AMATH 483/583
High Performance Scientific Computing

Lecture 17:
Distributed memory, communicating sequential processes

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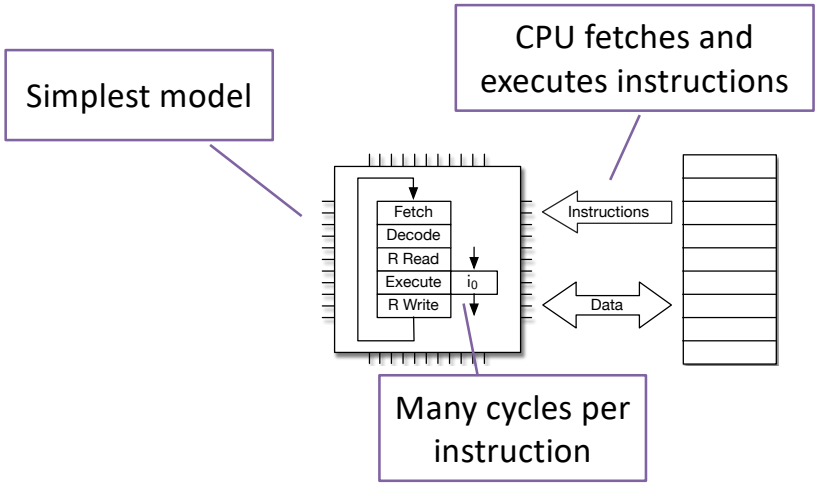
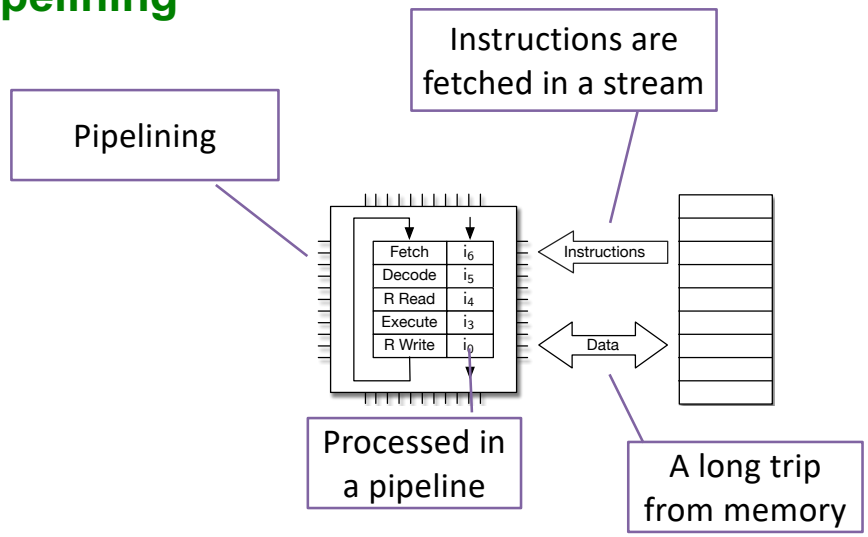
Overview

- Distributed memory systems
- Communicating sequential processes
- Message passing
- The message passing interface

Scaling progression of CPUs

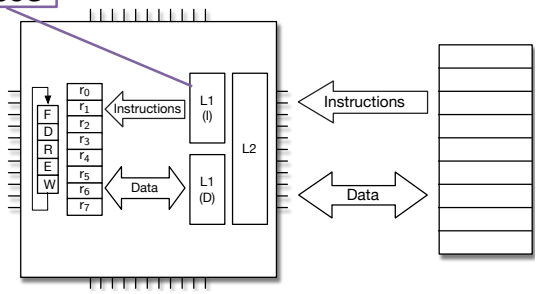


Pipelining



Hierarchical memory

Use special, fast memory to keep data and instructions close

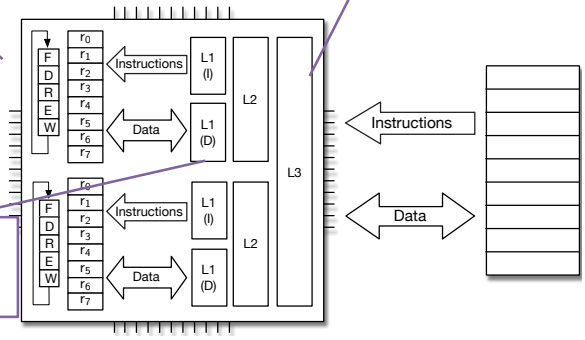


Multicore CPUs

Replicate 2X

Cores share slower memory

Caches need to be kept coherent

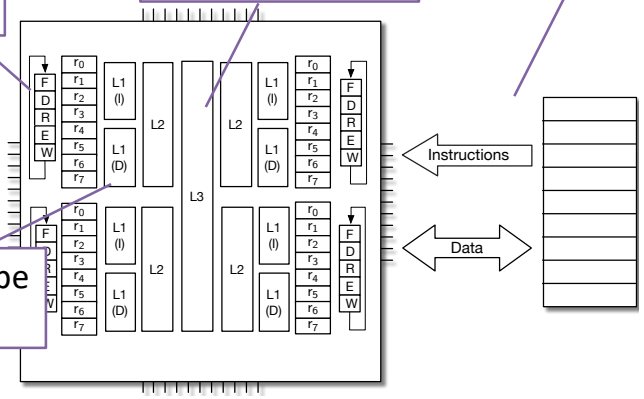


Even more cores

Replicate 4X

Cores share slower memory

Include super-slow DRAM

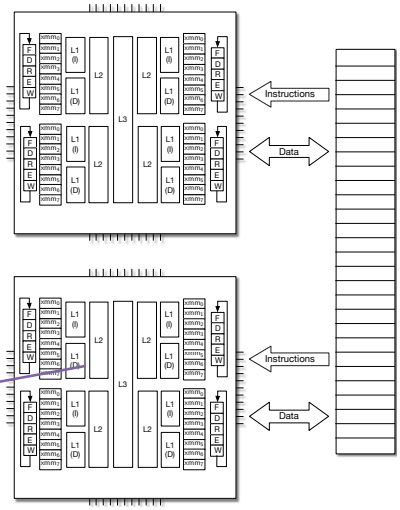


Caches need to be kept coherent

Symmetric Multi-Processor (SMP)

Multiple CPU chips

Memory may be uniformly shared among sockets



AKA “sockets”

Uniform memory access (UMA)

Caches still need to be kept (somewhat) coherent

Asymmetric

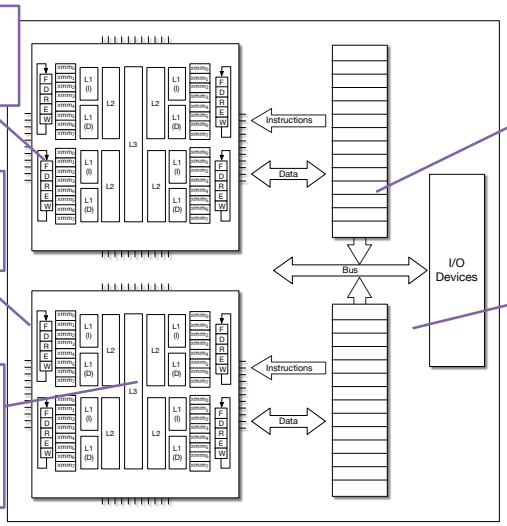
Multiple CPU chips

AKA “sockets”

Caches still need to be kept (somewhat) coherent: CC-NUMA

Memory may be non-uniformly shared among sockets

Non-uniform memory access (NUMA – most common)



