AMATH 483/583
High Performance Scientific Computing

Lecture 18:
Message Passing w/CSP/SPMD, MPI

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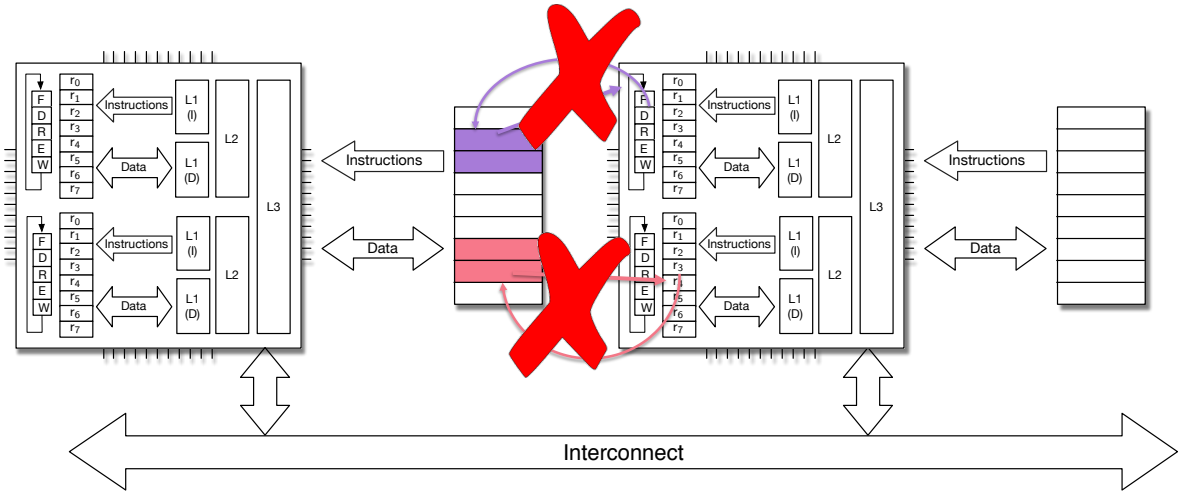
Overview

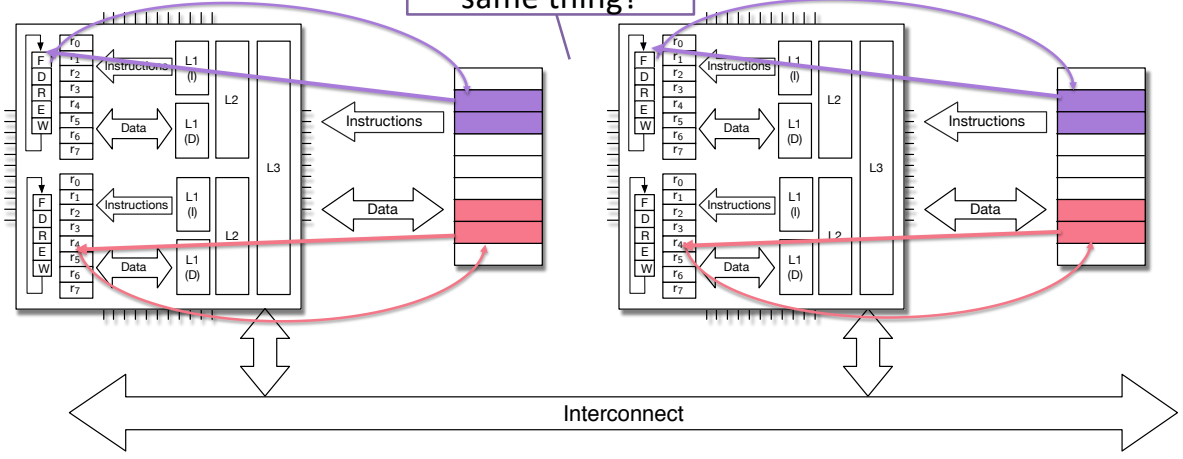
- SPMD / CSP recap
- MPI mental model recap
- Basic MPI recap
- Laplace's equation on a regular grid

Distributed memory

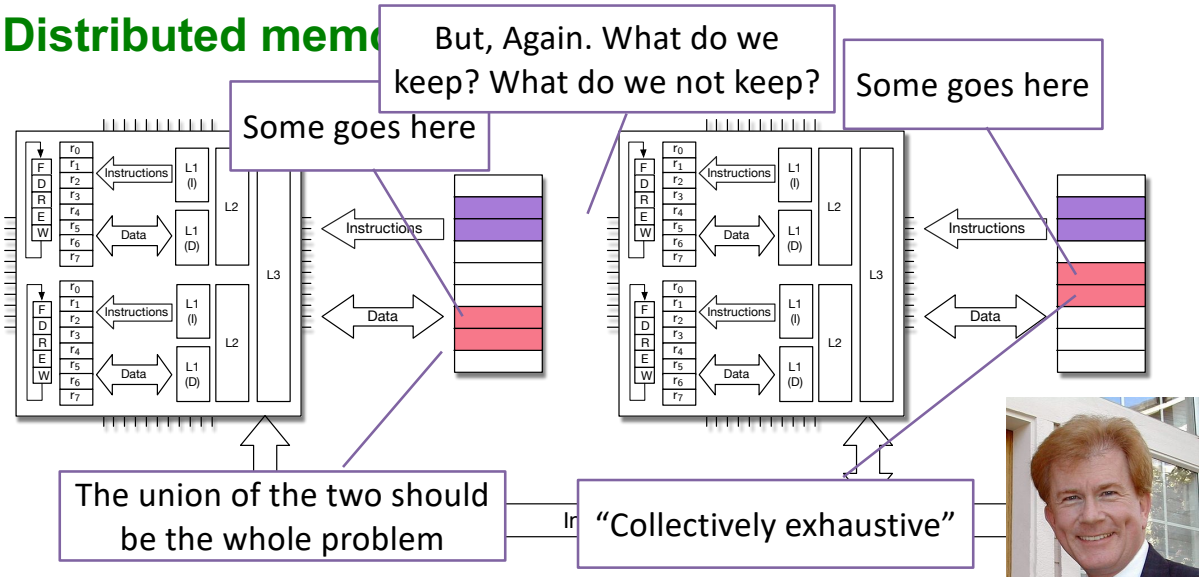


Distributed memor

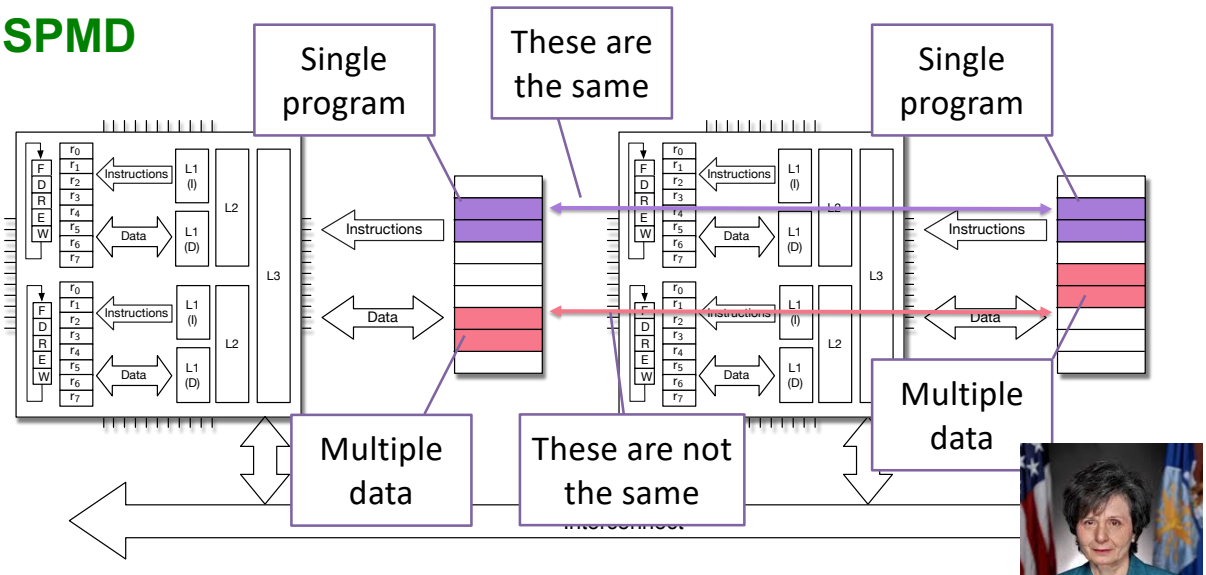
Do we want these doing the exact same thing?



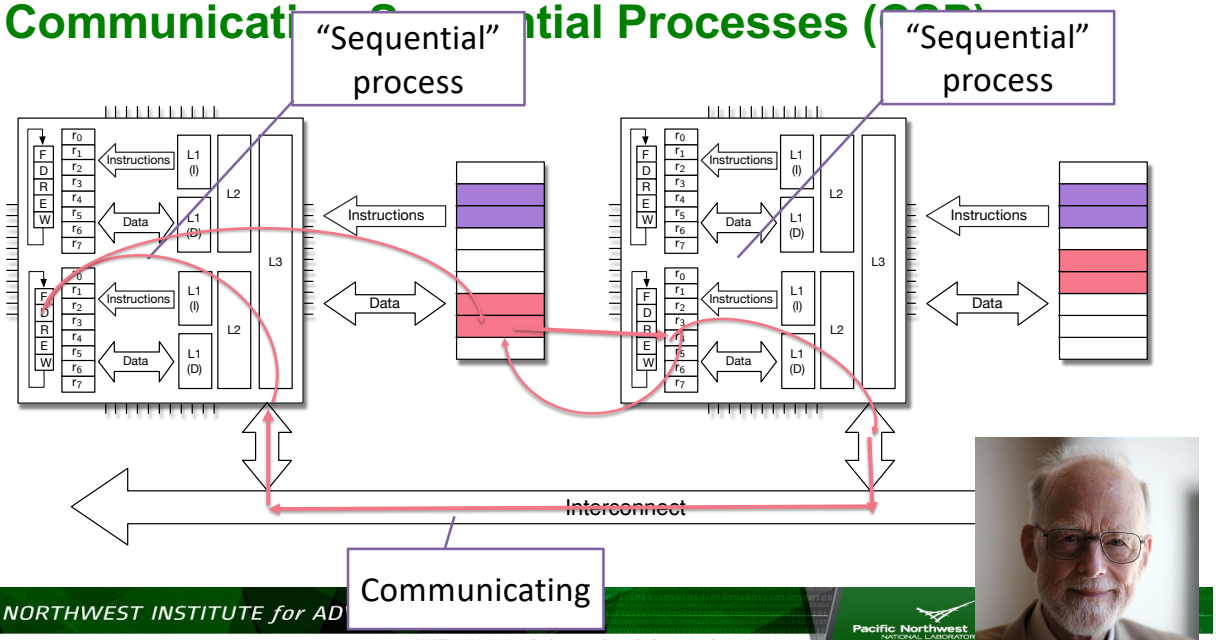
Distributed memo



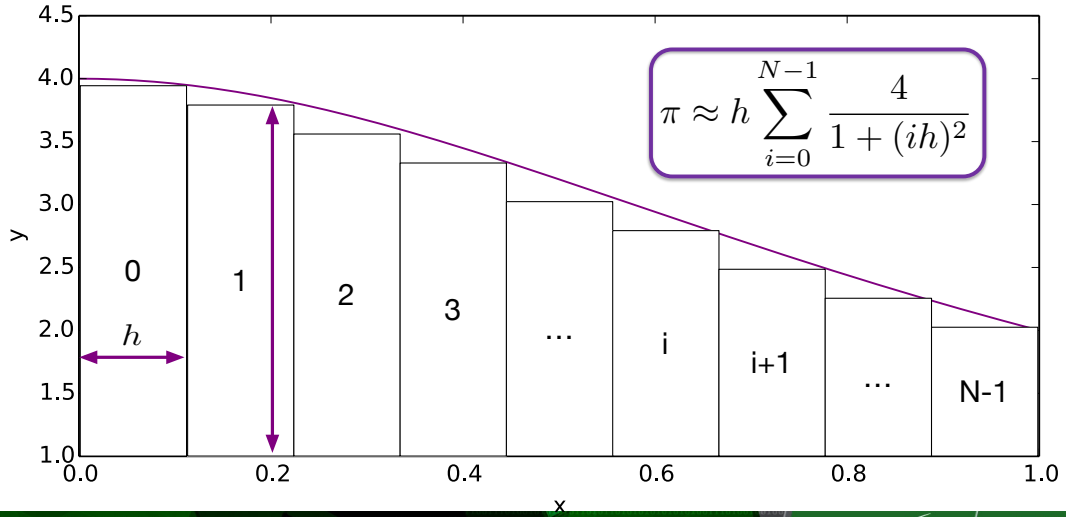
SPMD



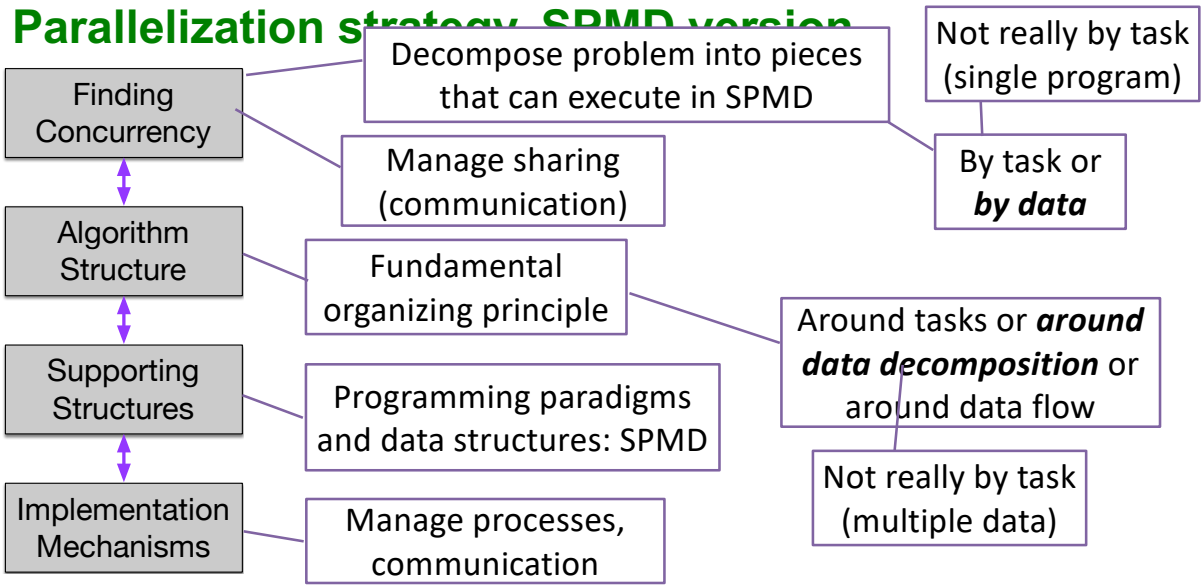
Communicati… …tial Processes (…)



Numerical Quadrature



Parallelization strategy, SPMD version



Timothy Mattson, Beverly Sanders, and Berna Massingill. 2004. *Patterns for Parallel Programming* (First ed.). Addison-Wesley Professional.

