nominal conformance: in addition to structural conformance, an explicit declaration is required to establish conformance.

# C++ Concepts

- Why change templates?
  - Templates enable generic programming in C++
  - Overloading permits natural abstractions
  - Instantiation eliminates cost of abstractions
  - Many successful, generic libraries in C++
- Big problems remain
  - Expression of ideas of generic programming not direct
  - C++ generic libraries can be very brittle
- Goal for concepts:
  - Improve support for generic programming in C++, retaining performance and flexibility of templates

# Unconstrained

## std::accumulate

Defined in header `<numeric>`

```
template< class InputIt, class T >
T accumulate( InputIt first, InputIt last, T init );           (1)
```

```
template< class InputIt, class T, class BinaryOperation >
T accumulate( InputIt first, InputIt last, T init,
              BinaryOperation op );                          (2)
```

We use name "InputIt" to hint to programmer that this should be an InputIterator

Computes the sum of the given value `init` and the elements in the range `[first, last)`. The first version uses `operator+` to sum up the elements, the second version uses the given binary function `op`, both applying `std::move` to their operands on the left hand side (since C++20).

`op` must not have side effects.

(until C++11)

`op` must not invalidate any iterators, including the end iterators, or modify any elements of the range involved.

(since C++11)

What is this?

### Type requirements

- `InputIt` must meet the requirements of `InputIterator`.
- `T` must meet the requirements of `CopyAssignable` and `CopyConstructible`.

# InputIterator

## C++ concepts: InputIterator

An InputIterator is an `Iterator` that can read from the pointed-to element. InputIterators only guarantee validity for single pass algorithms: once an InputIterator `i` has been incremented, all copies of its previous value may be invalidated.

### Requirements

The type `It` satisfies InputIterator if

- The type `It` satisfies `Iterator`
- The type `It` satisfies `EqualityComparable`

And, given

- `i` and `j`, values of type `It` or `const It`
- `reference`, the type denoted by `std::iterator_traits<It>::reference`
- `value_type`, the type denoted by `std::iterator_traits<It>::value_type`

# InputIterator

Expression	Return	Equivalent expression	Notes
<code>i != j</code>	contextually convertible to <code>bool</code>	<code>!(i == j)</code>	<b>Precondition:</b> <code>(i, j)</code> is in the domain of <code>==</code> .
<code>*i</code>	reference, convertible to <code>value_type</code>	If <code>i == j</code> and <code>(i, j)</code> is in the domain of <code>==</code> then this is equivalent to <code>*j</code> .	<b>Precondition:</b> <code>i</code> is dereferenceable. The expression <code>(void)*i, *i</code> is equivalent to <code>*i</code> .
<code>i-&gt;m</code>		<code>(*i).m</code>	<b>Precondition:</b> <code>i</code> is dereferenceable.
<code>++i</code>	<code>It&amp;</code>		<b>Precondition:</b> <code>i</code> is dereferenceable. <b>Postcondition:</b> <code>i</code> is dereferenceable or <code>i</code> is past-the-end. <b>Postcondition:</b> Any copies of the previous value of <code>i</code> are no longer required to be either dereferenceable or to be in the domain of <code>==</code> .
<code>(void)i++</code>		<code>(void)++i</code>	
<code>*i++</code>	convertible to <code>value_type</code>	<code>value_type x = *i; ++i; return x;</code>	

# Unconstrained

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### Type requirements

- `InputIt` must meet the requirements of `InputIterator`.
- `T` must meet the requirements of `CopyAssignable` and `CopyConstructible`.

# Iterator Concepts

<b>Iterator</b>	general concept to access data within some data structure (concept)
<b>InputIterator</b>	iterator that can be used to read data (concept)
<b>OutputIterator</b>	iterator that can be used to write data (concept)
<b>ForwardIterator</b>	iterator that can be used to read data multiple times (concept)
<b>BidirectionalIterator</b>	iterator that can be both incremented and decremented (concept)
<b>RandomAccessIterator</b>	iterator that can be advanced in constant time (concept)

# Sort

## std::sort

Defined in header `<algorithm>`

```
template< class RandomIt >  
void sort( RandomIt first, RandomIt last );  
  
template< class ExecutionPolicy, class RandomIt >  
void sort( ExecutionPolicy&& policy, RandomIt first, RandomIt last );  
  
template< class RandomIt, class Compare >  
void sort( RandomIt first, RandomIt last, Compare comp );  
  
template< class ExecutionPolicy, class RandomIt, class Compare >  
void sort( ExecutionPolicy&& policy, RandomIt first, RandomIt last, Compare comp );
```

Wait, what's  
this?