The Next Step

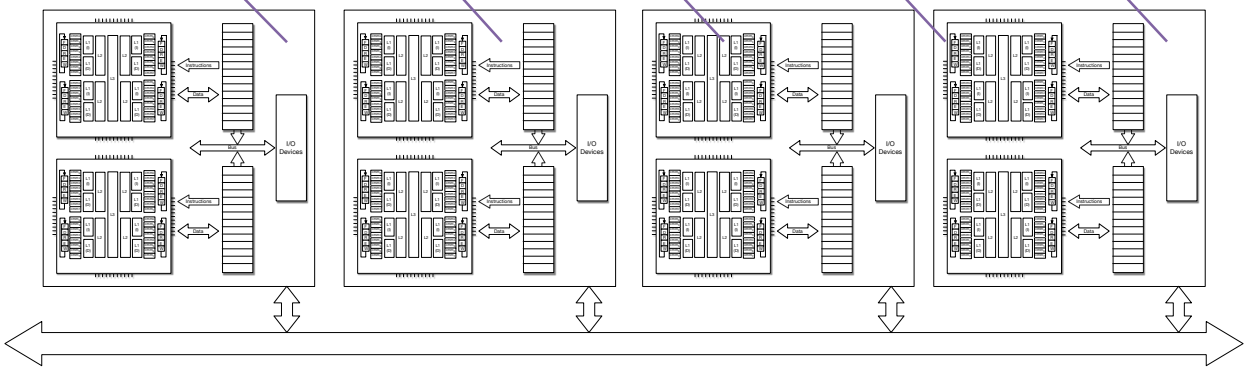
Put sockets on a blade

Put blades in a chassis

Put chassis in a rack

Put racks in a center

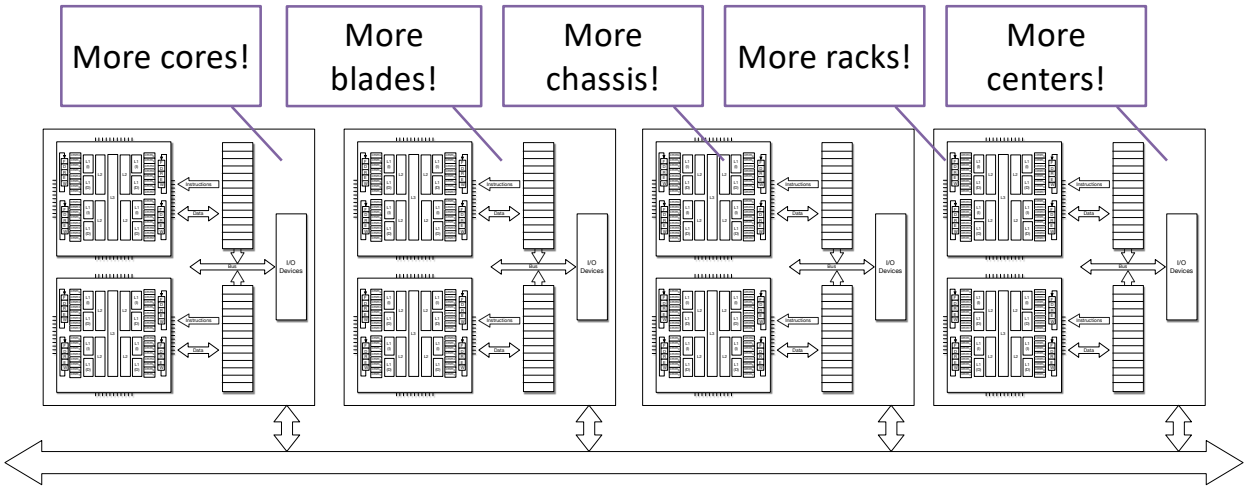
Put centers in the cloud



Then you have a supercomputer

But how do you use it?

Need More Power? Buy More Hardware!



Top500 November 2018

Rank	Site	System	Cores	Rmax (TFlop/s)	Rpeak (TFlop/s)	Power (kW)
1	DOE/SC/Oak Ridge National Laboratory United States	**Summit** - IBM Power System AC922, IBM POWER9 22C 3.07GHz, NVIDIA Volta GV100, Dual-rail Mellanox EDR Infiniband IBM	2,397,824	143,500.0	200,794.9	9,783
2	DOE/NNSA/LLNL United States	**Sierra** - IBM Power System S922LC, IBM POWER9 22C 3.1GHz, NVIDIA Volta GV100, Dual-rail Mellanox EDR Infiniband IBM / NVIDIA / Mellanox	1,572,480	94,640.0	125,712.0	7,438
3	National Supercomputing Center in Wuxi China	**Sunway TaihuLight** - Sunway MPP, Sunway SW26010 260C 1.45GHz, Sunway NRCPC	10,649,600	93,014.6	125,435.9	15,371
4	National Super Computer Center in Guangzhou China	**Tianhe-2A** - TH-IVB-FEP Cluster, Intel Xeon E5-2692v2 12C 2.2GHz, TH Express-2, Matrix-2000 NUDT	4,981,760	61,444.5	100,678.7	18,482

2.4M cores

1.5M cores

10M cores

There are no parallel computers

It's really just a bunch of computers

Separate memory

(Each has its own memory)

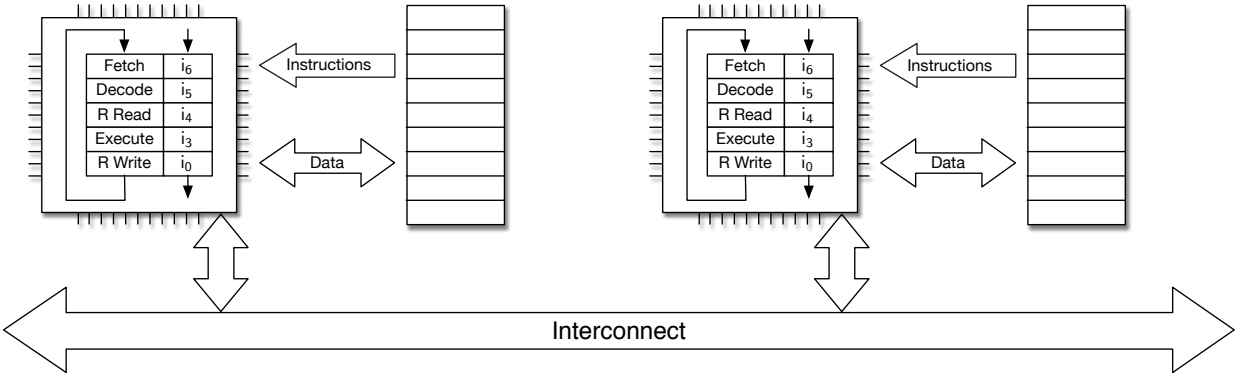
(Each has its own storage)

(Each has its own OS)