Finding Concurrency

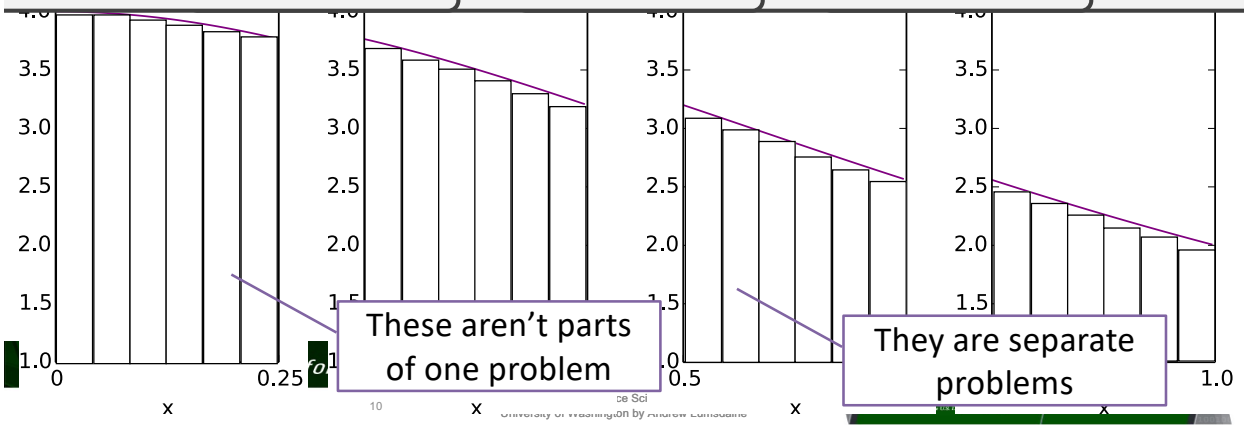
```
for (int i = begin; i < end; ++i) {
  pi += h * 4.0 / (1 + i*h*i*h);
}
```

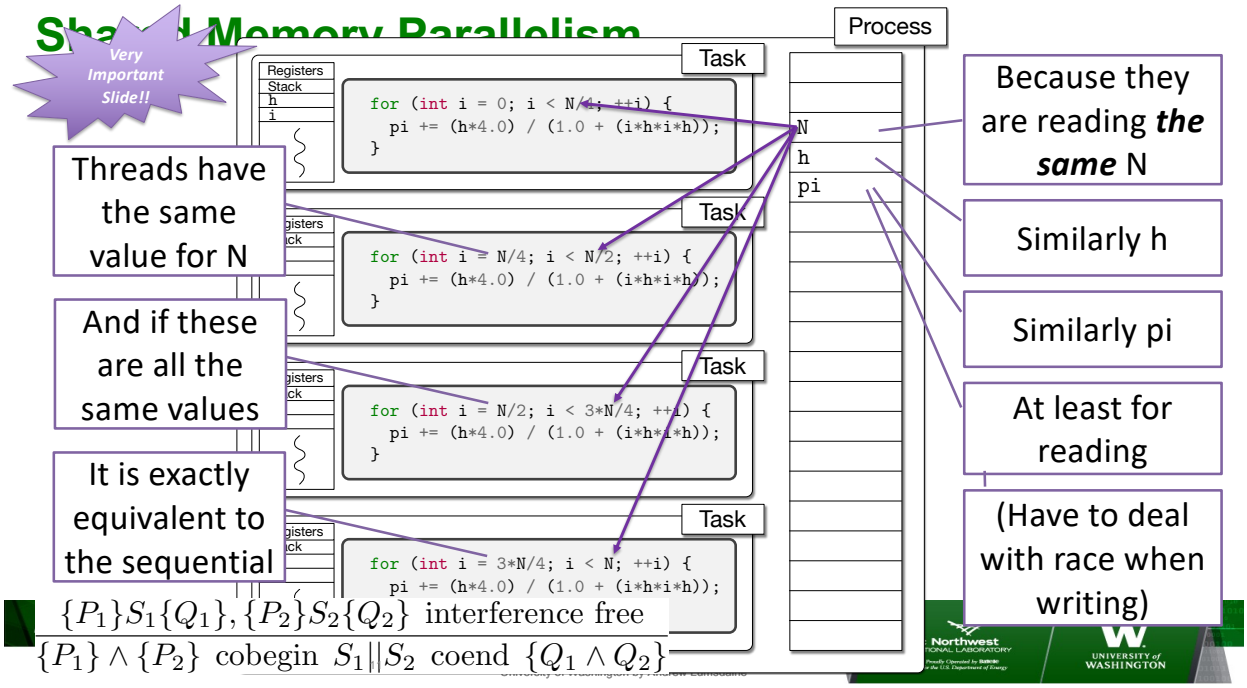
```
int i = 0; i < N/4; ++i) {
+= h * 4.0 / (1 + i*h*i*h);
```

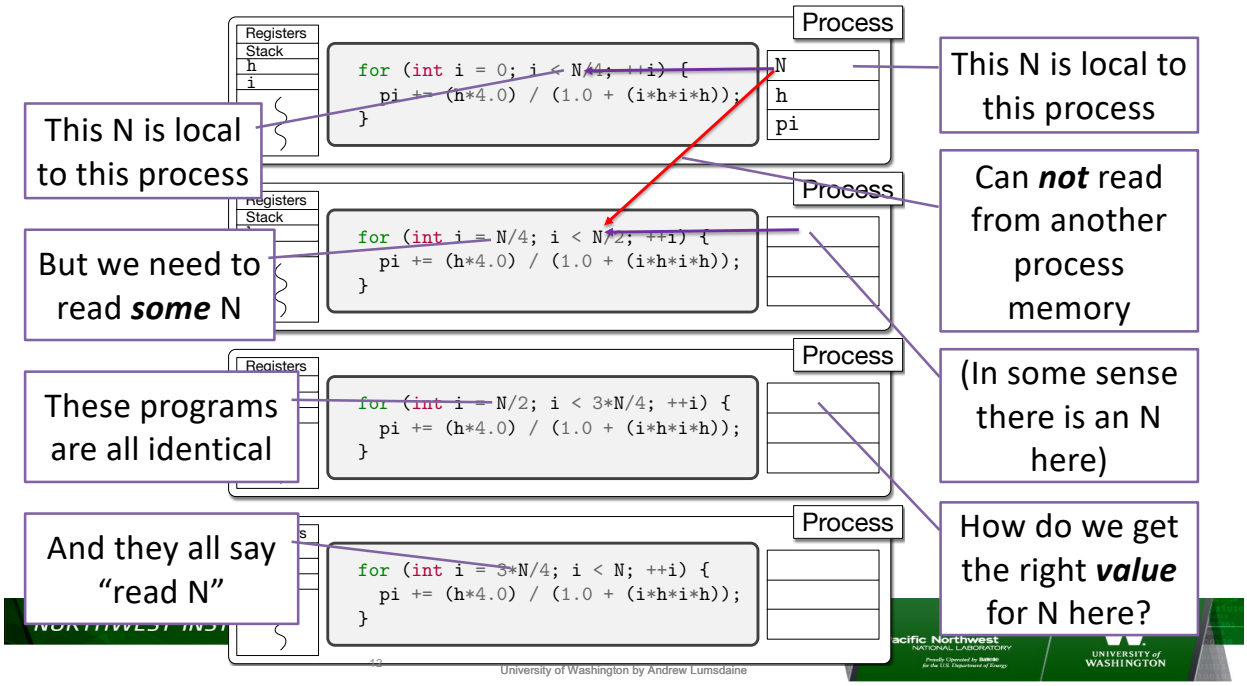
```
4; i < N/2; ++i) {
/ (1 + i*h*i*h);
```

```
/2; i < 3*N/4; ++i) {
0 / (1 + i*h*i*h);
```

```
N/4; i < N
/ (1 + i*h
```




Shared Memory Parallelism
Very Important Slide!!
Process
Task
Registers
Stack
h
i
for (int i = 0; i < N/4; ++i) {
pi += (h*4.0) / (1.0 + (i*h*i*h));
}
for (int i = N/4; i < N/2; ++i) {
pi += (h*4.0) / (1.0 + (i*h*i*h));
}
for (int i = N/2; i < 3*N/4; ++i) {
pi += (h*4.0) / (1.0 + (i*h*i*h));
}
for (int i = 3*N/4; i < N; ++i) {
pi += (h*4.0) / (1.0 + (i*h*i*h));
}
N
h
pi
Threads have the same value for N
And if these are all the same values
It is exactly equivalent to the sequential
Because they are reading *the same* N
Similarly h
Similarly pi
At least for reading
(Have to deal with race when writing)
$\{P_1\}S_1\{Q_1\}, \{P_2\}S_2\{Q_2\}$ interference free
$\{P_1\} \wedge \{P_2\}$ cobegin $S_1 || S_2$ coend $\{Q_1 \wedge Q_2\}$


Process
Registers
Stack
h
i
for (int i = 0; i < N/4; ++i) {
pi += (h*4.0) / (1.0 + (i*h*i*h));
}
for (int i = N/4; i < N/2; ++i) {
pi += (h*4.0) / (1.0 + (i*h*i*h));
}
for (int i = N/2; i < 3*N/4; ++i) {
pi += (h*4.0) / (1.0 + (i*h*i*h));
}
for (int i = 3*N/4; i < N; ++i) {
pi += (h*4.0) / (1.0 + (i*h*i*h));
}
N
h
pi
This N is local to this process
But we need to read *some* N
These programs are all identical
And they all say "read N"
This N is local to this process
Can *not* read from another process memory
(In some sense there is an N here)
How do we get the right *value* for N here?
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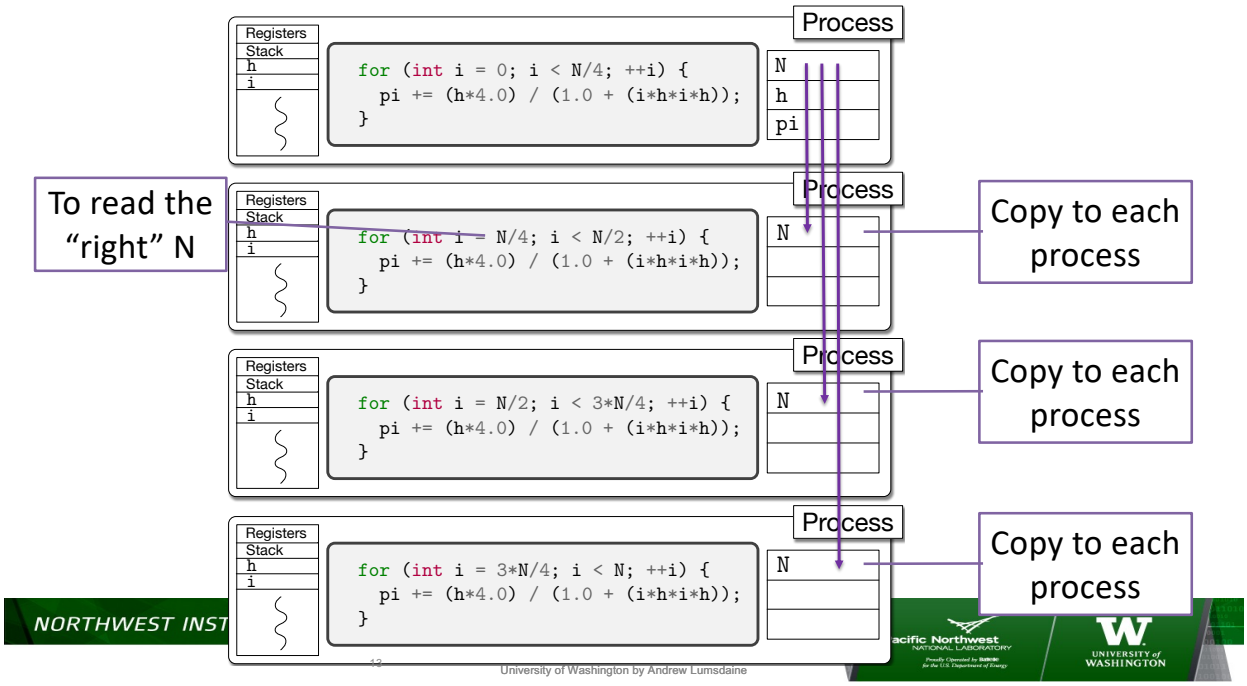
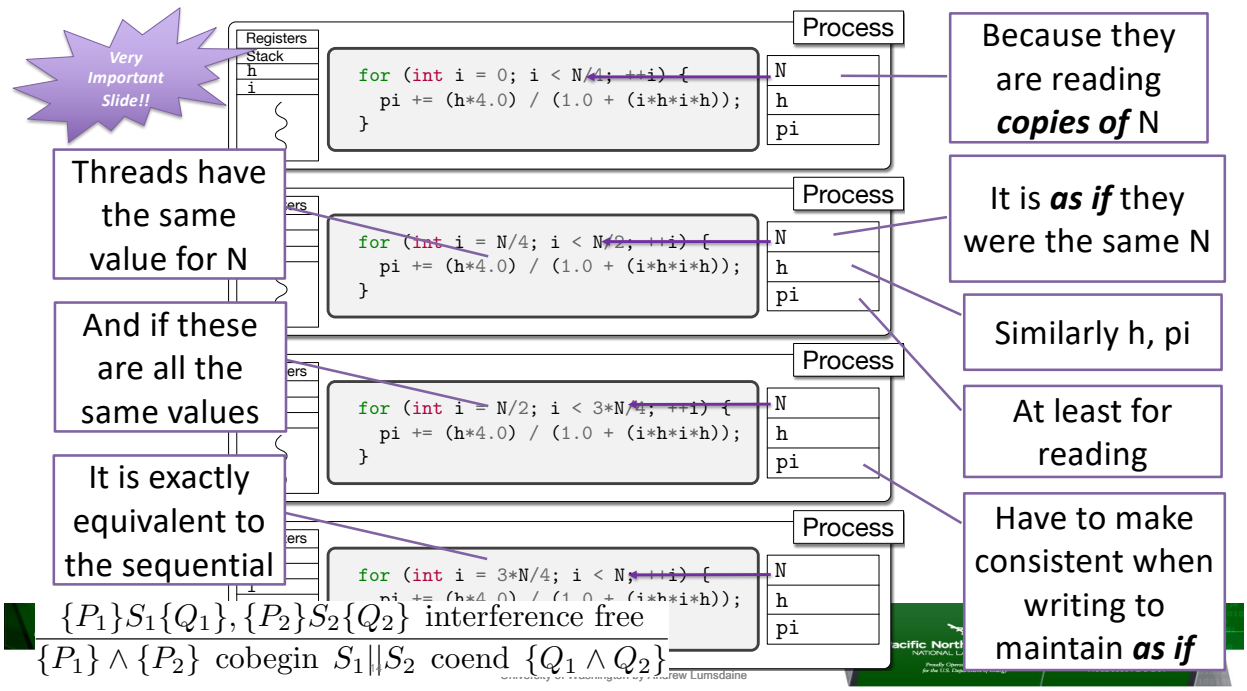

Process
Registers
Stack
h
i
for (int i = 0; i < N/4; ++i) {
pi += (h*4.0) / (1.0 + (i*h*i*h));
}
N
h
pi
Process
for (int i = N/4; i < N/2; ++i) {
pi += (h*4.0) / (1.0 + (i*h*i*h));
}
N
Process
for (int i = N/2; i < 3*N/4; ++i) {
pi += (h*4.0) / (1.0 + (i*h*i*h));
}
N
Process
for (int i = 3*N/4; i < N; ++i) {
pi += (h*4.0) / (1.0 + (i*h*i*h));
}
N
To read the "right" N
Copy to each process
Copy to each process
Copy to each process
NORTHWEST INST
University of Washington by Andrew Lumsdaine


Very Important Slide!!
Process
Registers
Stack
h
i
for (int i = 0; i < N/4; ++i) {
pi += (h*4.0) / (1.0 + (i*h*i*h));
}
N
h
pi
Process
for (int i = N/4; i < N/2; ++i) {
pi += (h*4.0) / (1.0 + (i*h*i*h));
}
N
h
pi
Process
for (int i = N/2; i < 3*N/4; ++i) {
pi += (h*4.0) / (1.0 + (i*h*i*h));
}
N
h
pi
Process
for (int i = 3*N/4; i < N; ++i) {
pi += (h*4.0) / (1.0 + (i*h*i*h));
}
N
h
pi
Because they are reading copies of N
It is as if they were the same N
Similarly h, pi
At least for reading
Have to make consistent when writing to maintain as if
Threads have the same value for N
And if these are all the same values
It is exactly equivalent to the sequential
$\{P_1\}S_1\{Q_1\}, \{P_2\}S_2\{Q_2\}$ interference free
$\{P_1\} \wedge \{P_2\}$ cobegin $S_1 || S_2$ coend $\{Q_1 \wedge Q_2\}$