Defaultx

Customizable

### Type requirements

- RandomIt must meet the requirements of [ValueSwappable](#) and [RandomAccessIterator](#).
- The type of dereferenced RandomIt must meet the requirements of [MoveAssignable](#) and [MoveConstructible](#).
- Compare must meet the requirements of [Compare](#).

# Execution Policies

`std::execution::seq`, `std::execution::par`, `std::execution::par_unseq`

Defined in header `<execution>`

```
inline constexpr std::execution::sequenced_policy seq { /* unspecified */ };
```

```
inline constexpr std::execution::parallel_policy par { /* unspecified */ };
```

```
inline constexpr std::execution::parallel_unsequenced_policy par_unseq { /* unspecified */ };
```

# Parallel standard library algorithms

- `std::adjacent_difference`
- `std::adjacent_find`
- `std::all_of`
- `std::any_of`
- `std::copy`
- `std::copy_if`
- `std::copy_n`
- `std::count`
- `std::count_if`
- `std::equal`
- `std::fill`
- `std::fill_n`
- `std::find`
- `std::find_end`
- `std::find_first_of`
- `std::find_if`
- `std::find_if_not`
- `std::generate`
- `std::generate_n`
- `std::includes`
- `std::inner_product`
- `std::inplace_merge`
- `std::is_heap`
- `std::is_heap_until`
- `std::is_partitioned`
- `std::is_sorted`
- `std::is_sorted_until`
- `std::lexicographical_compare`
- `std::max_element`
- `std::merge`
- `std::min_element`
- `std::minmax_element`
- `std::mismatch`
- `std::move`
- `std::none_of`
- `std::nth_element`
- `std::partial_sort`
- `std::partial_sort_copy`
- `std::partition`
- `std::partition_copy`
- `std::remove`
- `std::remove_copy`
- `std::remove_copy_if`
- `std::remove_if`
- `std::replace`
- `std::replace_copy`
- `std::replace_copy_if`
- `std::replace_if`
- `std::reverse`
- `std::reverse_copy`
- `std::rotate`
- `std::rotate_copy`
- `std::search`
- `std::search_n`
- `std::set_difference`
- `std::set_intersection`
- `std::set_symmetric_difference`
- `std::set_union`
- `std::sort`
- `std::stable_partition`
- `std::stable_sort`
- `std::swap_ranges`
- `std::transform`
- `std::uninitialized_copy`
- `std::uninitialized_copy_n`
- `std::uninitialized_fill`
- `std::uninitialized_fill_n`
- `std::unique`
- `std::unique_copy`

Where is accumulate?

There is no parallel accumulate

Why not?

# New parallel algorithms

Instead of  
accumulate

<b>for_each</b>	similar to <code>std::for_each</code> except returns void (function template)
<b>for_each_n</b> Defined in header <code>&lt;experimental/numeric&gt;</code>	applies a function object to the first n elements of a sequence (function template)
<b>reduce</b> (parallelism TS)	similar to <code>std::accumulate</code> , except out of order (function template)
<b>exclusive_scan</b>	similar to <code>std::partial_sum</code> , excludes the ith input element from the ith sum (function template)
<b>inclusive_scan</b>	similar to <code>std::partial_sum</code> , includes the ith input element in the ith sum (function template)
<b>transform_reduce</b> (parallelism TS)	applies a functor, then reduces out of order (function template)
<b>transform_exclusive_scan</b>	applies a functor, then calculates exclusive scan (function template)
<b>transform_inclusive_scan</b>	applies a functor, then calculates inclusive scan (function template)