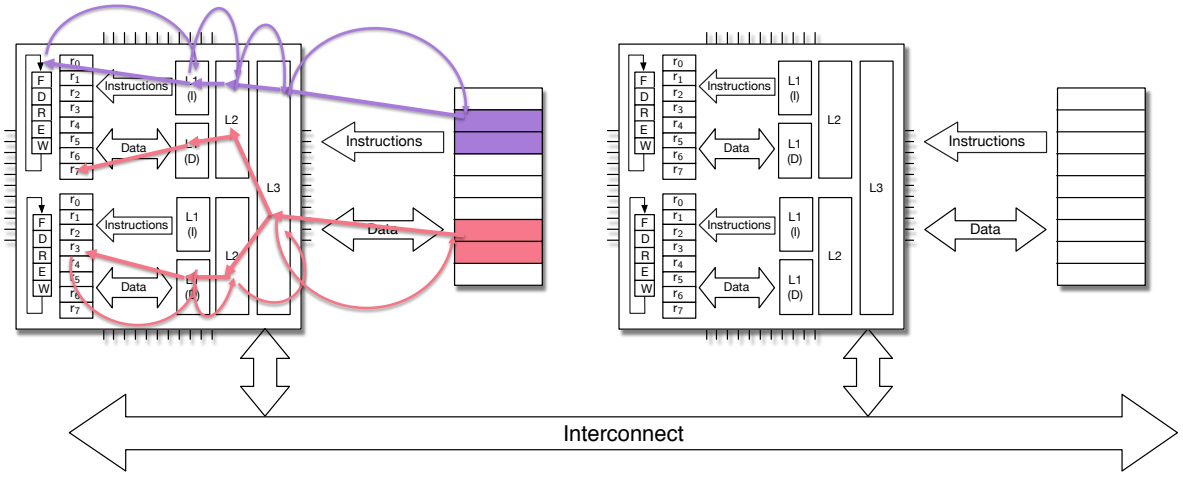
There are no parallel programs

It's really just a bunch of programs

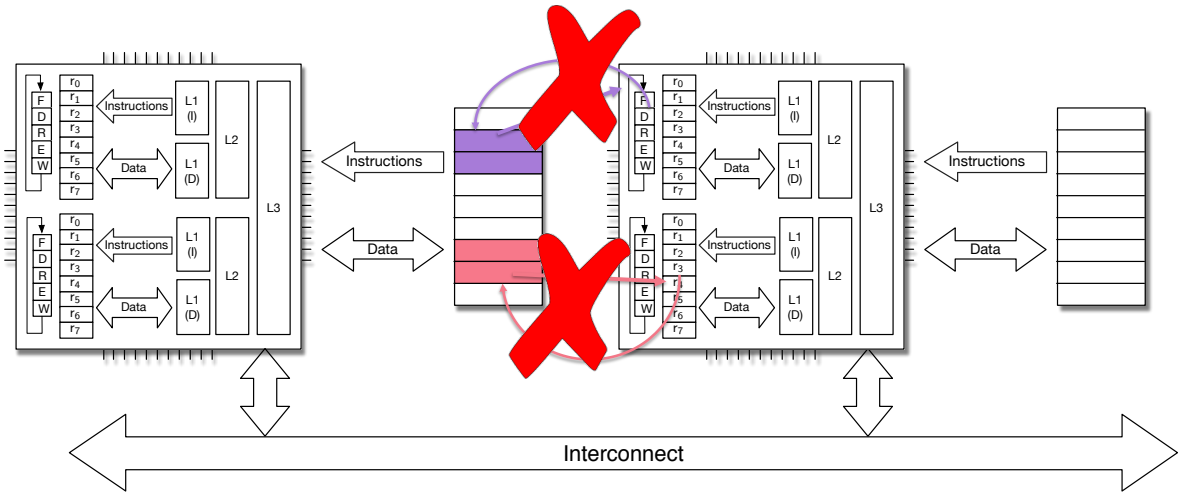
Distributed memory



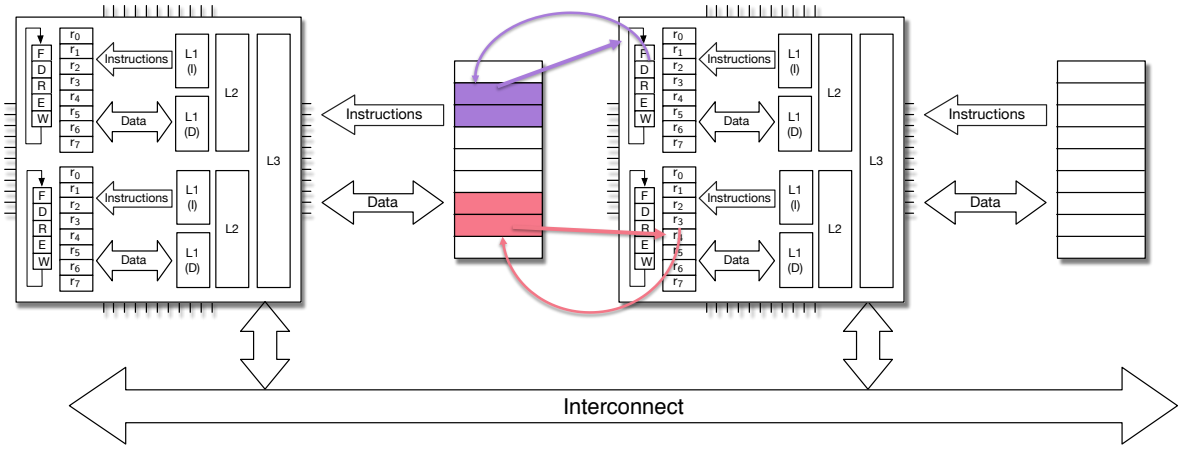
Distributed memory



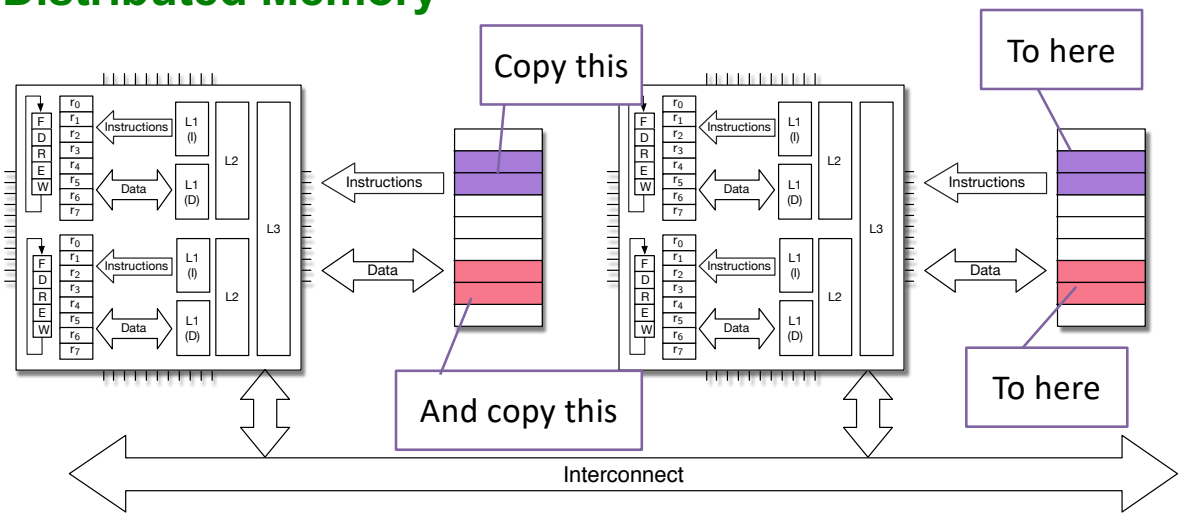
Distributed memory