MPI

Get our id and number of other nodes

id 0 gets N

This pattern is ubiquitous

id 0 shares N

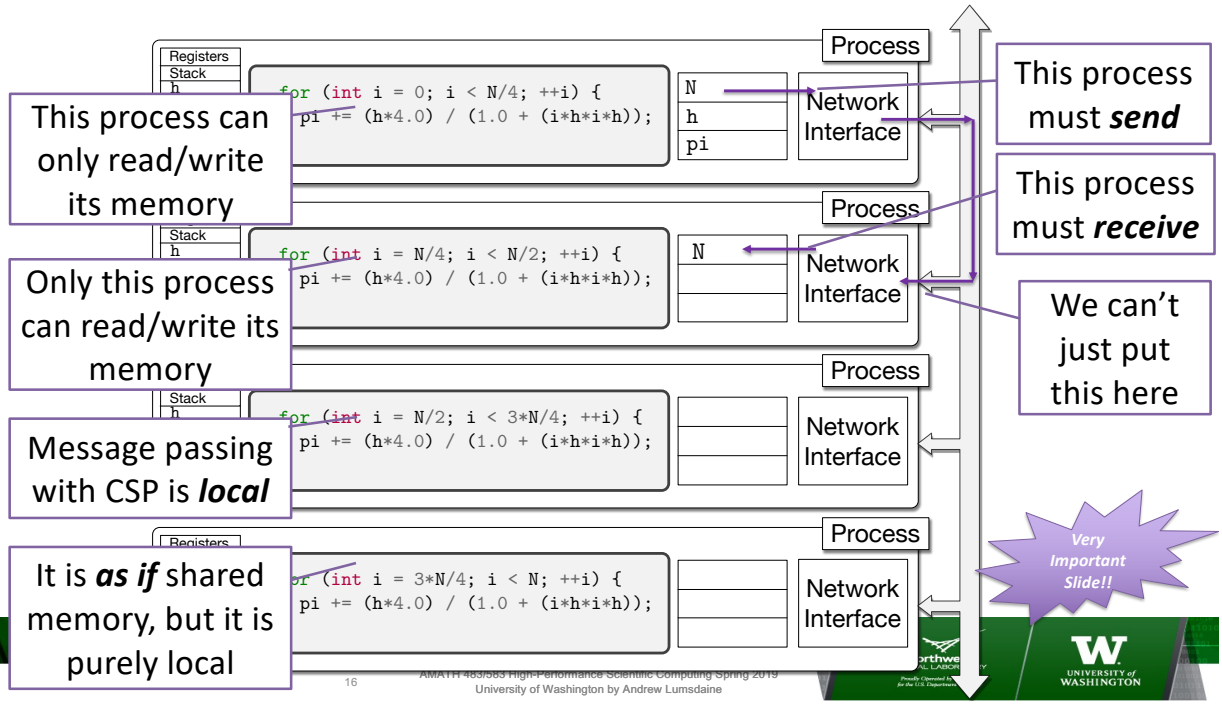
Everyone has same N

Everyone computes their own partial pi

id 0 collects all partials, adds them, and prints

id 0 is root

```
int main(int argc, char* argv[]) {
  size_t intervals = 1024 * 1024;

  MPI::Init();

  int myrank = MPI::COMM_WORLD.Get_rank();
  int mysize = MPI::COMM_WORLD.Get_size();

  if (0 == myrank) {
    if (argc >= 2) intervals = std::atol(argv[1]);
  }

  MPI::COMM_WORLD.Bcast(&intervals, 1, MPI::UNSIGNED_LONG, 0);

  size_t blocksize = intervals / mysize;
  size_t begin     = blocksize * myrank;
  size_t end       = blocksize * (myrank + 1);
  double h         = 1.0 / ((double)intervals);

  double pi        = 0.0;
  for (size_t i = begin; i < end; ++i) {
    pi += 4.0 / (1.0 + (i * h * i * h));

  MPI::COMM_WORLD.Reduce(&mypi, &pi, 1, MPI::DOUBLE, MPI::SUM, 0);

  if (0 == myrank) {
    std::cout << "pi is approximately " << pi << std::endl;
  }

  MPI::Finalize();

  return 0;
}
```

This process can only read/write its memory

Only this process can read/write its memory

Message passing with CSP is ***local***

It is ***as if*** shared memory, but it is purely local

This process must ***send***

This process must ***receive***

We can't just put this here

Very Important Slide!!

When we run this "parallel program" we aren't running a parallel program

We are running multiple copies of this sequential program

All copies execute exactly this same code (not in lock step)

```
int main(int argc, char* argv[]) {
  size_t intervals = 1024 * 1024;

  MPI::Init();

  int myrank = MPI::COMM_WORLD.Get_rank();
  int mysize = MPI::COMM_WORLD.Get_size();

  if (0 == myrank) if (argc >= 2) intervals = std::atol(argv[1]);

  MPI::COMM_WORLD.Bcast(&intervals, 1, MPI::UNSIGNED_LONG, 0);

  size_t blocksize = intervals / mysize;
  size_t begin     = blocksize * myrank;
  size_t end       = blocksize * (myrank + 1);
  double h         = 1.0 / ((double)intervals);

  double pi        = 0.0;
  for (size_t i = begin; i < end; ++i)
    pi += 4.0 / (1.0 + (i * h * i * h));

  MPI::COMM_WORLD.Reduce(&mypi, &pi, 1, MPI::DOUBLE, MPI::SUM, 0);

  if (0 == myrank) std::cout << "pi is approximately " << pi << std::endl;

  MPI::Finalize();

  return 0;
}
```

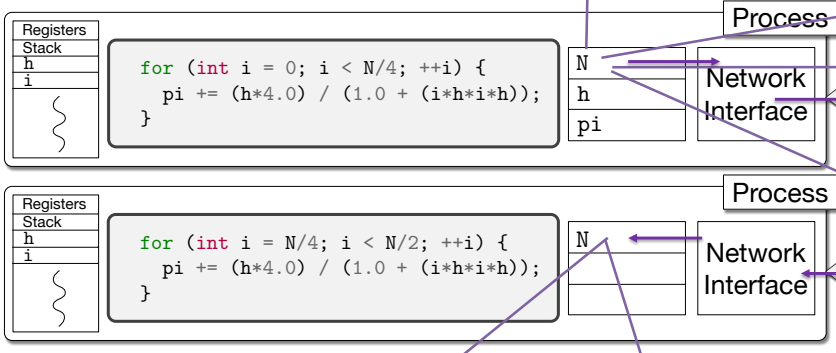
All copies call this communication function

And `intervals` gets copied to all processes

Only because each process calls this

First question: What are we sending?

"N" only has meaning in source code



We are sending the ***value*** of "N"

The bits in the memory location

And we need to be able to id other processes

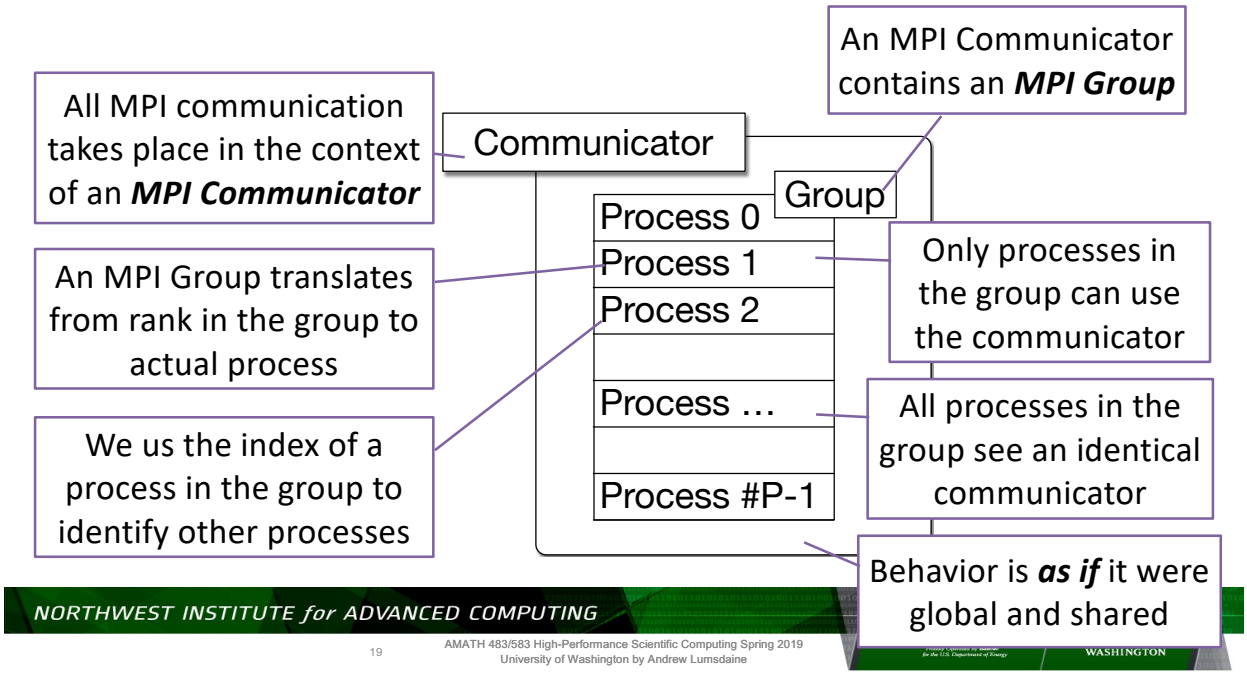
Second question: Where are we sending the bits?

How do we say "N@other_process"?

"N" only has meaning in source code

And its location only makes sense locally

But sender and receiver can agree on an alias


An MPI Communicator contains an MPI Group
All MPI communication takes place in the context of an MPI Communicator
Communicator
Group
Process 0
Process 1
Process 2
Process …
Process #P-1
Only processes in the group can use the communicator
An MPI Group translates from rank in the group to actual process
All processes in the group see an identical communicator
We us the index of a process in the group to identify other processes
Behavior is as if it were global and shared
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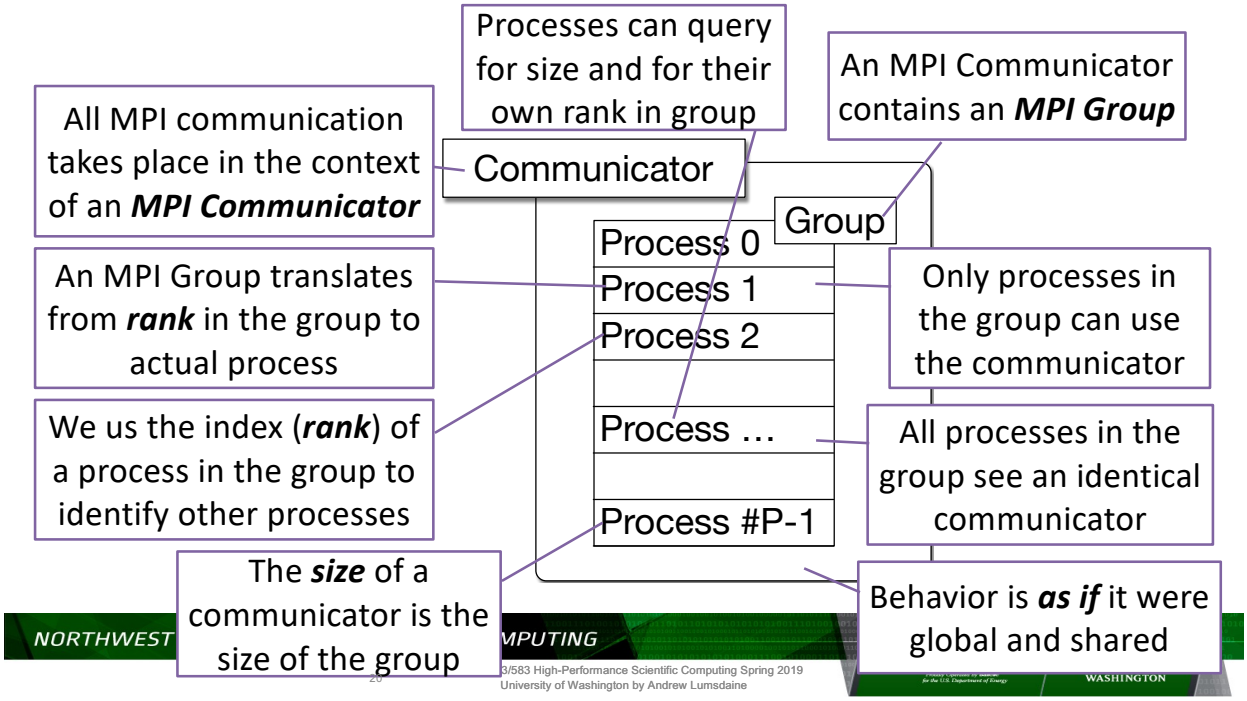
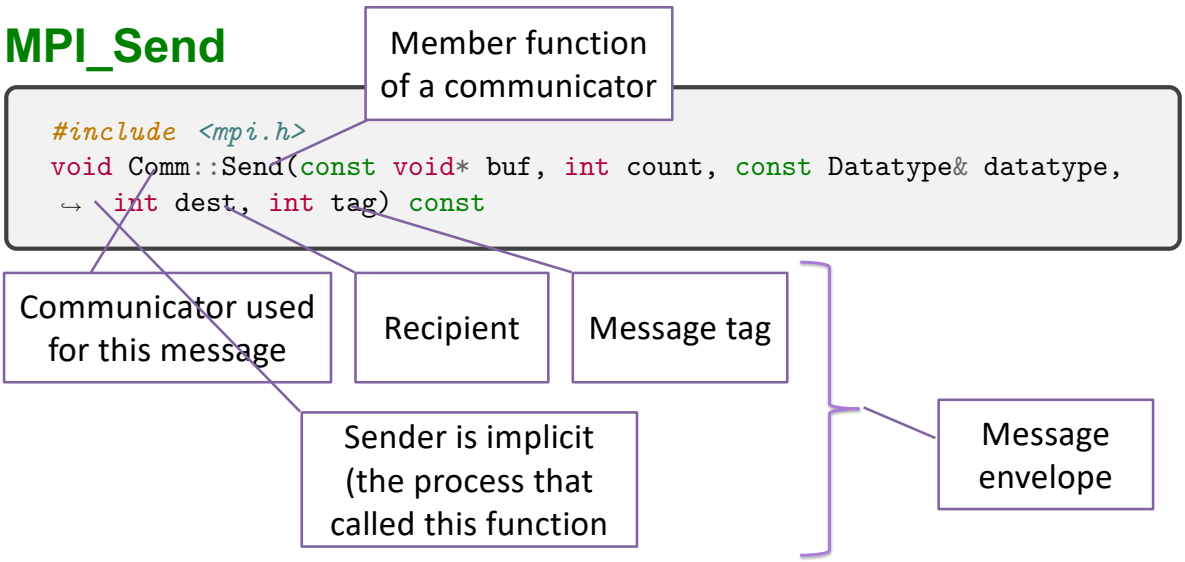
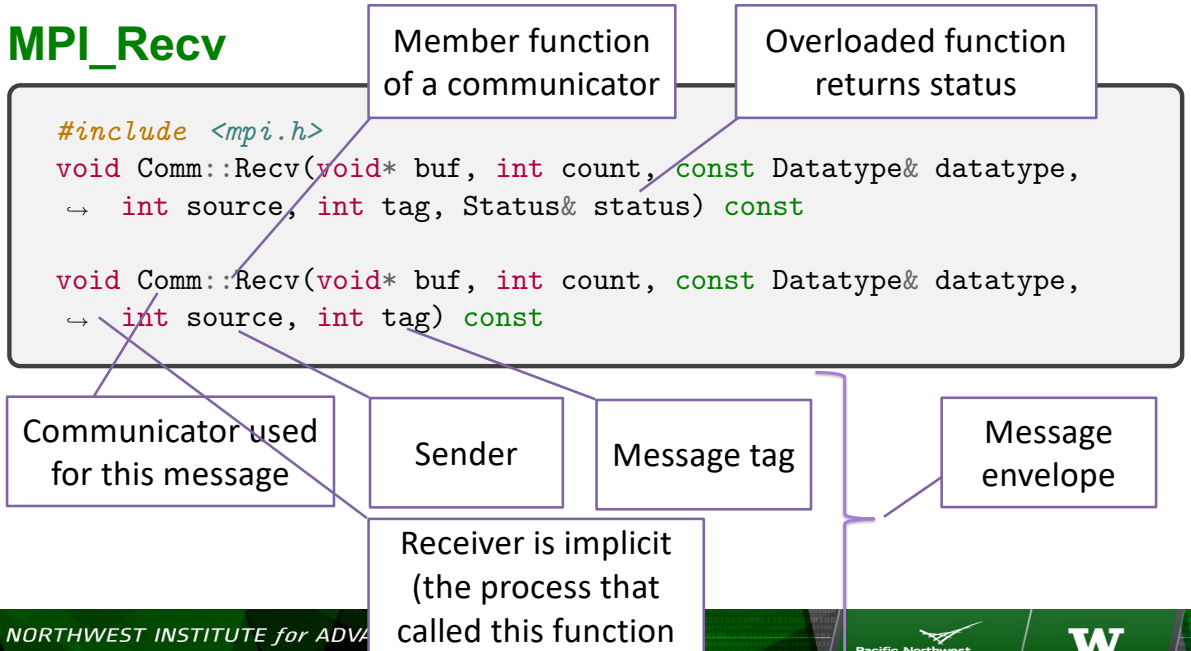
WASHINGTON