# Reduce

## std::experimental::parallel::reduce

Defined in header `<experimental/numeric>`

```
template<class InputIt>  
typename std::iterator_traits<InputIt>::value_type reduce(  
    InputIt first, InputIt last);
```

```
template<class ExecutionPolicy, class InputIterator>  
typename std::iterator_traits<InputIt>::value_type reduce(  
    ExecutionPolicy&& policy, InputIt first, InputIt last);
```

```
template<class InputIt, class T>  
T reduce(InputIt first, InputIt last, T init);
```

```
template<class ExecutionPolicy, class InputIt, class T>  
T reduce(ExecutionPolicy&& policy, InputIt first, InputIt last, T init);
```

```
template<class InputIt, class T, class BinaryOp>  
T reduce(InputIt first, InputIt last, T init, BinaryOp binary_op);
```

```
template<class ExecutionPolicy, class InputIt, class T, class BinaryOp>  
T reduce(ExecutionPolicy&& policy,  
    InputIt first, InputIt last, T init, BinaryOp binary_op);
```

# Notes

- If `policy` is an instance of `sequential_execution_policy`, all operations are performed in the calling thread.
- If `policy` is an instance of `parallel_execution_policy`, operations may be performed in unspecified number of threads, indeterminately sequenced with each other
- If `policy` is an instance of `parallel_vector_execution_policy`, execution may be both parallelized and vectorized: function body boundaries are not respected and user code may be overlapped and combined in arbitrary manner (in particular, this implies that a user-provided Callable must not acquire a mutex to access a shared resource)

# Example

```
{ Timer t; t.start();  
for (size_t k = 0; k < loops; ++k)  
    result = std::accumulate(&v(0), &v(v.num_rows()), 0.0);  
t.stop();  
std::cout << "std::accumulate result " << result << " took " << t.elapsed()  
↳ << " ms\n"; }
```

Regular  
accumulate

```
{ Timer t; t.start();  
for (size_t k = 0; k < loops; ++k)  
    result = std::reduce(pstl::execution::seq, &v(0), &v(v.num_rows()), 0.0);  
t.stop();  
std::cout << "std::reduce result " << result << " took " << t.elapsed() <<  
↳ " ms\n"; }
```

Sequential  
execution

# Example

```
{ Timer t; t.start();  
for (size_t k = 0; k < loops; ++k)  
    result = std::reduce(pstl::execution::par, &v(0), &v(v.num_rows()), 0.0);  
t.stop();  
std::cout << "std::reduce result " << result << " took " << t.elapsed() <<  
    << " ms\n"; }
```

Parallel  
execution

```
{ Timer t; t.start();  
for (size_t k = 0; k < loops; ++k)  
    result = std::reduce(pstl::execution::par_unseq, &v(0), &v(v.num_rows()),  
        << 0.0);  
t.stop();  
std::cout << "std::reduce result " << result << " took " << t.elapsed() <<  
    << " ms\n"; }
```

Parallel  
execution

# Results

```
std::accumulate result -2310.8 took 1155 ms  
std::reduce result -2310.8 took 1167 ms  
std::reduce result -2310.8 took 329 ms  
std::reduce result -2310.8 took 337 ms
```

Accumulate

Sequential  
reduce

Parallel  
execution

Parallel  
execution

## Example II

```
std::list<double> v(&x(0), &x(x.num_rows()));

{ Timer t; t.start();
for (size_t k = 0; k < loops; ++k)
    result = std::accumulate(v.begin(), v.end(), 0.0);
t.stop();
std::cout << "std::accumulate result " << result << " took " << t.elapsed()
↳ << " ms\n"; }

{ Timer t; t.start();
for (size_t k = 0; k < loops; ++k)
    result = std::reduce(pstl::execution::seq, v.begin(), v.end(), 0.0);
t.stop();
std::cout << "std::reduce result " << result << " took " << t.elapsed() <<
↳ " ms\n"; }
```