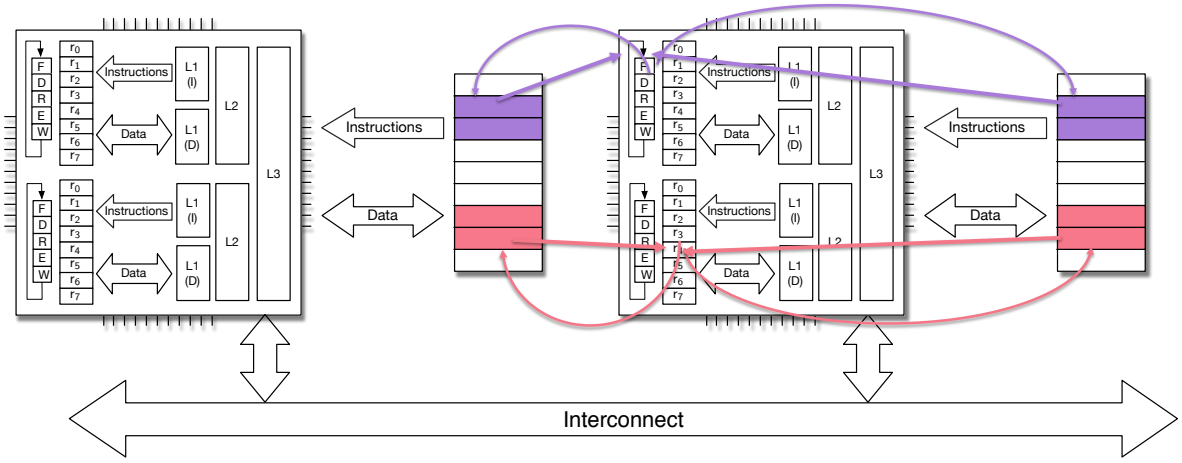
Distributed memory



Distributed Memory

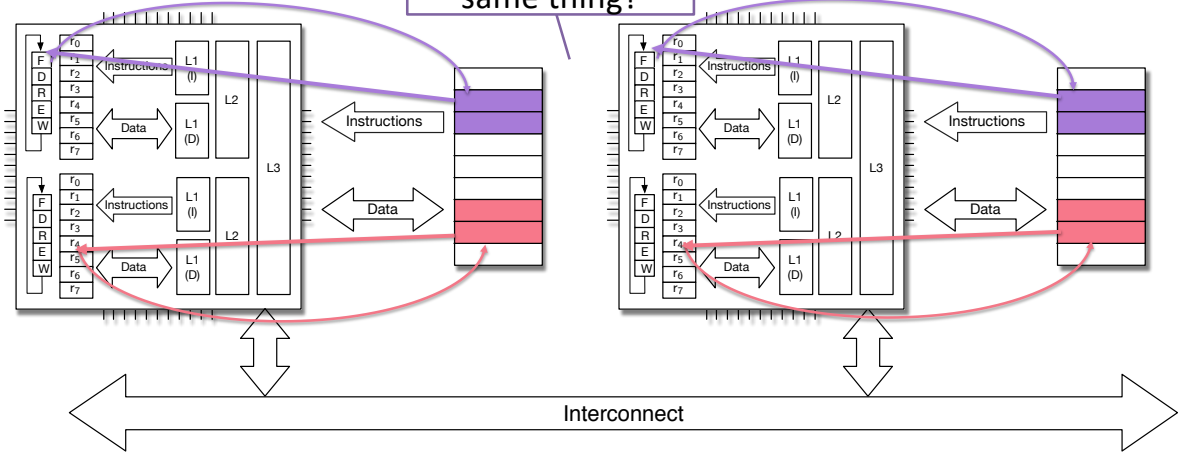


Distributed memory

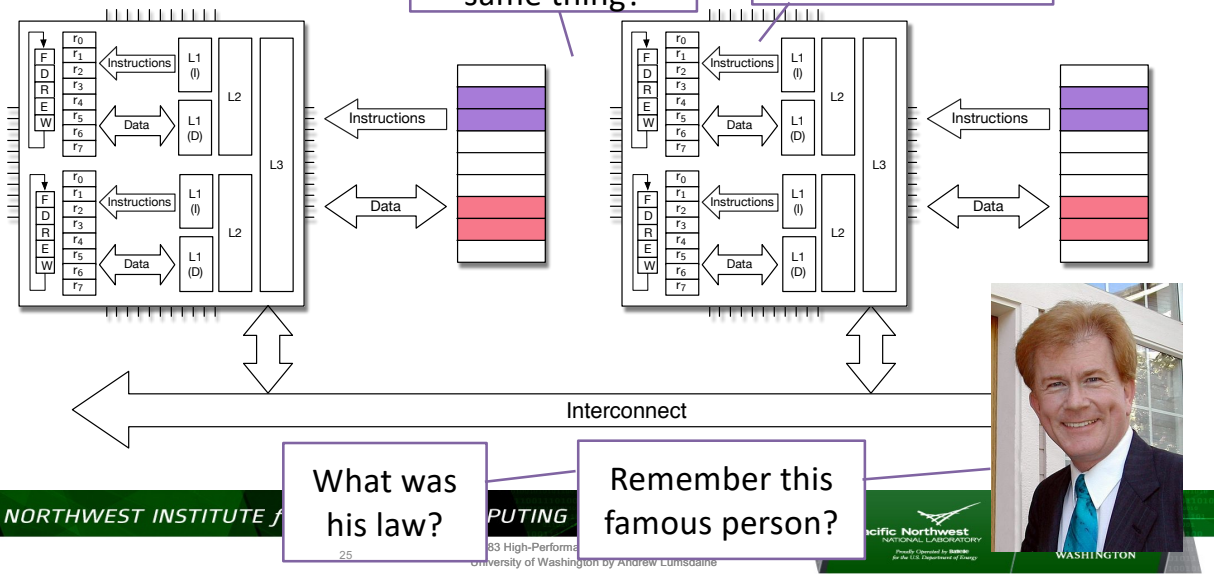


Distributed memor

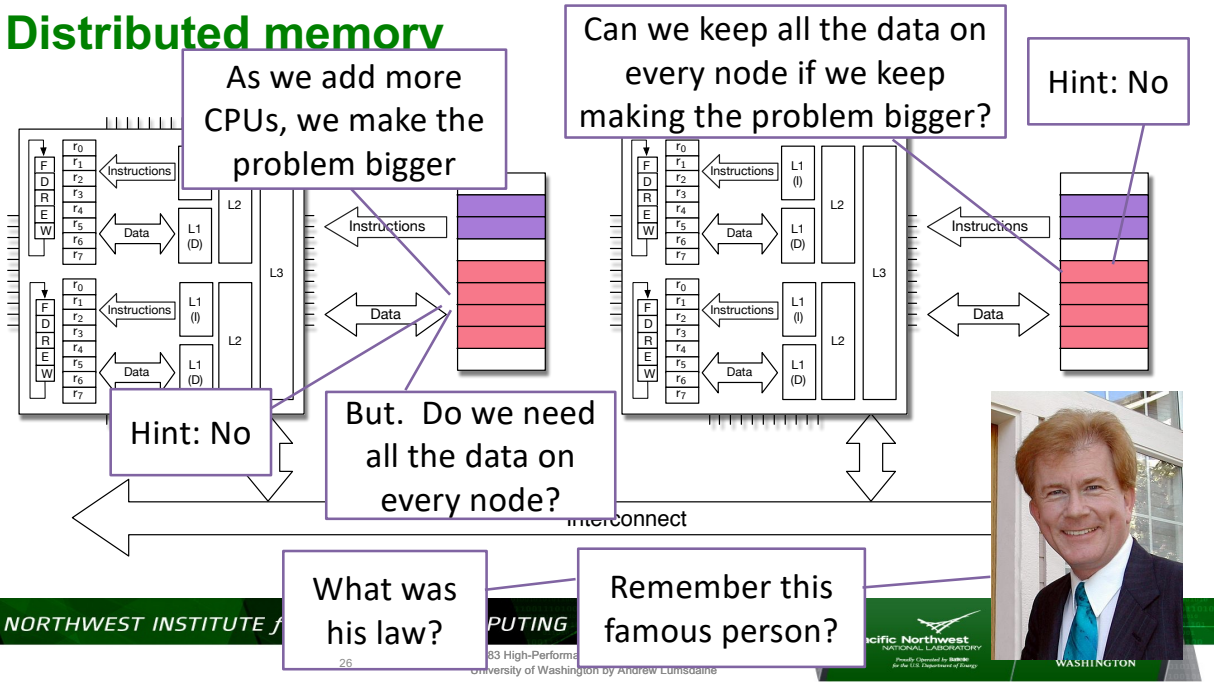
Do we want these doing the exact same thing?