Processes can query for size and for their own rank in group
An MPI Communicator contains an MPI Group
All MPI communication takes place in the context of an MPI Communicator
Communicator
Group
Process 0
Process 1
Process 2
Process …
Process #P-1
Only processes in the group can use the communicator
An MPI Group translates from rank in the group to actual process
All processes in the group see an identical communicator
We us the index (rank) of a process in the group to identify other processes
The size of a communicator is the size of the group
Behavior is as if it were global and shared
NORTHWEST
MPUTING
WASHINGTON

MPI_Send



MPI_Recv



MPI_Send and MPI_Recv

```
#include <mpi.h>
void Comm::Send(const void* buf, int count, const Datatype& datatype,
↪ int dest, int tag) const
```

?

Contents

```
#include <mpi.h>
void Comm::Recv(void* buf, int count, const Datatype& datatype,
↪ int source, int tag, Status& status) const

void Comm::Recv(void* buf, int count, const Datatype& datatype,
↪ int source, int tag) const
```

Match these for message delivery

NB: SPMD

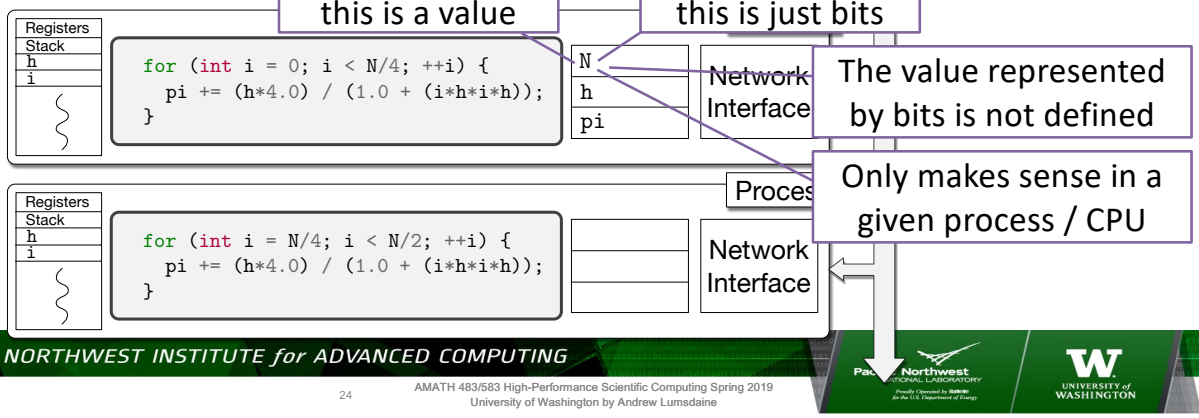
Message Contents

```
#include <mpi.h>
void Comm::Send(const void* buf, int count, const Datatype& datatype,
↪ int dest, int tag) const
```

Contents

In the program this is a value

In the computer this is just bits



The value represented by bits is not defined

Only makes sense in a given process / CPU

Message Contents

```
#include <mpi.h>
void Comm::Send(const void* buf, int count, const Datatype& datatype,
↪  int dest, int tag) const
```

Contents

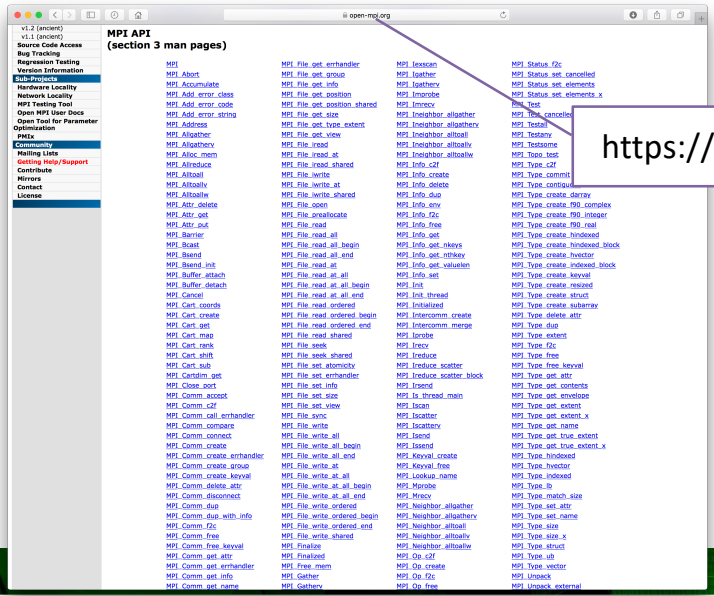
The location in memory of the bits we want to send

How many of the elements are in the message

How to interpret the bits as a data element

Note that contents are not part of envelope

Documentation of All MPI Functions



https://www.open-mpi.org/doc/v1.8/

Six Function MPI (Point to Point)

```
#include <mpi.h>

void MPI::Init(int& argc, char**& argv)
void MPI::Init()

int MPI::Comm::Get_size() const

int MPI::Comm::Get_rank() const

void MPI::Comm::Send(const void* buf, int count, const Datatype&
↪  datatype, int dest, int tag) const

void MPI::Comm::Recv(void* buf, int count, const Datatype& datatype,
↪  int source, int tag, Status& status) const
void MPI::Comm::Recv(void* buf, int count, const Datatype& datatype,
↪  int source, int tag) const

void MPI::Finalize()
```

Initialize MPI environment

Get size of communicator

Get rank of communicator

Send

Receive

Shut down and leave MPI environment

Aside

```
#include <mpi.h>

int MPI_Recv(void *buf, int count, MPI_Datatype datatype,
↪  int source, int tag, MPI_Comm comm, MPI_Status *status)
```

MPI has C bindings for all functions

```
INCLUDE 'mpif.h'

MPI_RECV(BUF, COUNT, DATATYPE, SOURCE, TAG, COMM, STATUS,
↪  IERROR)
    <type>    BUF(*)
    INTEGER    COUNT, DATATYPE, SOURCE, TAG, COMM
    INTEGER    STATUS(MPI_STATUS_SIZE), IERROR
```

And Fortran bindings

MPI Functions

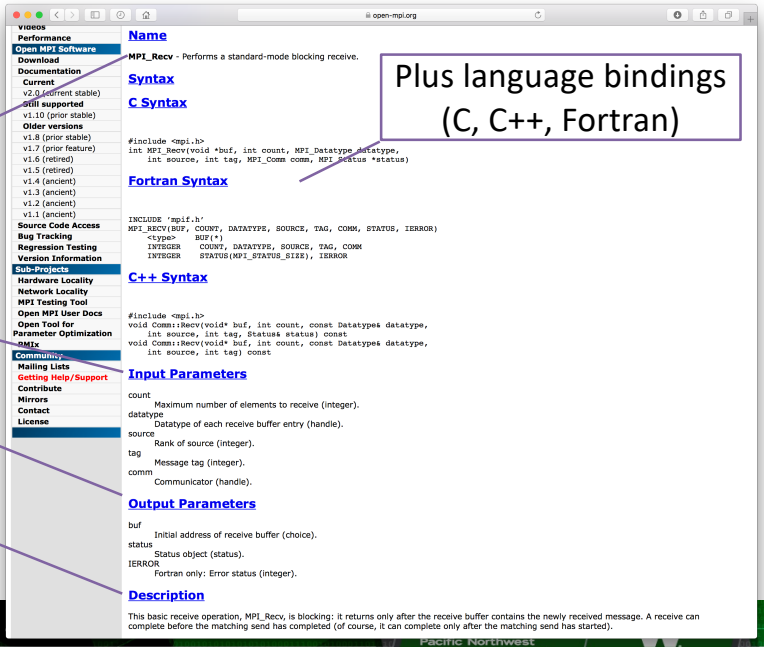
Functions are defined independently of any language

Including parameters

Including parameters

And semantics

Plus language bindings (C, C++, Fortran)



Hello MPI World

Include mpi.h

NB: namespace MPI

Initialize

Get size of communicator

Get rank of calling process

Finalize

```
#include <iostream>
#include <mpi.h>

int main() {

  MPI::Init();

  int mysize = MPI::COMM_WORLD.Get_size();
  int myrank = MPI::COMM_WORLD.Get_rank();

  std::cout << "Hello World!";
  std::cout << "I am " << myrank << " of " << mysize << std::endl;

  MPI::Finalize();

  return 0;
}
```

Hello MPI World

```
#include <iostream>
#include <mpi.h>

int main() {

  MPI::Init();

  int mysize = MPI::COMM_WORLD.Get_size();
  int myrank = MPI::COMM_WORLD.Get_rank();

  std::cout << "Hello World!";
  std::cout << "I am " << myrank << " of " << mysize << std::endl;

  MPI::Finalize();

  return 0;
}
```

MPI defines that a default communicator exists after MPI::Init()

Recall that send, receive, etc all referred to a communicator

Creating other communicators is done by program

Named MPI::COMM_WORLD

MPI::COMM_WORLD is usually sufficient for many programs

Compiling and Running

```
$ mpic++ hello.cpp
```

Usually we use a compiler wrapper set up for local development environment

```
$ mpirun -np 4 ./a.out
```

Launch 4 copies of a.out

```
Hello World! I am 0 of 4
Hello World! I am 3 of 4
Hello World! I am 1 of 4
Hello World! I am 2 of 4
```

Output (printed from all processes since this was local on my laptop)

Compiling and Running

```
$ mpic++ hello.cpp
```

Where did compiler come from? mpi.h? The actual MPI functions?

```
$ mpirun -np 4 ./a.out
```

Where did mpirun come from

- MPI is just a library interface specification (with language bindings)
- It is up the community (researchers, vendors, et al) to provide implementations the conform to the standard specification
- High-quality implementations have useful extensions

Open MPI, MPICH, Intel MPI

Ping Pong

```
int main() {

  MPI::Init();

  int myrank = MPI::COMM_WORLD.Get_rank();
  int mysize = MPI::COMM_WORLD.Get_size();

  int ballsent = 42, ballreceived = 0;
  MPI::COMM_WORLD.Send(&ballsent, 1, MPI::INT, 1, 321);
  MPI::COMM_WORLD.Recv(&ballsent, 1, MPI::INT, 0, 321);

  MPI::COMM_WORLD.Send(&ballreceived, 1, MPI::INT, 0, 321);
  MPI::COMM_WORLD.Recv(&ballreceived, 1, MPI::INT, 1, 321);
  std::cout << "Received " << ballreceived << std::endl;

  MPI::Finalize();

  return 0;
}
```

Ping Pong

```
$ mpic++ pingpong.cpp

$ mpirun -np 2 ./a.out

Received 42
... ^C ....  Process terminated
```


Ping Pong – What Went Wrong?

```
int main() {

  MPI::Init();

  int myrank = MPI::COMM_WORLD.Get_rank();
  int mysize = MPI::COMM_WORLD.Get_size();

  int ballsent = 42, ballreceived = 0;
  MPI::COMM_WORLD.Send(&ballsent, 1, MPI::INT, 1, 321);
  MPI::COMM_WORLD.Recv(&ballsent, 1, MPI::INT, 0, 321);

  MPI::COMM_WORLD.Send(&ballreceived, 1, MPI::INT, 0, 3
  MPI::COMM_WORLD.Recv(&ballreceived, 1, MPI::INT, 1, 3
  std::cout << "Received " << ballreceived << std::endl

  MPI::Finalize();

  return 0;
}
```

All processes run this same program

Both processes send this

And try to receive

Ping

Only process 0 sends this

Only process 0 receives this

Only process 1 receives this

Only process 1 sends this

```
main() {

PI::Init();

int myrank = MPI::COMM_WORLD.Get_rank();
int mysize = MPI::COMM_WORLD.Get_size();

int ballsent = 42, ballreceived = 0;
if (0 == myrank) {
  MPI::COMM_WORLD.Send(&ballsent, 1, MPI::INT, 1, 321);
  MPI::COMM_WORLD.Recv(&ballreceived, 1, MPI::INT, 1, 321);
  std::cout << "Received " << ballreceived << std::endl;
}
if (1 == myrank) {
  MPI::COMM_WORLD.Recv(&ballreceived, 1, MPI::INT, 0, 321);
  MPI::COMM_WORLD.Send(&ballsent, 1, MPI::INT, 0, 321);
}

MPI::Finalize();

return 0;
}
```


Ping Pong 2.0

```
$ mpic++ pingpong.cpp

$ mpirun -np 2 ./a.out

Received 42

$
```

Ping Pong 2.0

```
$ mpic++ pingpong.cpp

$ mpirun -np 8 ./a.out

Received 42

$
```


Six Function MPI (Point to Point)

```
#include <mpi.h>

void MPI::Init(int& argc, char**& argv)
void MPI::Init()

int MPI::Comm::Get_size() const

int MPI::Comm::Get_rank() const

void MPI::Comm::Send(const void* buf, int count, const Datatype&
↪  datatype, int dest, int tag) const

void MPI::Comm::Recv(void* buf, int count, const Datatype& datatype,
↪  int source, int tag, Status& status) const
void MPI::Comm::Recv(void* buf, int count, const Datatype& datatype,
↪  int source, int tag) const

void MPI::Finalize()
```

Initialize MPI environment

Get size of communicator

Get rank of communicator

Send

Receive

Shut down and leave MPI environment

Ping

Only process 0 sends this

Only process 0 receives this

Only process 1 receives this

Only process 1 sends this

```
main() {

PI::Init();

int myrank = MPI::COMM_WORLD.Get_rank();
int mysize = MPI::COMM_WORLD.Get_size();

int ballsent = 42, ballreceived = 0;
if (0 == myrank) {
  MPI::COMM_WORLD.Send(&ballsent, 1, MPI::INT, 1, 321);
  MPI::COMM_WORLD.Recv(&ballreceived, 1, MPI::INT, 1, 321);
  std::cout << "Received " << ballreceived << std::endl;
}
if (1 == myrank) {
  MPI::COMM_WORLD.Recv(&ballreceived, 1, MPI::INT, 0, 321);
  MPI::COMM_WORLD.Send(&ballsent, 1, MPI::INT, 0, 321);
}

MPI::Finalize();

return 0;
}
```