## Example II

```
{ Timer t; t.start();  
for (size_t k = 0; k < loops; ++k)  
    result = std::reduce(pstl::execution::par, v.begin(), v.end(), 0.0);  
t.stop();  
std::cout << "std::reduce result " << result << " took " << t.elapsed() <<  
↳ " ms\n"; }
```

```
{ Timer t; t.start();  
for (size_t k = 0; k < loops; ++k)  
    result = std::reduce(pstl::execution::par_unseq, v.begin(), v.end(),  
↳ 0.0);  
t.stop();  
std::cout << "std::reduce result " << result << " took " << t.elapsed() <<  
↳ " ms\n"; }
```

## Results II

```
std::accumulate result 694.824 took 1107 ms  
std::reduce result 694.824 took 1997 ms  
std::reduce result 694.824 took 2042 ms  
std::reduce result 694.824 took 1980 ms
```

Accumulate

Sequential  
reduce

Parallel  
execution

Parallel  
execution

Why no  
speedup?

# Recall

```
for (size_t k = 0; k < parts; ++k) {  
    futs.push_back(std::async(std::launch::async, [&, k]() -> double {  
        double sum = 0.0;  
        for (size_t i = k*blocksize; i < (k+1)*blocksize; ++i)  
            sum += x(i) * x(i);  
        return sum;  
    }));  
}
```

What is the first access in each thread?

How long should it take to get there?

# std::list

## Member types

Member type	Definition
value_type	T
allocator_type	Allocator
size_type	Unsigned integer type (usually <code>std::size_t</code> )
difference_type	Signed integer type (usually <code>std::ptrdiff_t</code> )
reference	Allocator::reference (until C++11) value_type& (since C++11)
const_reference	Allocator::const_reference (until C++11) const value_type& (since C++11)
pointer	Allocator::pointer (until C++11) <code>std::allocator_traits&lt;Allocator&gt;::pointer</code> (since C++11)
const_pointer	Allocator::const_pointer (until C++11) <code>std::allocator_traits&lt;Allocator&gt;::const_pointer</code> (since C++11)
iterator	<code>BidirectionalIterator</code>
const_iterator	Constant <code>BidirectionalIterator</code>
reverse_iterator	<code>std::reverse_iterator&lt;iterator&gt;</code>
const_reverse_iterator	<code>std::reverse_iterator&lt;const_iterator&gt;</code>

## C++ concepts: BidirectionalIterator

A `BidirectionalIterator` is a `ForwardIterator` that can be moved in both directions (i.e. incremented and decremented).

# std::vector

## Member types

Member type	Definition
value_type	T
allocator_type	Allocator
size_type	Unsigned integer type (usually <code>std::size_t</code> )
difference_type	Signed integer type (usually <code>std::ptrdiff_t</code> )
reference	Allocator::reference (until C++11) value_type& (since C++11)
const_reference	Allocator::const_reference (until C++11) const value_type& (since C++11)
pointer	Allocator::pointer (until C++11) <code>std::allocator_traits&lt;Allocator&gt;::pointer</code> (since C++11)
const_pointer	Allocator::const_pointer (until C++11) <code>std::allocator_traits&lt;Allocator&gt;::const_pointer</code> (since C++11)
iterator	RandomAccessIterator
const_iterator	Constant RandomAccessIterator
reverse_iterator	<code>std::reverse_iterator&lt;iterator&gt;</code>
const_reverse_iterator	<code>std::reverse_iterator&lt;const_iterator&gt;</code>

## C++ concepts: RandomAccessIterator

A RandomAccessIterator is a BidirectionalIterator that can be moved to point to any element in constant time.

A pointer to an element of an array satisfies all requirements of RandomAccessIterator

## Example II

Random  
Access!

```
{ Timer t; t.start();  
for (size_t k = 0; k < loops; ++k)  
    result = std::reduce(pstl::execution::par, v.begin(), v.end(), 0.0);  
t.stop();  
std::cout << "std::reduce result " << result << " took " << t.elapsed() <<  
↳ " ms\n"; }
```

```
{ Timer t; t.start();  
for (size_t k = 0; k < loops; ++k)  
    result = std::reduce(pstl::execution::par_unseq, v.begin(), v.end(),  
↳ 0.0);  
t.stop();  
std::cout << "std::reduce result " << result << " took " << t.elapsed() <<  
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```

# Thank you!

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