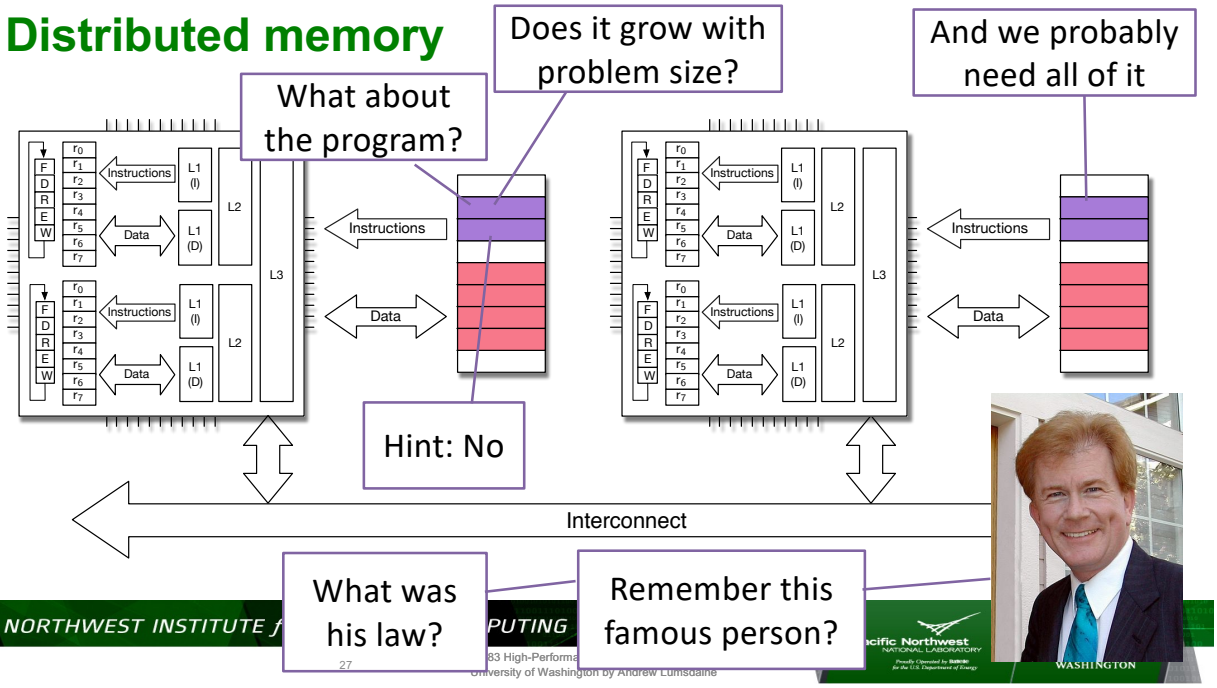
Distributed memor



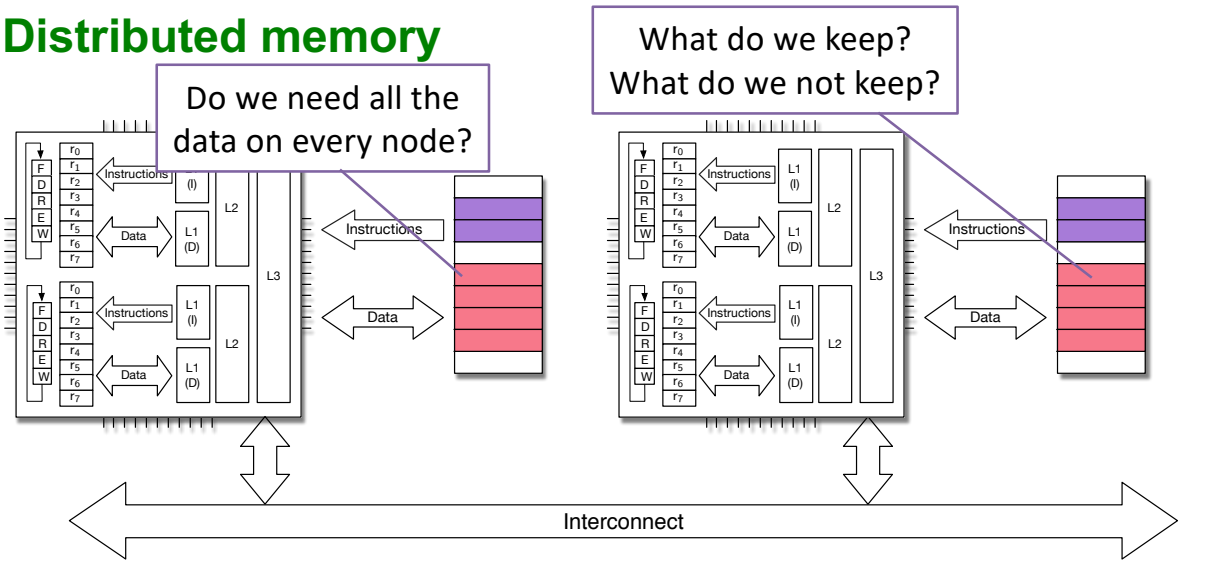
Distributed memory



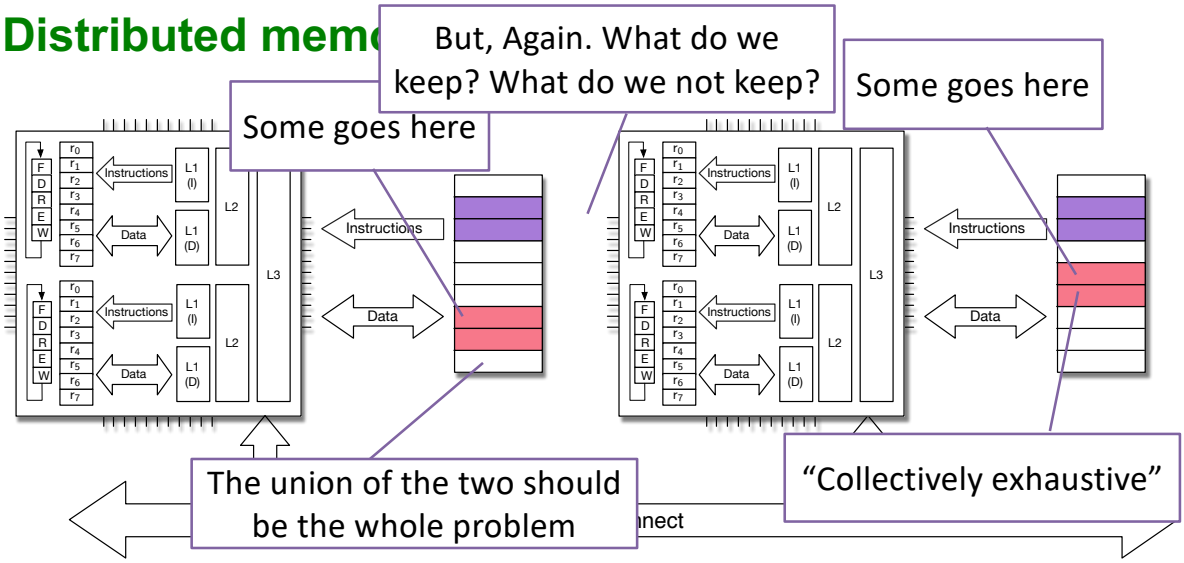
Distributed memory



Distributed memory

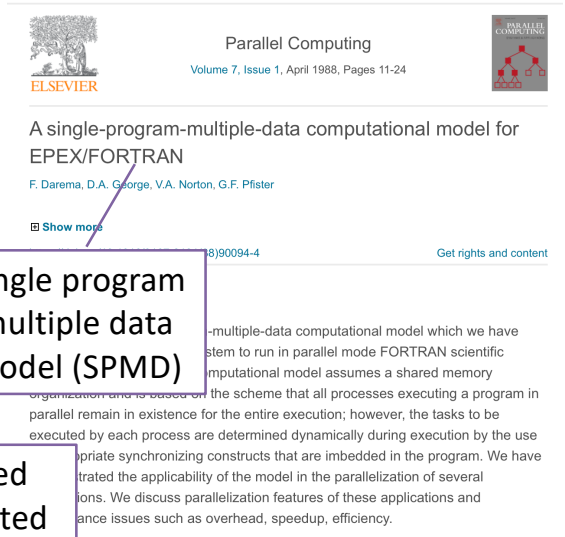


Distributed meme



Name this famous person

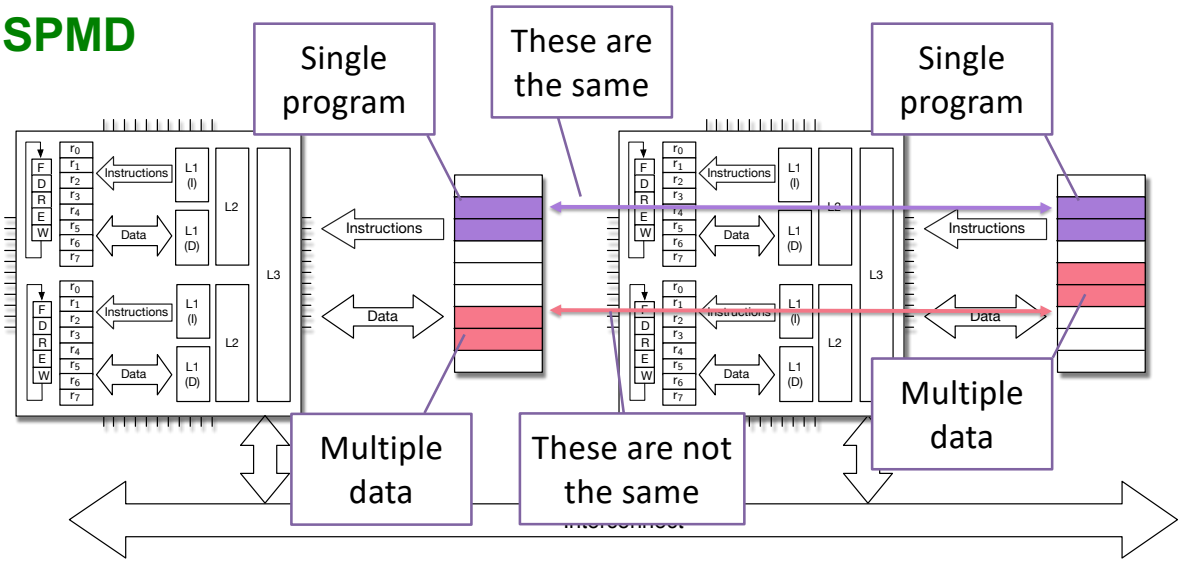