Six Function MPI Point to Point Version

```
#include <mpi.h>

void MPI::Init(int& argc, char**& argv)
void MPI::Init()

int MPI::Comm::Get_size() const

int MPI::Comm::Get_rank() const

void MPI::Comm::Send(const void* buf, int count, const Datatype&
↪  datatype, int dest, int tag) const

void MPI::Comm::Recv(void* buf, int count, const Datatype& datatype,
↪  int source, int tag, Status& status) const
void MPI::Comm::Recv(void* buf, int count, const Datatype& datatype,
↪  int source, int tag) const

void MPI::Finalize()
```

The Other Six Fun

Broadcast values to all nodes

All nodes to exactly this

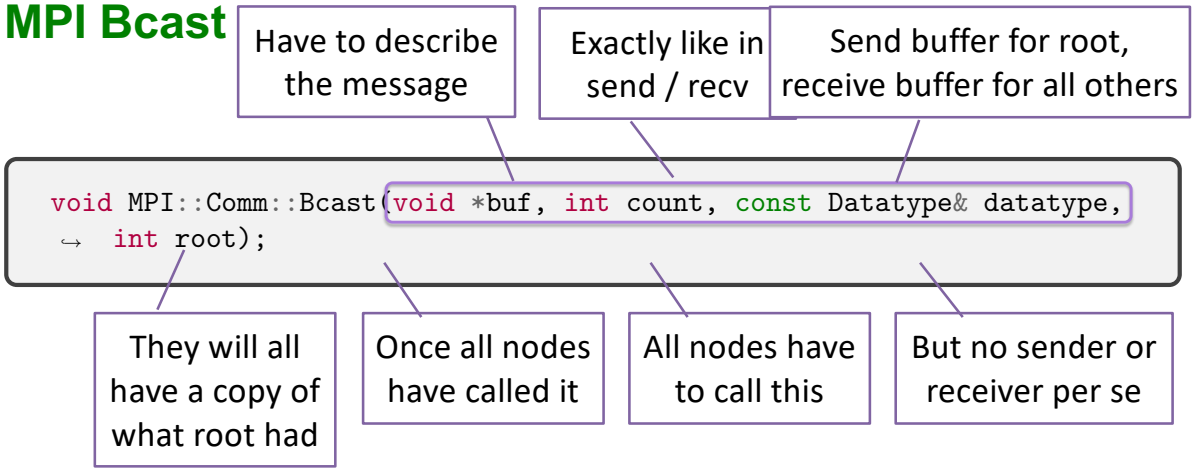
Collect results from all nodes

```
int main(int argc, char* argv[]) {
  size_t intervals = 1024 * 1024;

  MPI::Init();

  int myrank = MPI::COMM_WORLD.Get_rank();
  int mysize = MPI::COMM_WORLD.Get_size();

  if (0 == myrank) if (argc >= 2) intervals = std::atol(argv[1]);

  MPI::COMM_WORLD.Bcast(&intervals, 1, MPI::UNSIGNED_LONG, 0);

  size_t blocksize = intervals / mysize;
  size_t begin     = blocksize * myrank;
  size_t end       = blocksize * (myrank + 1);
  double h         = 1.0 / ((double)intervals);

  double pi        = 0.0;
  for (size_t i = begin; i < end; ++i)
    pi += 4.0 / (1.0 + (i * h * i * h));

  MPI::COMM_WORLD.Reduce(&mypi, &pi, 1, MPI::DOUBLE, MPI::SUM, 0);

  if (0 == myrank) std::cout << "pi is approximately " << pi << std::endl;

  MPI::Finalize();

  return 0;
}
```

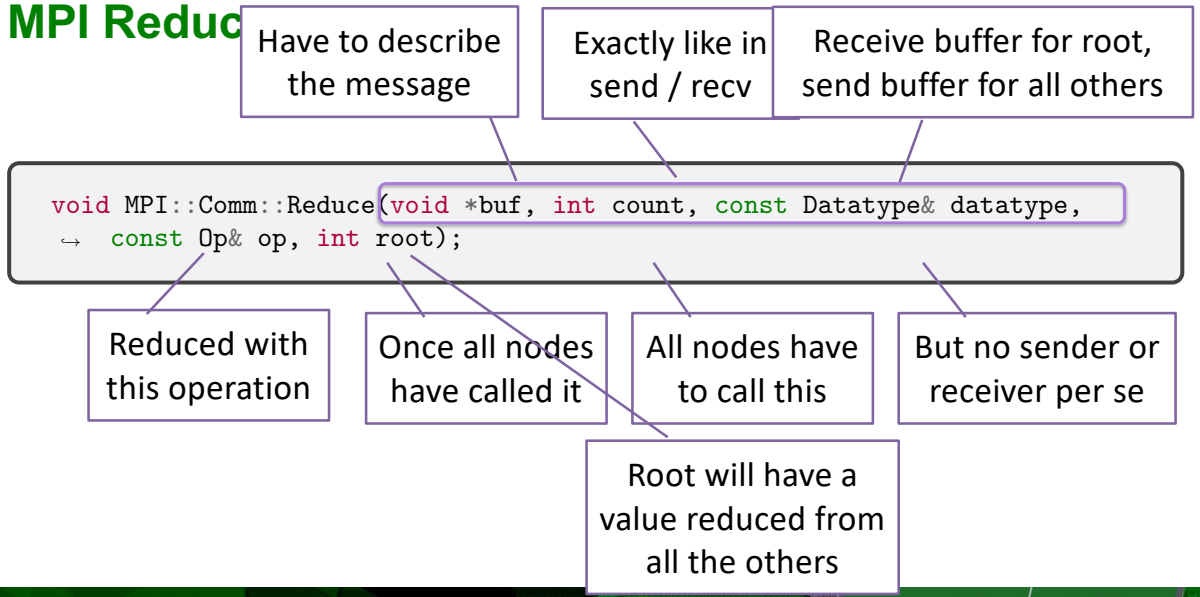

Six Function MPI Collective Version

```
#include <mpi.h>

void MPI::Init(int& argc, char**& argv)
void MPI::Init()

int MPI::Comm::Get_size() const
int MPI::Comm::Get_rank() const

void MPI::Comm::Bcast(void *buf, int count, const Datatype& datatype,
↪  int root);
void MPI::Comm::Reduce(void *buf, int count, const Datatype&
↪  datatype, const Op& op, int root);

void MPI::Finalize()
```

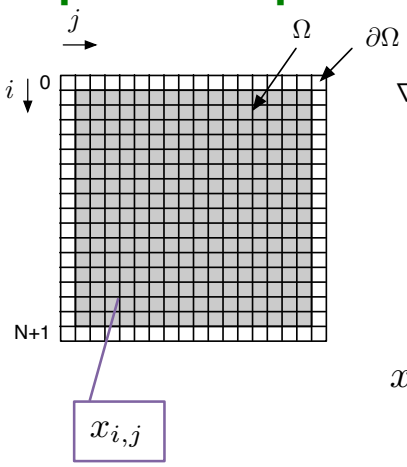
MPI Bcast



MPI Reduc



Laplace's Equation on a Regular Grid



$$\begin{aligned} \nabla^2 \phi &= 0 \quad \text{on } \Omega \\ \phi &= f \quad \text{on } \partial\Omega \end{aligned}$$

Discretization

$$x_{i-1,j} + x_{i+1,j} + x_{i,j-1} + x_{i,j+1} - 4x_{i,j} = 0$$

$$\frac{1}{h^2}\begin{bmatrix} 4 & -1 & \cdots & -1 & & \\ -1 & \ddots & \ddots & \ddots & \ddots & \\ \vdots & \ddots & \ddots & \ddots & \ddots & -1 \\ -1 & \ddots & \ddots & \ddots & \ddots & \vdots \\ & \ddots & \ddots & \ddots & \ddots & -1 \\ & & -1 & \cdots & -1 & 4 \end{bmatrix} \begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ \vdots \\ \end{bmatrix} = \begin{bmatrix} b_0 \\ b_1 \\ b_2 \\ \vdots \\ \end{bmatrix}$$

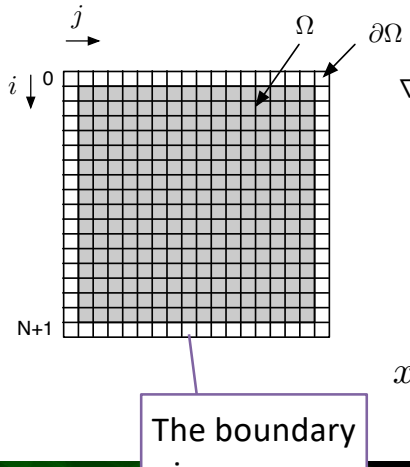
$$x_{i,j} = (x_{i-1,j} + x_{i+1,j} + x_{i,j-1} + x_{i,j+1})/4$$

The value of each point on the grid

The average of its neighbors

Laplace's Equation on a Regular Grid

Why isn't 0 the solution?



$$\begin{aligned} \nabla^2 \phi &= 0 \quad \text{on } \Omega \\ \phi &= f \quad \text{on } \partial\Omega \end{aligned}$$

Discret

The boundary is non-zero

$$\frac{1}{h^2}\begin{bmatrix} 4 & -1 & \cdots & -1 & & \\ -1 & \ddots & \ddots & \ddots & \ddots & \\ \vdots & \ddots & \ddots & \ddots & \ddots & -1 \\ -1 & \ddots & \ddots & \ddots & \ddots & \vdots \\ & \ddots & \ddots & \ddots & \ddots & -1 \\ & & -1 & \cdots & -1 & 4 \end{bmatrix} \begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ \vdots \\ \end{bmatrix} = \begin{bmatrix} b_0 \\ b_1 \\ b_2 \\ \vdots \\ \end{bmatrix}$$

Non-zeros in here due to boundary

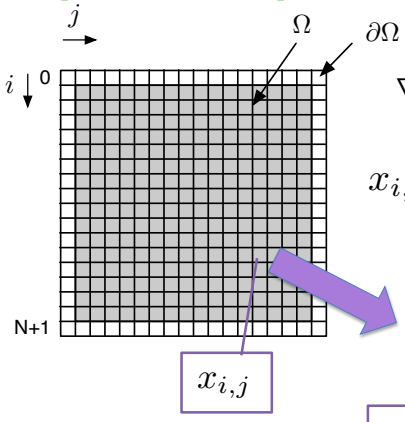
$$x_{i,j} = (x_{i-1,j} + x_{i+1,j} + x_{i,j-1} + x_{i,j+1})/4$$

The boundary is non-zero

The value of each point on the grid

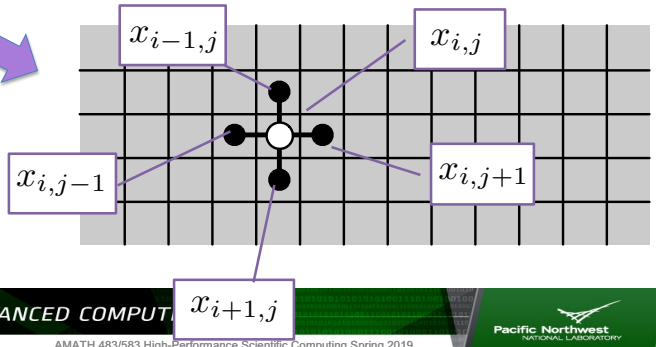
The average of its neighbors

Laplace's Equation on a Regular Grid



$$\begin{aligned} \nabla^2\phi &= 0 \quad \text{on } \Omega \\ \phi &= f \quad \text{on } \partial\Omega \end{aligned}$$

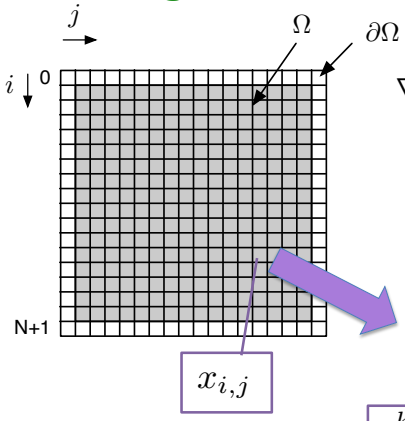
$$x_{i,j} = (x_{i-1,j} + x_{i+1,j} + x_{i,j-1} + x_{i,j+1})/4$$



Iterating for a solution

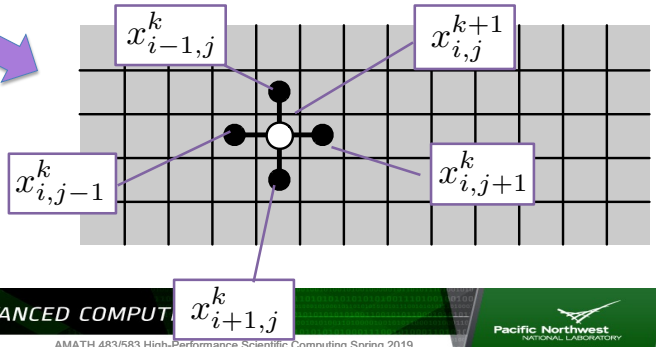
Approximation at iteration $k+1$

Average of approximation at iteration k



$$\begin{aligned} \nabla^2\phi &= 0 \quad \text{on } \Omega \\ \phi &= f \quad \text{on } \partial\Omega \end{aligned}$$

$$x^{k+1}_{i,j} = (x^k_{i-1,j} + x^k_{i+1,j} + x^k_{i,j-1} + x^k_{i,j+1})/4$$



Iterating for a soluti

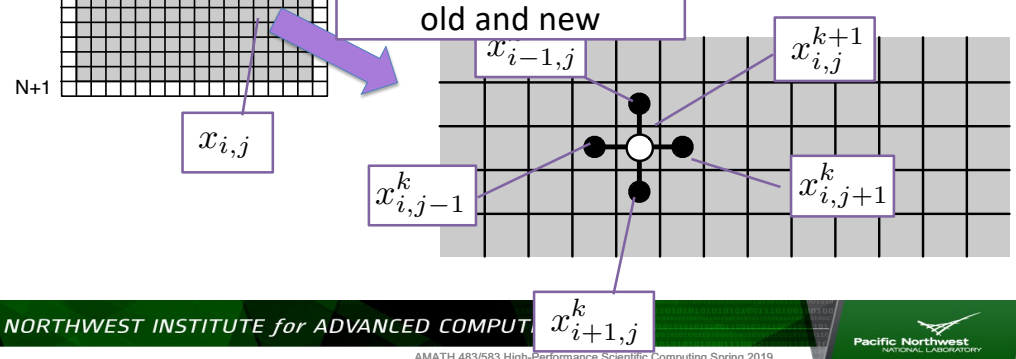
```
while (! converged())
  for (size_t i = 1; i < N+1; ++i)
    for (size_t j = 1; j < N+1; ++j)
      y(i,j) = (x(i-1,j) + x(i+1,j) + x(i,j-
  swap(x,y);
}
```

Approximation at iteration k+1

Average of approximation at iteration k

At end of each outer iteration: new becomes old (and v.v.)

Only need to use two arrays to do iteration: old and new



class Grid

```
class Grid {

public:
  explicit Grid(size_t x, size_t y)                :
      xPoints(x+2), yPoints(y+2), arrayData(xPoints*yPoints) {}

        double &operator()(size_t i, size_t j)
            { return arrayData[i*yPoints + j]; }
  const double &operator()(size_t i, size_t j) const
            { return arrayData[i*yPoints + j]; }

  size_t numX() const { return xPoints; }
  size_t numY() const { return yPoints; }

private:
  size_t xPoints, yPoints;
  std::vector<double> arrayData;
};
```

Grid is a 2D array

Constructor

Accessor

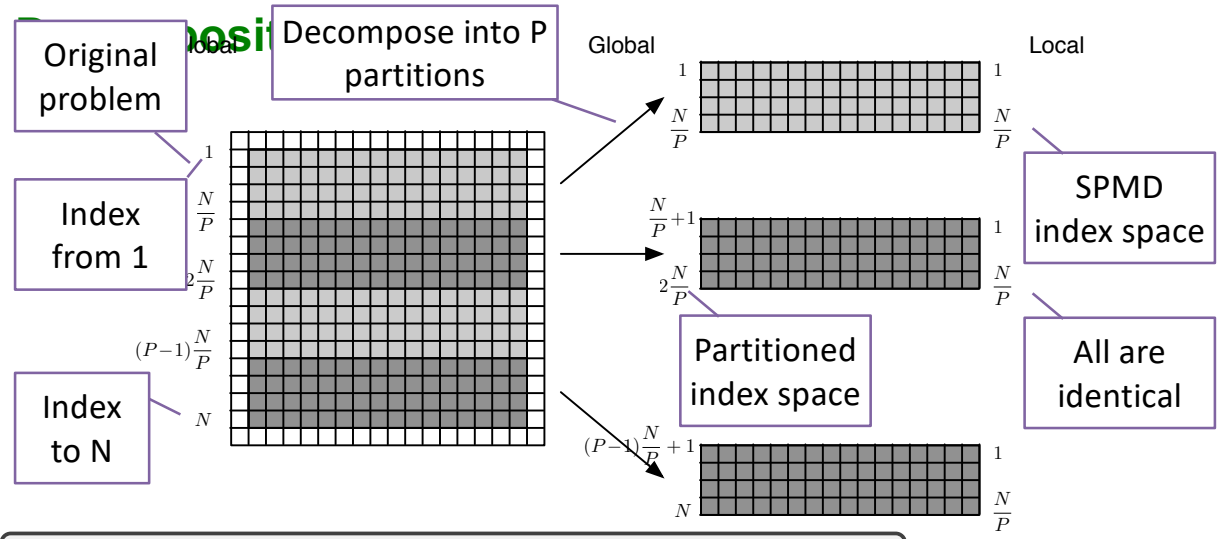
Storage

Main Sequential Jacobi Sweep

```
double jacobiStep(const Grid& x, Grid& y) {
  assert(x.numX() == y.numX() && x.numY() == y.numY());
  double rnorm = 0.0;

  for (size_t i = 1; i < x.numX()-1; ++i) {
    for (size_t j = 1; j < x.numY()-1; ++j) {
      y(i, j) = (x(i-1, j) + x(i+1, j) + x(i, j-1) + x(i, j+1))/4.0;
      rnorm += (y(i, j) - x(i, j)) * (y(i, j) - x(i, j));
    }
  }

  return std::sqrt(rnorm);
}
```


Sequential Jacobi Solver

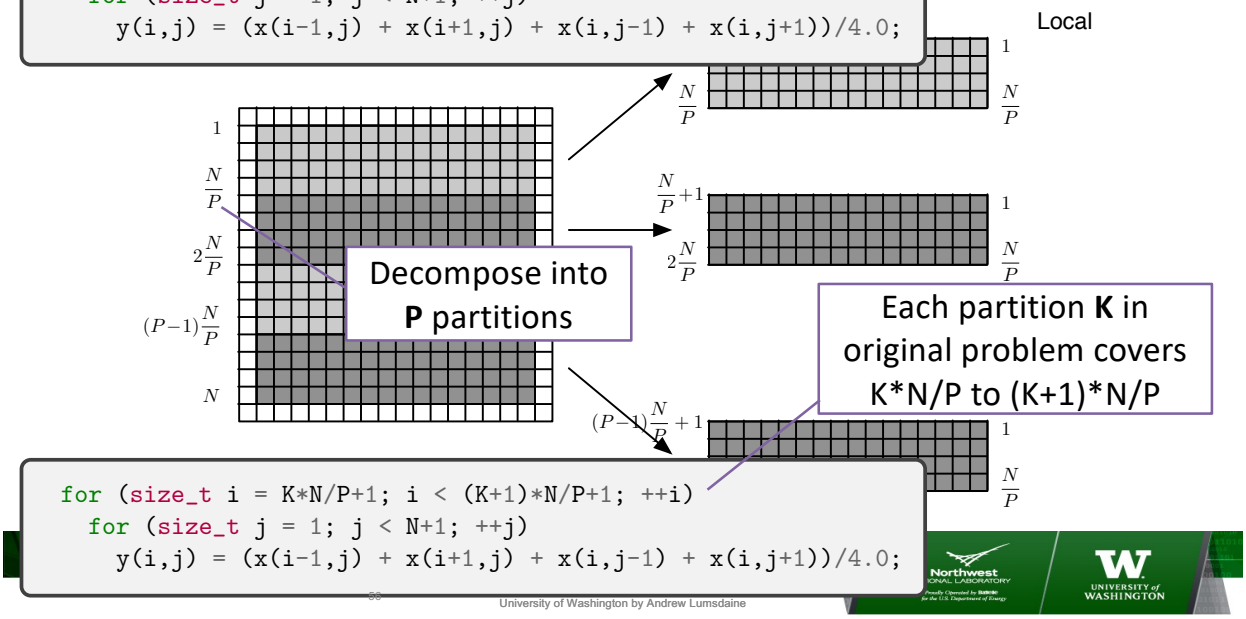
```
int jacobi(Grid& X0, Grid& X1, size_t max_iters, double tol) {
  for (size_t iter = 0; iter < max_iters; ++iter) {
    double rnorm = jacobiStep(X0, X1);
    if (rnorm < tol) return 0;
    swap(X0, X1);
  }
  return -1;
}
```




```
for (size_t i = 1; i < N+1; ++i)
  for (size_t j = 1; j < N+1; ++j)
    y(i,j) = (x(i-1,j) + x(i+1,j) + x(i,j-1) + x(i,j+1))/4.0;
```

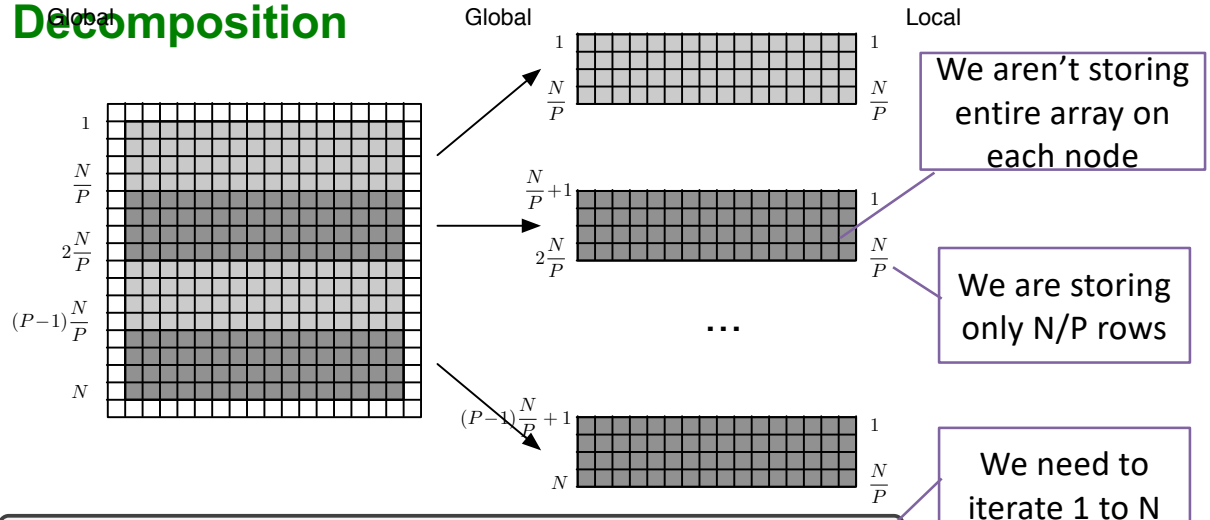


```
for (size_t i = 1; i < N+1; ++i)
  for (size_t j = 1; j < N+1; ++j)
    y(i,j) = (x(i-1,j) + x(i+1,j) + x(i,j-1) + x(i,j+1))/4.0;
```



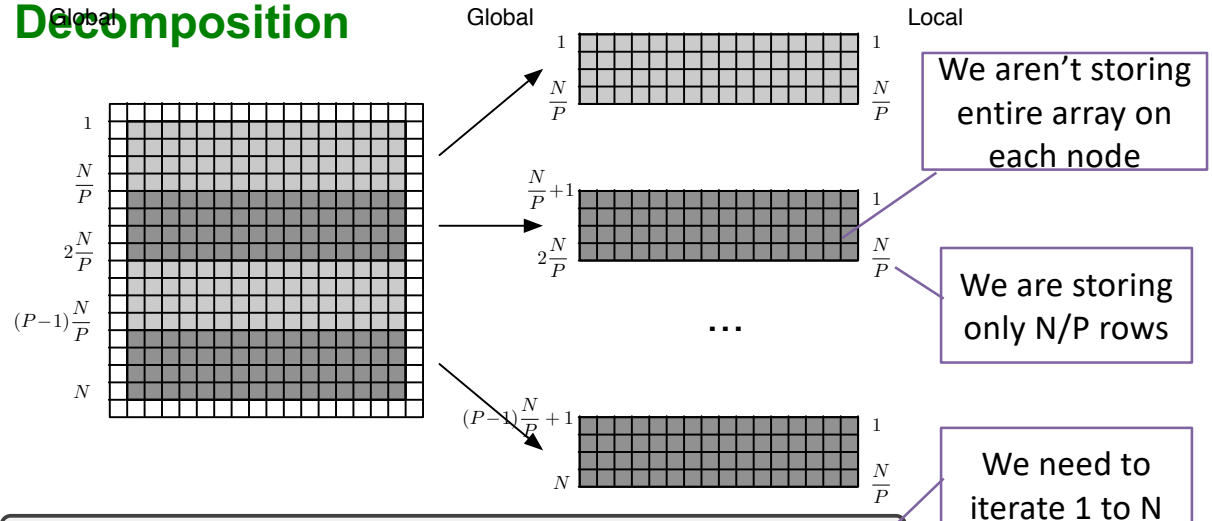

```
for (size_t i = K*N/P+1; i < (K+1)*N/P+1; ++i)
  for (size_t j = 1; j < N+1; ++j)
    y(i,j) = (x(i-1,j) + x(i+1,j) + x(i,j-1) + x(i,j+1))/4.0;
```

Decomposition




```
for (size_t i = K*N/P+1; i < (K+1)*N/P+1; ++i)
  for (size_t j = 1; j < N+1; ++j)
    y(i,j) = (x(i-1,j) + x(i+1,j) + x(i,j-1) + x(i,j+1))/4.0;
```

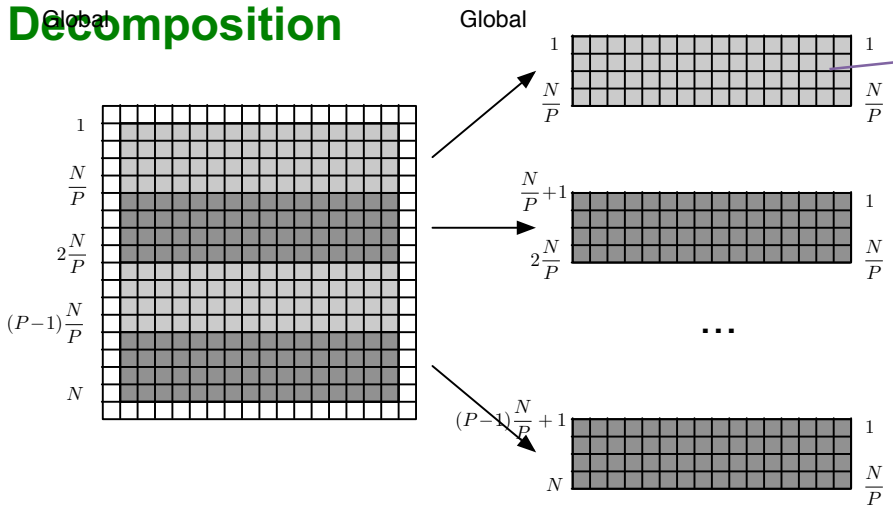

Decomposition




```
for (size_t i = K*N/P+1; i < (K+1)*N/P+1; ++i)
  for (size_t j = 1; j < N+1; ++j)
    y(i,j) = (x(i-1,j) + x(i+1,j) + x(i,j-1) + x(i,j+1))/4.0;
```

University of Washington by Andrew Lumsdaine

Decomposition

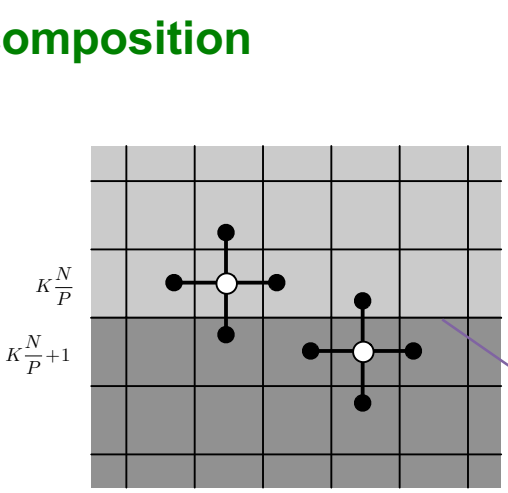


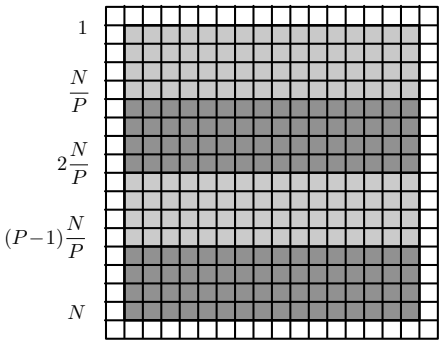
Any obvious problems?