Frederica Darema (Director, Air Force Office of Scientific Research)



Single program multiple data model (SPMD)

Most widely used model in distributed memory programming

SPMD



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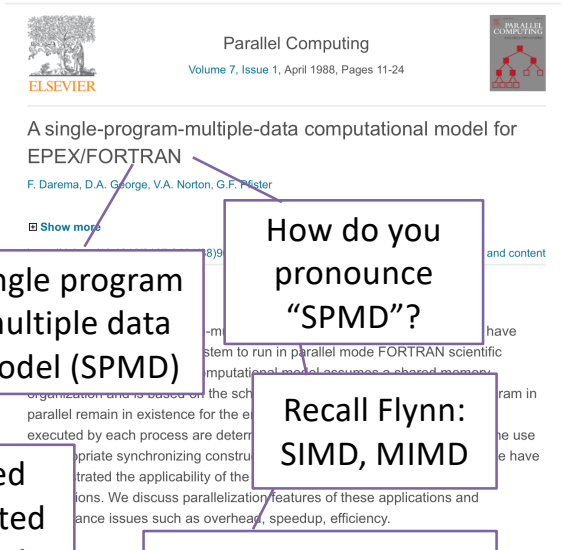
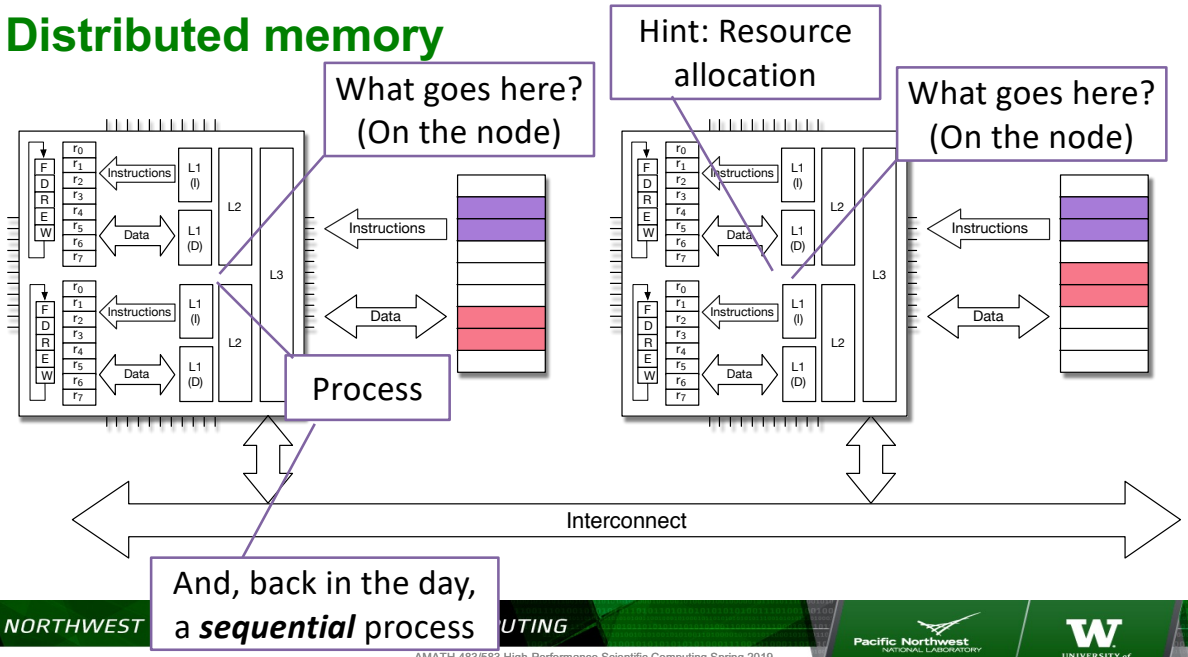
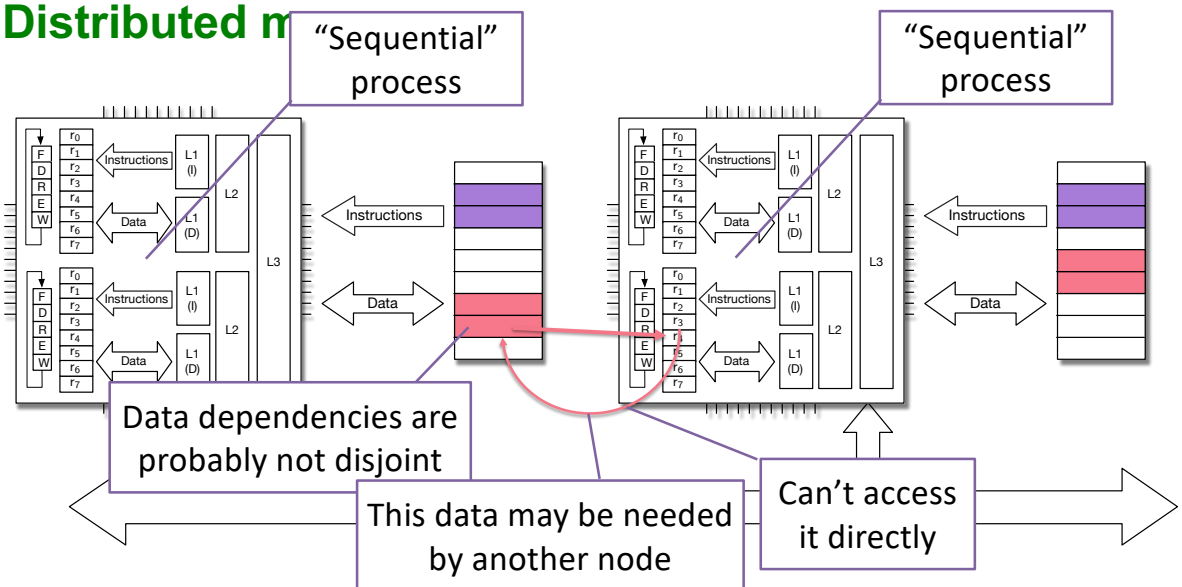