We need to iterate 1 to N

```
for (size_t i = 1; i < N/P+1; ++i)
  for (size_t j = 1; j < N+1; ++j)
    y(i,j) = (x(i-1,j) + x(i+1,j) + x(i,j-1) + x(i,j+1))/4.0;
```

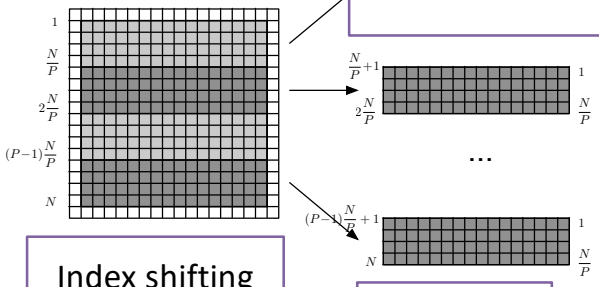

Decomposition





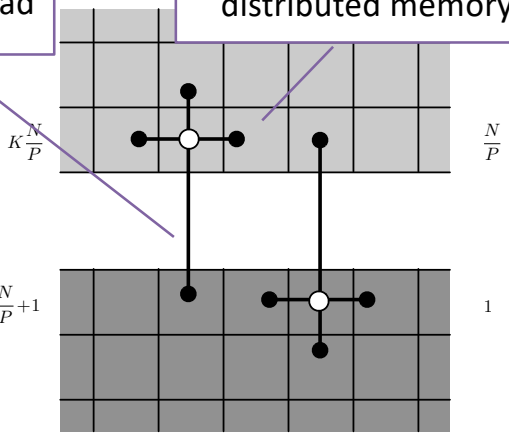
Data dependencies for stencil crosses partition boundary in original problem

Decomposition



This is not a valid read

Which is a problem in distributed memory



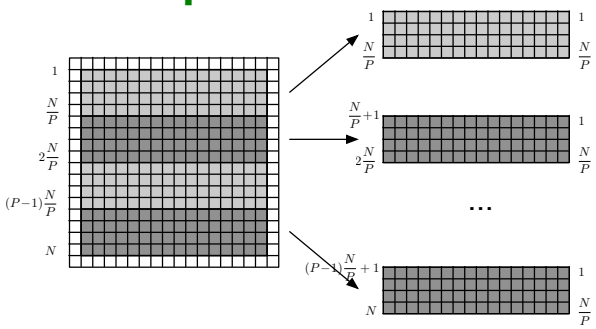
Index shifting doesn't help

This is just a program

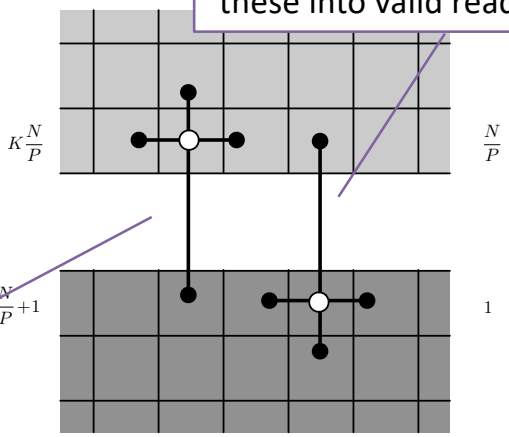
```
for (size_t i = 1; i < N/P+1; ++i)
  for (size_t j = 1; j < N+1; ++j)
    y(i,j) = (x(i-1,j) + x(i+1,j) + x(i,j-1) + x(i,j+1))/4
```

It reads/write local data, just like any other

Decomposition



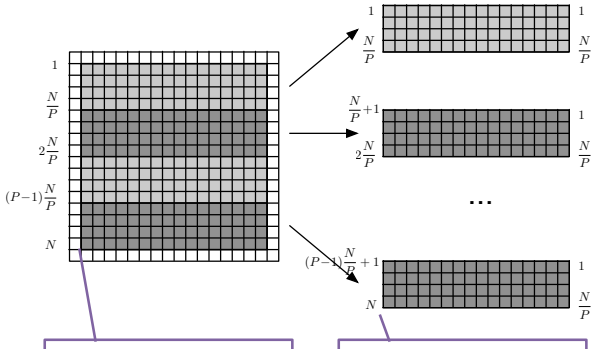
We need to make these into valid reads



And preserve the "as-if" property

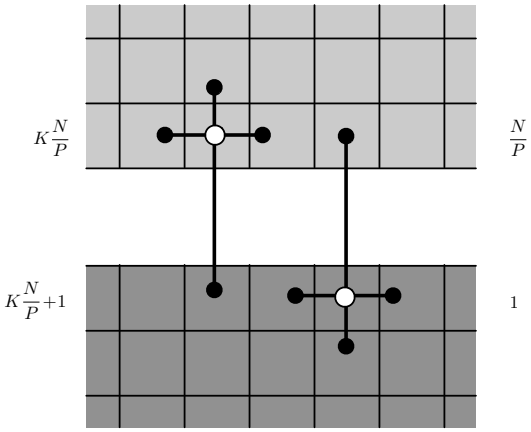
```
for (size_t i = 1; i < N/P+1; ++i)
  for (size_t j = 1; j < N+1; ++j)
    y(i,j) = (x(i-1,j) + x(i+1,j) + x(i,j-1) + x(i,j+1))/4.0;
```

Decomposition



Where did the boundary go

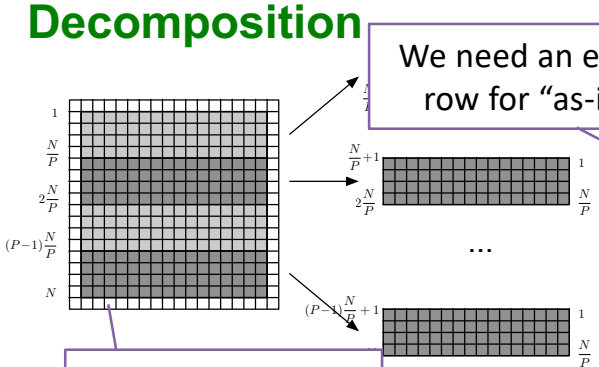
When we partitioned?



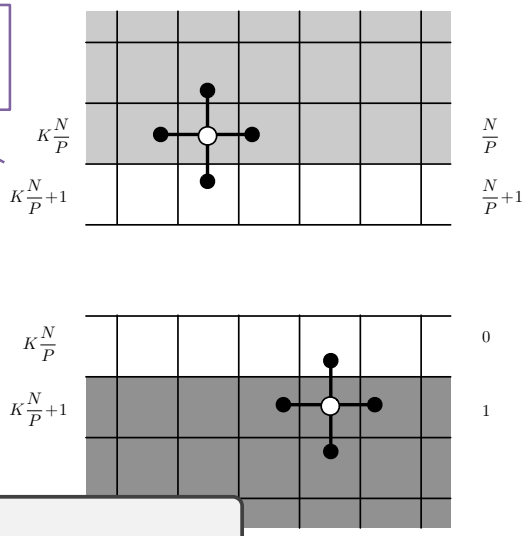

```
for (size_t i = 1; i < N/P+1; ++i)
  for (size_t j = 1; j < N+1; ++j)
    y(i,j) = (x(i-1,j) + x(i+1,j) + x(i,j-1) + x(i,j+1))/4.0;
```


Decomposition

We need an extra row for “as-if”

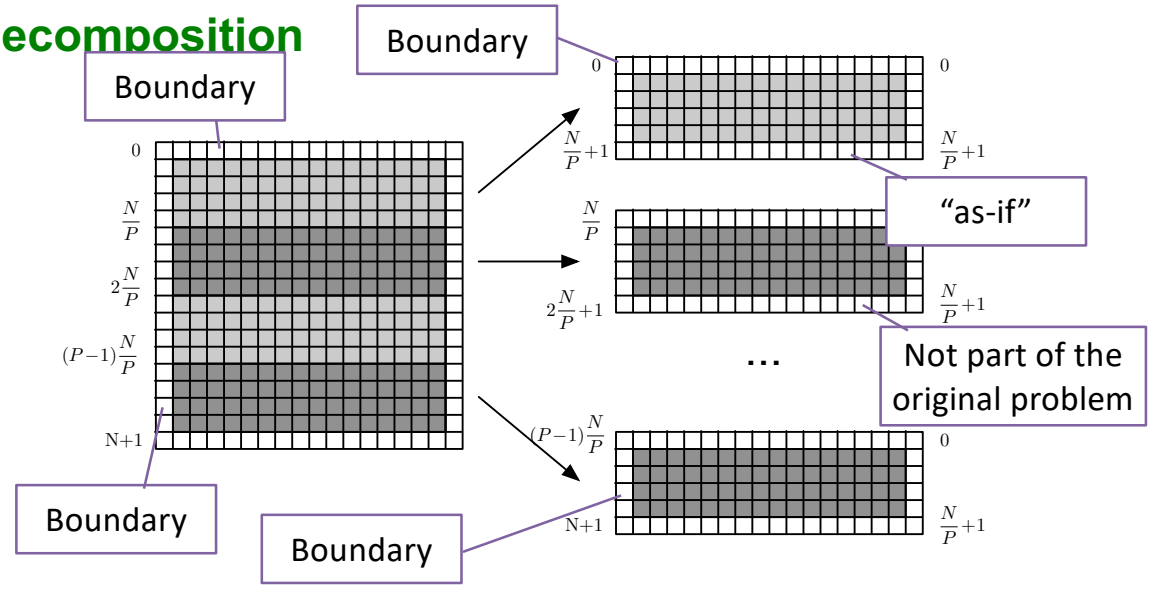


And an extra row (on some nodes) for the boundary

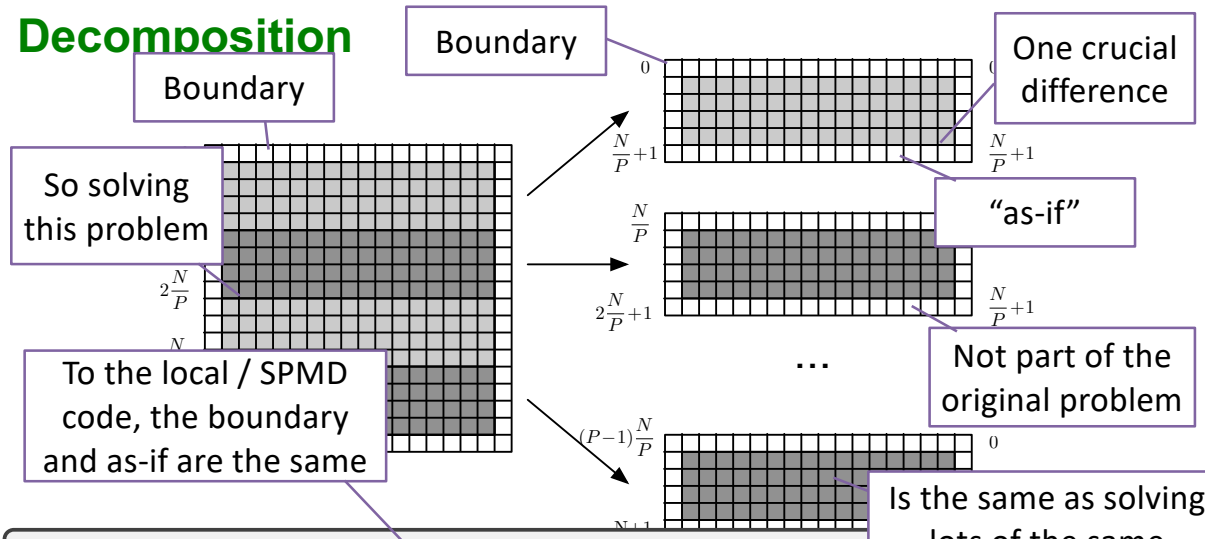



```
for (size_t i = 1; i < N/P+1; ++i)
  for (size_t j = 1; j < N+1; ++j)
    y(i,j) = (x(i-1,j) + x(i+1,j) + x(i,j-1) + x(i,j+1))/4.0;
```

Decomposition

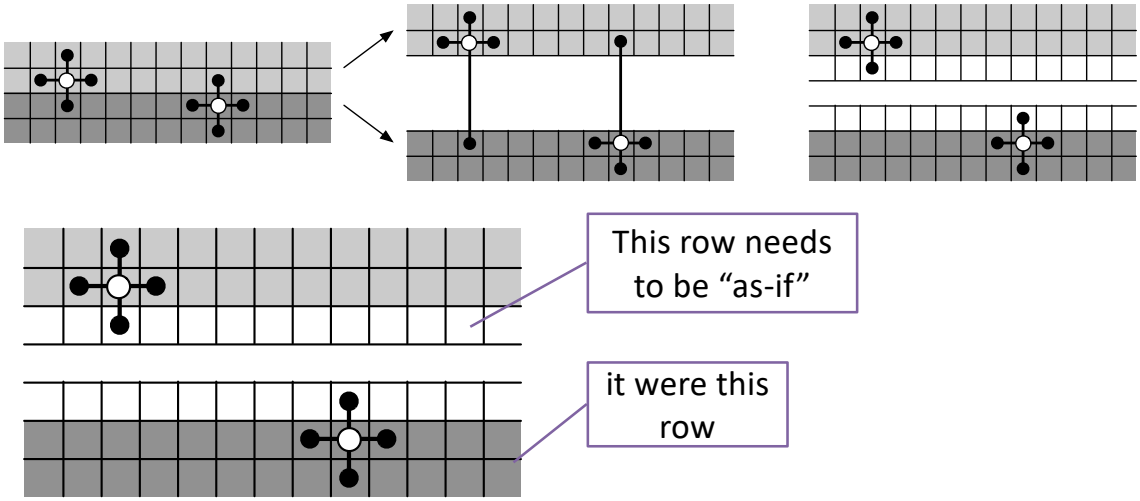


Decomposition

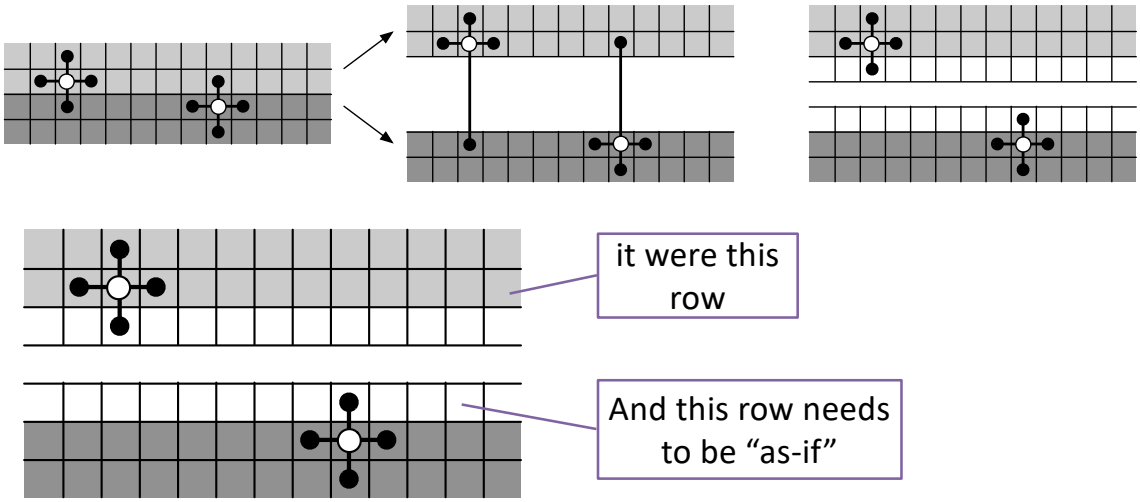



```
for (size_t i = 1; i < N/P+1; ++i)
  for (size_t j = 1; j < N+1; ++j)
    y(i,j) = (x(i-1,j) + x(i+1,j) + x(i,j-1) + x(i,j+1))/4.0;
```

As-If

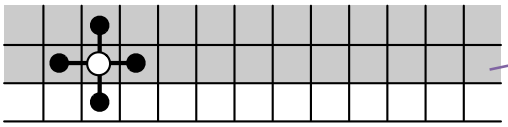

This row needs to be “as-if”
it were this row

As-If


it were this row
And this row needs to be “as-if”

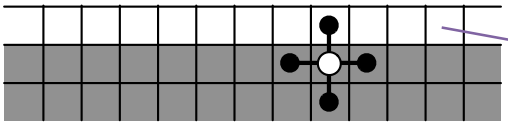
As-If

```
for (size_t i = 1; i < N/P+1; ++i)
  for (size_t j = 1; j < N+1; ++j)
    y(i,j) = (x(i-1,j) + x(i+1,j) + x(i,j-1) + x(i,j+1))/4.0;
```



it were this row

This is the computation

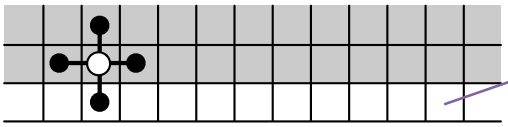


And this row needs to be “as-if”

As-If

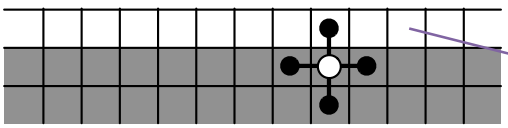
This is the computation

```
for (size_t i = 1; i < N/P+1; ++i)
  for (size_t j = 1; j < N+1; ++j)
    y(i,j) = (x(i-1,j) + x(i+1,j) + x(i,j-1) + x(i,j+1))/4.0;
```



Note these are not changed during an iteration

Only when it is read



Does this row ***always*** have to have the same value as the other row?

```
while (! converged()) {
  for (size_t i = 1; i < N+1; ++i)
    for (size_t j = 1; j < N+1; ++j)
      y(i,j) = (x(i-1,j) + x(i+1,j) + x(i,j-1) + x(i,j+1))/4.0;
  swap(x,y);
}
```

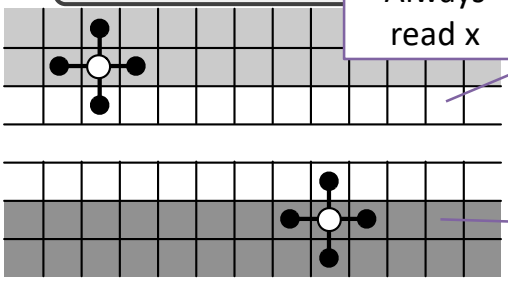
Always write y

This is the entire program

Always read x

Not changed during an iteration

Rows need to be as-if only during iteration



This changes only on every outer iteration (on the swap())

As-If

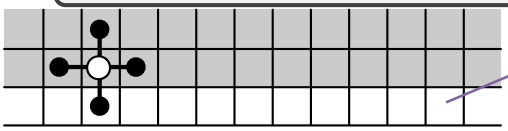
```
while (! conve
  for (size_t
    for (size_
      y(i,j) = (x(i-1,j) + x(i+1,j) + x(i,j-1) + x(i,j+1))/4.0;
  swap(x,y);
}
```

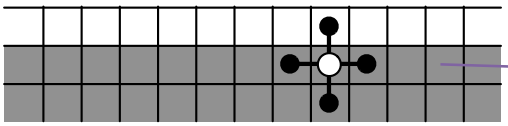
Here is where we need to make as-if true

This is the entire program

Not changed during an iteration

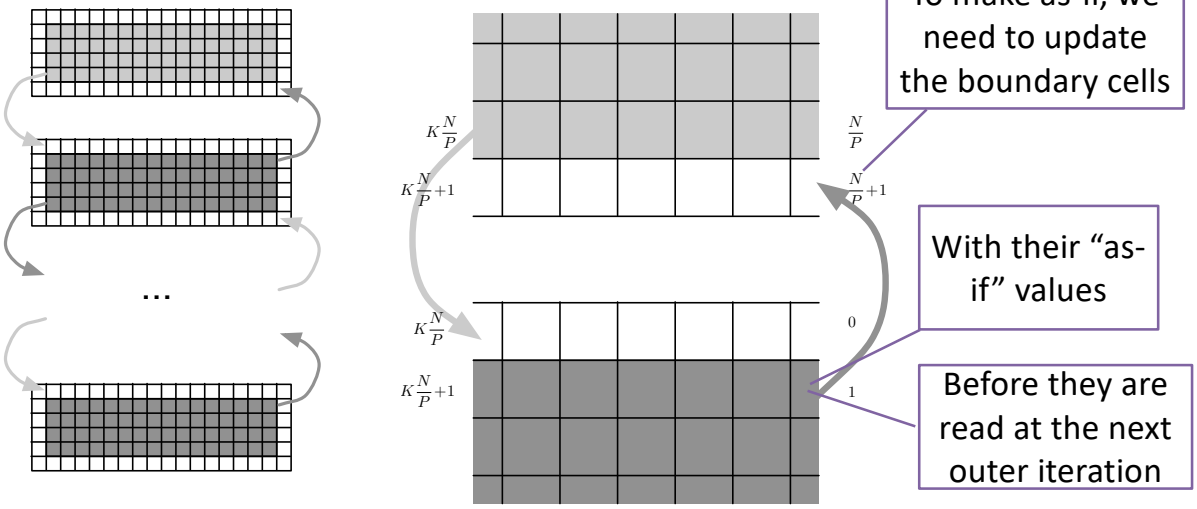
Rows need to be as-if only during iteration





This changes only on every outer iteration (on the swap())

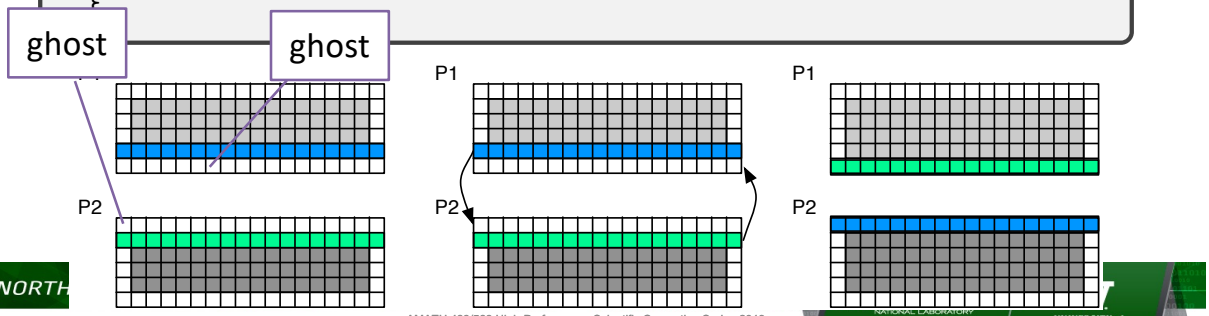
Compute / Communicate



Compute / Communicate

Standard terminology for as-if boundary is "ghost cell"

```
while (! converged()) {
  for (size_t i = 1; i < N+1; ++i)
    for (size_t j = 1; j < N+1; ++j)
      y(i,j) = (x(i-1,j) + x(i+1,j) + x(i,j-1) + x(i,j+1))/4.0;
  swap(x,y);
  make_as_if(x); // Communicate ghost cells
}
```



Compute / Communicate

```
while (! converged()) {
  for (size_t i = 1; i < N+1; ++i)
    for (size_t j = 1; j < N+1; ++j)
      y(i,j) = (x(i-1,j) + x(i+1,j) + x(i,j-1) + x(i,j+1))/4.0;
  swap(x,y);
  make_as_if(x); // Communicate ghost cells
}
```

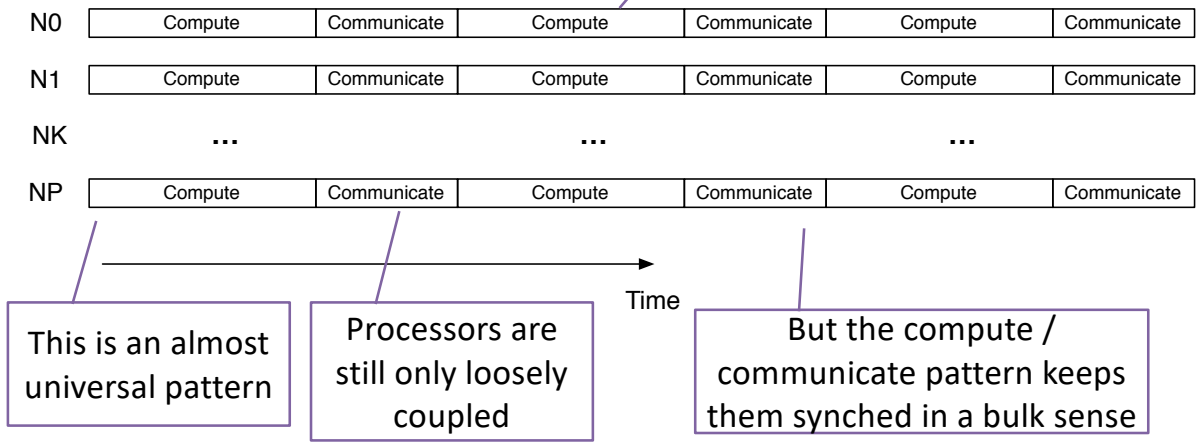
Compute

This is an almost universal pattern

Communicate

Compute / Communicate

"Bulk Synchronous Parallel" (BSP)



This is an almost universal pattern

Processors are still only loosely coupled

But the compute / communicate pattern keeps them synched in a bulk sense

Parallel Jacobi Solver

```
int jacobi(Grid& X0, Grid& X1, size_t max_iters, double tol) {
  for (size_t iter = 0; iter < max_iters; ++iter) {
    double rnorm = jacobiStep(X0, X1);
    if (rnorm < tol) return 0;
    swap(X0, X1);
  }
  return -1;
}
```

As-if: This needs to happen on all nodes all_reduce instead of reduce

As-if: Update ghost cells

Parallel Jacobi Step

```
double jacobiStep(const Grid& x, Grid& y) {
  assert(x.numX() == y.numX() && x.numY() == y.numY());
  double rnorm = 0.0;

  for (size_t i = 1; i < x.numX()-1; ++i) {
    for (size_t j = 1; j < x.numY()-1; ++j) {
      y(i, j) = (x(i-1, j) + x(i+1, j) + x(i, j-1) + x(i, j+1))/4.0;
      rnorm += (y(i, j) - x(i, j)) * (y(i, j) - x(i, j));
    }
  }

  return std::sqrt(rnorm);
}
```

MPI Allreduce

Just like reduce

So there are two buffers

But when call is completed ***all*** nodes have reduced value

```
void MPI::Comm::Allreduce(const void* sendbuf, void* recvbuf,
    int count, const MPI::Datatype& datatype, const MPI::Op& op)
```

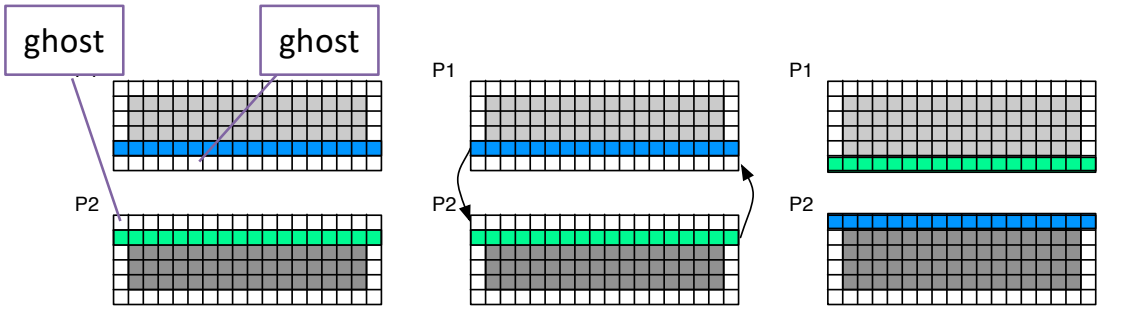

Parallel Jacobi Solver

```
int jacobi(Grid& X0, Grid& X1, size_t max_iters, double tol) {
  for (size_t iter = 0; iter < max_iters; ++iter) {
    double lnorm = jacobiStep(X0, X1);
    double rnorm = 0.0;
    MPI_COMM_WORLD.Allreduce(&rnorm, &lnorm, 1, MPI::DOUBLE, MPI::SUM);
    if (rnorm < tol) return 0;
    swap(X0, X1);
    update_ghosts(X0);
  }
  return -1;
}
```

As-if: This needs to happen on all nodes all_reduce instead of reduce

As-if: Update ghost cells

Updating Ghost Cells

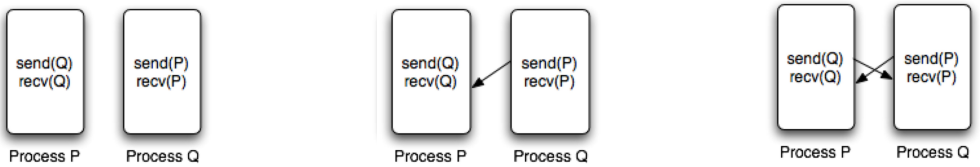



```
MPI_Send( ... );   // to upper neighbor
MPI_Send( ... );   // to lower neighbor
MPI_Send( ... );   // from lower neighbor
MPI_Send( ... );   // from upper neighbor
```

Works?

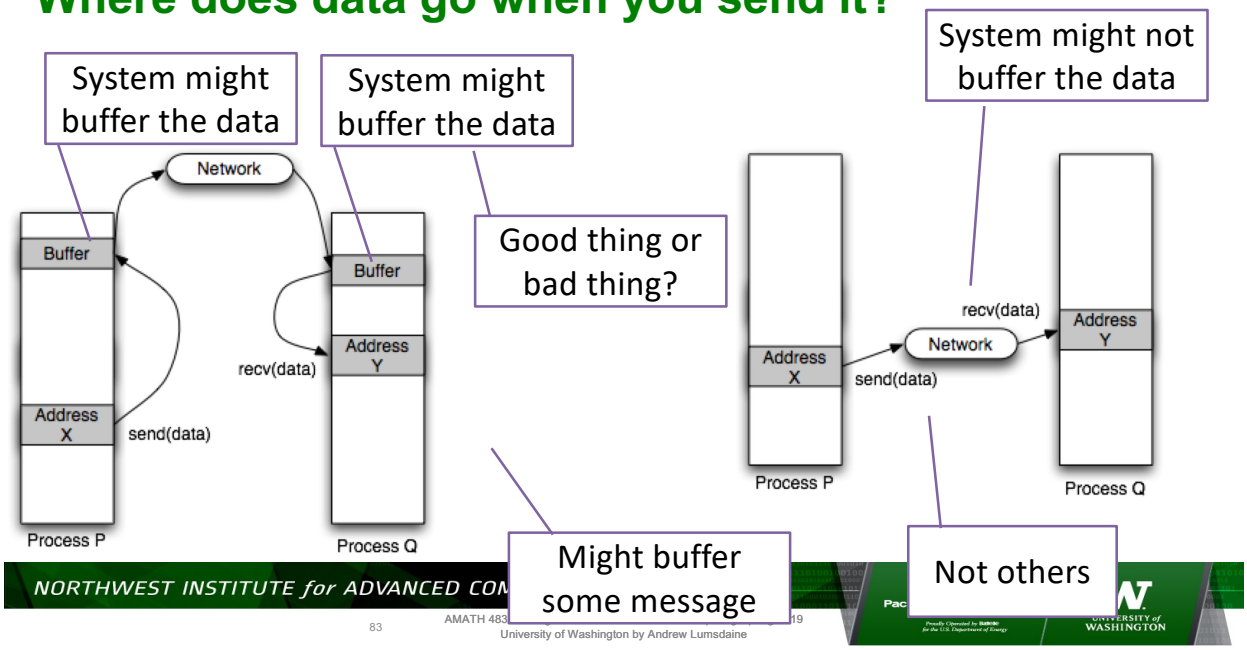
Updating ghost cells

- What happens with following sequence of communication operations?



- Have we seen this before?
- Behavior depends on availability (and size) of buffering
  - System dependent
  - MPI implementation (LAM, Open MPI, MPICH) have diagnostics for this

Where does data go when you send it?



MPI_Send

```
#include <mpi.h>
void Comm::Send(const void* buf, int count, const Datatype& datatype,
↪  int dest, int tag) const
```

- MPI_Send is sometimes called a “blocking send”
- Semantics (from the standard): Send MPI_Send returns, it is safe to reuse the buffer
- So it only blocks until buffer is safe to reuse
- (Recall we can only specify local semantics)

MPI_Recv

```
#include <mpi.h>
void Comm::Recv(void* buf, int count, const Datatype& datatype,
 ↪  int source, int tag, Status& status) const

void Comm::Recv(void* buf, int count, const Datatype& datatype,
 ↪  int source, int tag) const
```

- Blocking receive
- Semantics: Blocks until message is received. On return from call, buffer will have message data

Summary

- As-if is the most important principle in parallelization (correctness first)
- SPMD has high degree of self-similarity – solving global problem is same as solving local problem – communication enforces as-if
- Ubiquitous compute / communicate cycle

Next

- Performance models (LogP and BSP)
- Summary of collectives, datatypes, non-blocking operations
- Finish up CSP Jacobi iteration
- Briefly discuss amath583 cluster login, questions for docker portion

- Stay tuned

Thank You!

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