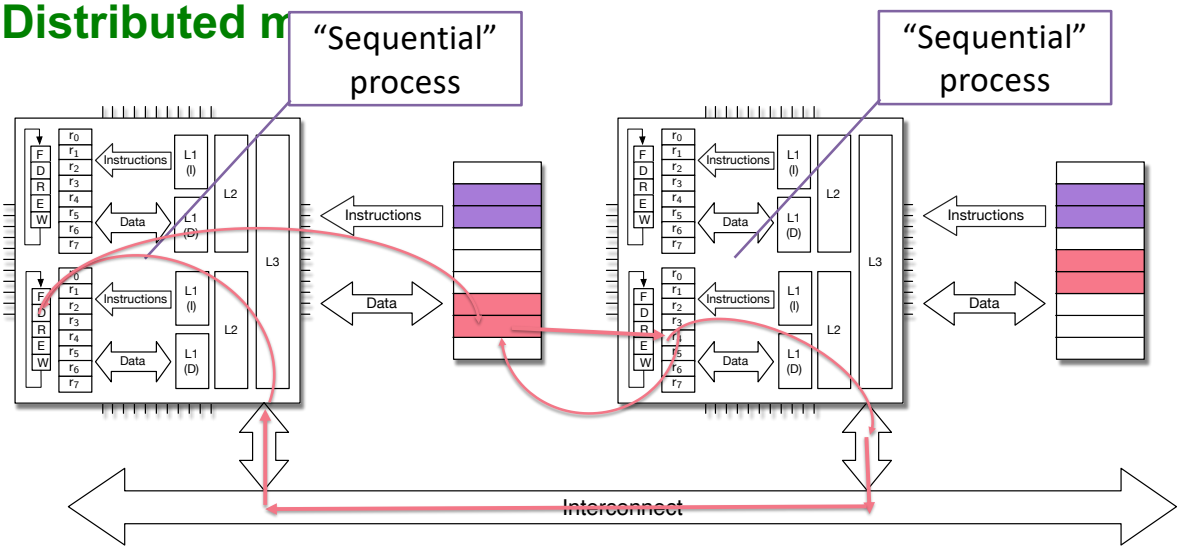
Distributed memory



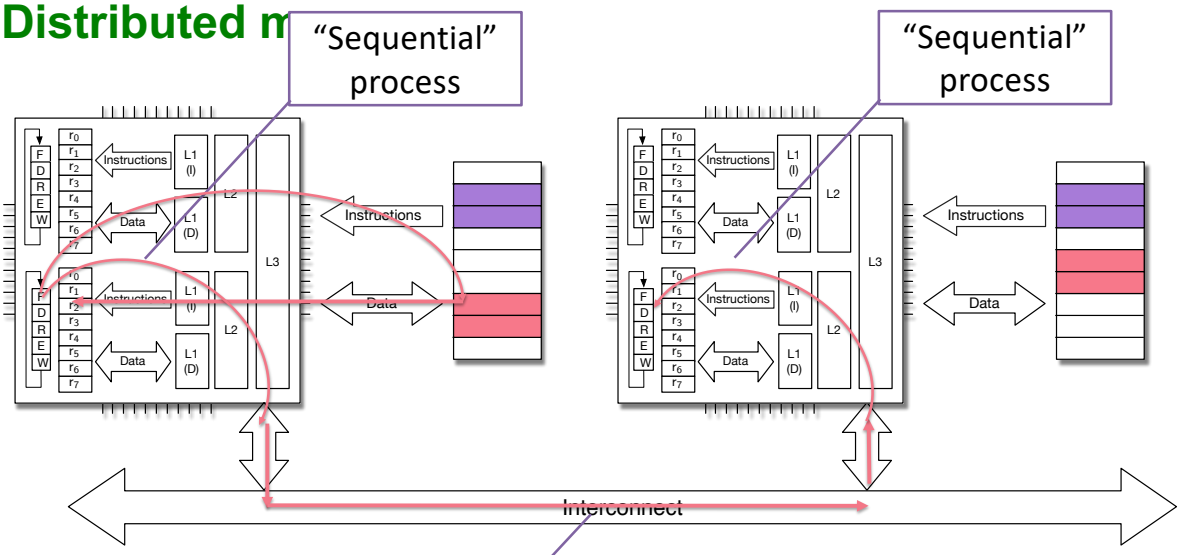
Distributed m



Distributed m

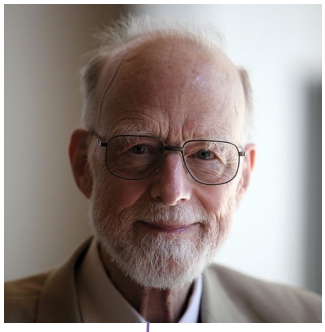


Distributed m



What are these sequential processes doing?

Recall this famous person



C.A.R (Tony) Hoare

An Axiomatic Basis for Computer Programming

C. A. R. Hoare
The Queen's University of Belfast, Northern Ireland*

In this paper an attempt is n
tions of computer progran
were first applied in the s
been extended to other
volves the elucidation of se
which can be used in proc
programs. Examples are given of such axioms and rules, and a formal proof of a simple theorem is displayed. Finally, it is argued that important advantages, both theoretical and practical, may follow from a pursuance of these topics.

"CSP" (pronounced see ess pea)

Programming Techniques — S. L. Graham, R. L. Rivest Editors

Communicating Sequential Processes

C.A.R. Hoare
The Queen's University
Belfast, Northern Ireland

This paper suggests that input and output are basic primitives of programming and that parallel composition of communicating sequential processes is a fundamental program structuring method. When combined with a development of Dijkstra's guarded command, these concepts are surprisingly versatile. Their use is illustrated by sample solutions of a variety of familiar programming exercises.

Key Words and Phrases: programming, programming languages, programming primitives, program structures, parallel programming, concurrency, input, output, guarded commands, nondeterminacy, coroutines, procedures, multiple entries, multiple exits, classes, data representations, recursion, conditional critical regions, monitors, iterative arrays
CR Categories: 4.20, 4.22, 4.32

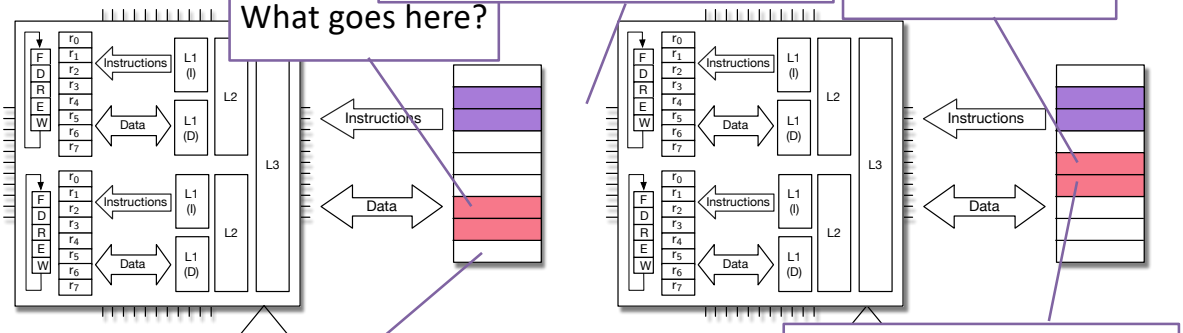
PS: These aren't even what he is most famous for

Distributed meme

What do we keep? What do we not keep?

What goes here?

What goes here?



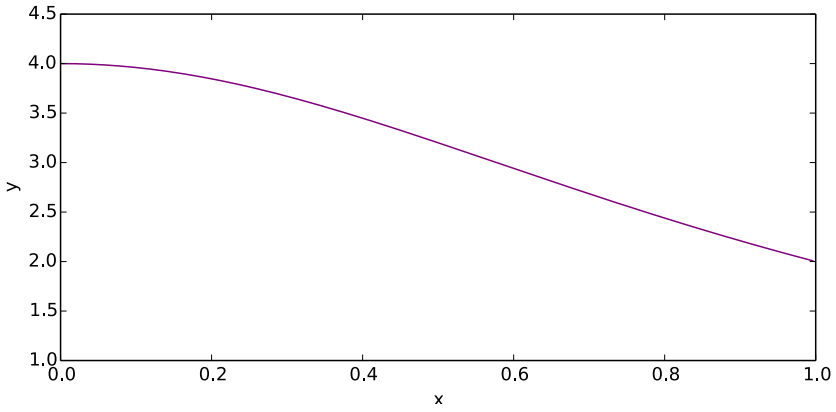
The union of the two should be the whole problem

"Collectively exhaustive"

Back to our trusty example (one of them)

- Find the value of π
- Using formula

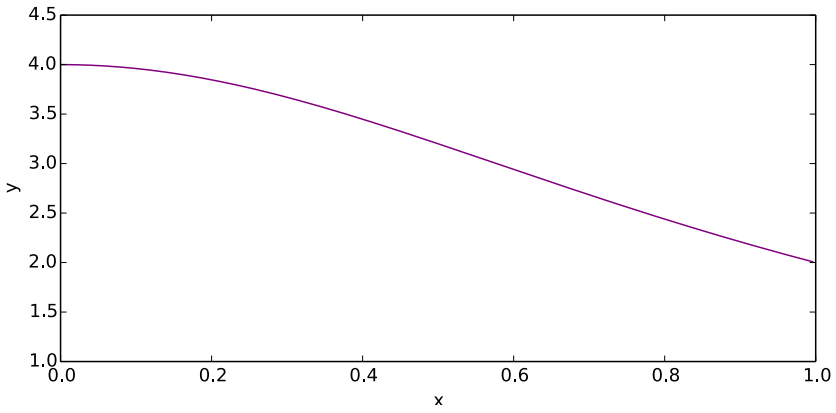
$$\pi = \int_0^1 \frac{4}{1+x^2} dx$$



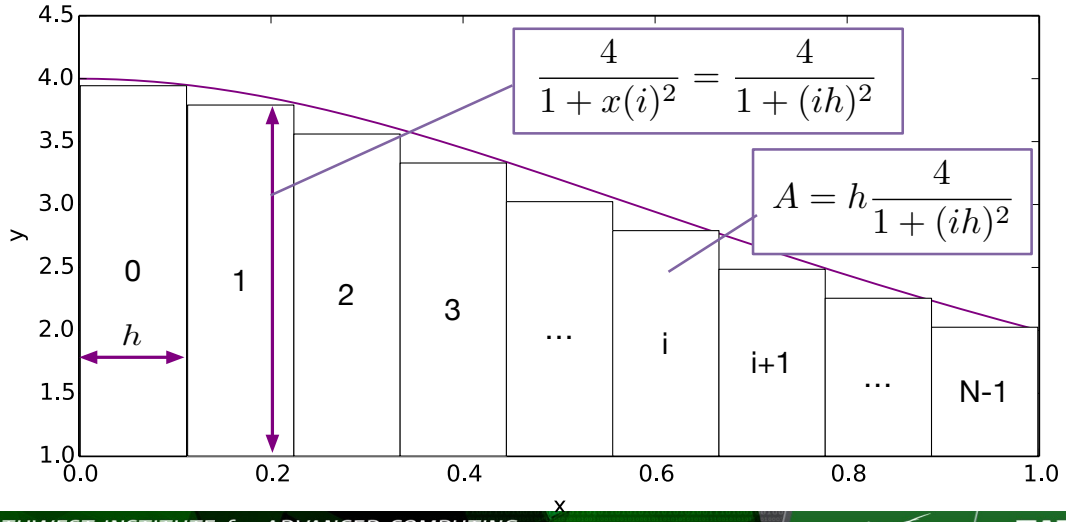
Example

- Find the value of π
- Using formula

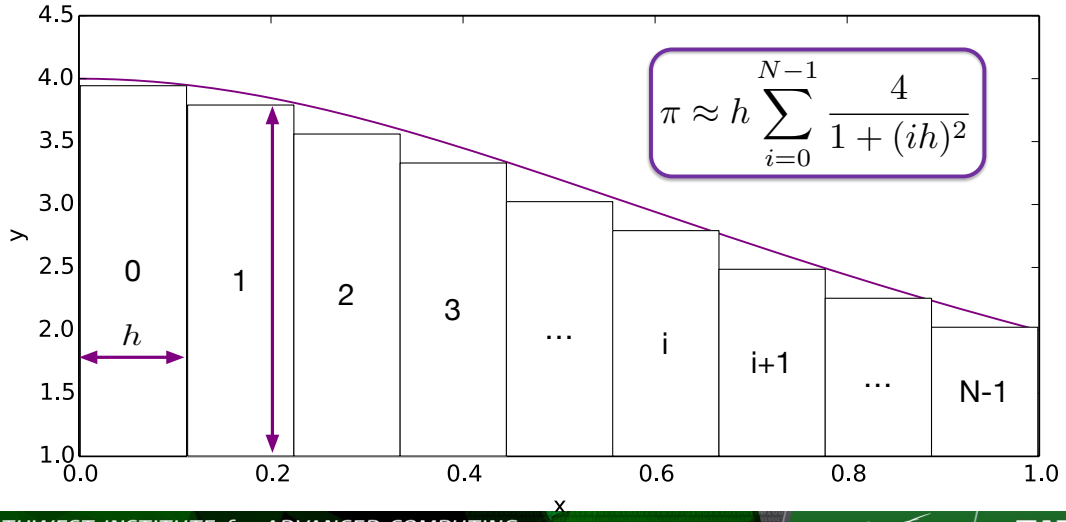
$$\pi = \int_0^1 \frac{4}{1+x^2} dx$$



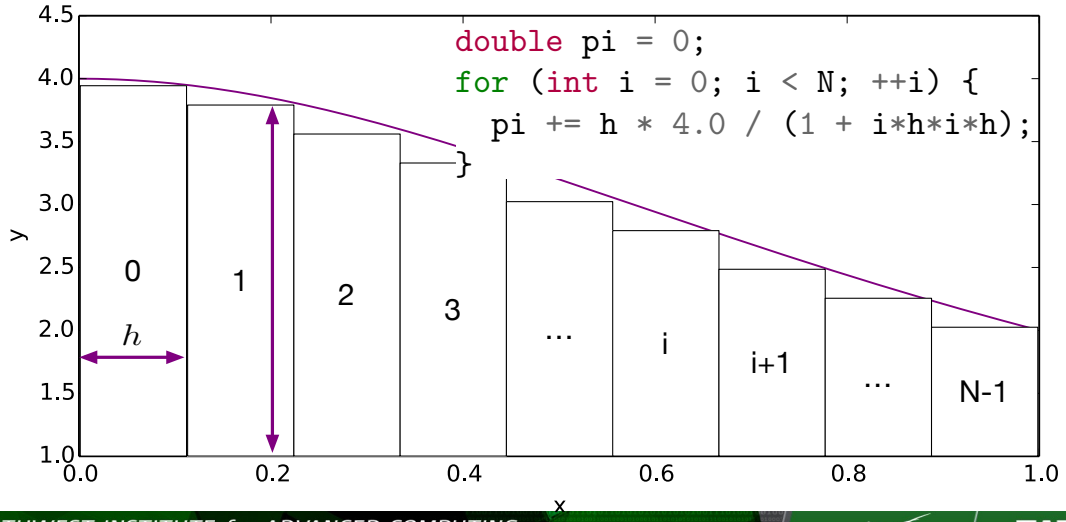
Numerical Quadrature



Numerical Quadrature

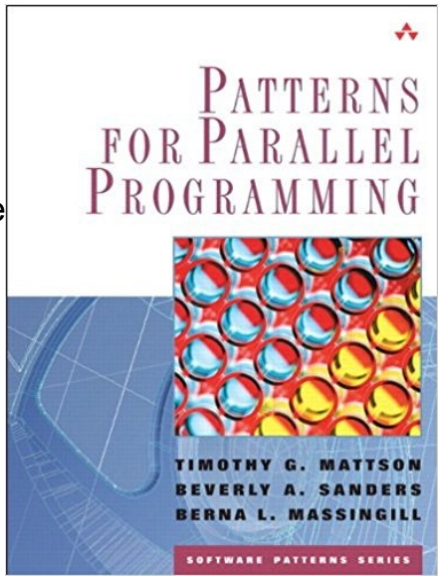


Numerical Quadrature (Sequential)



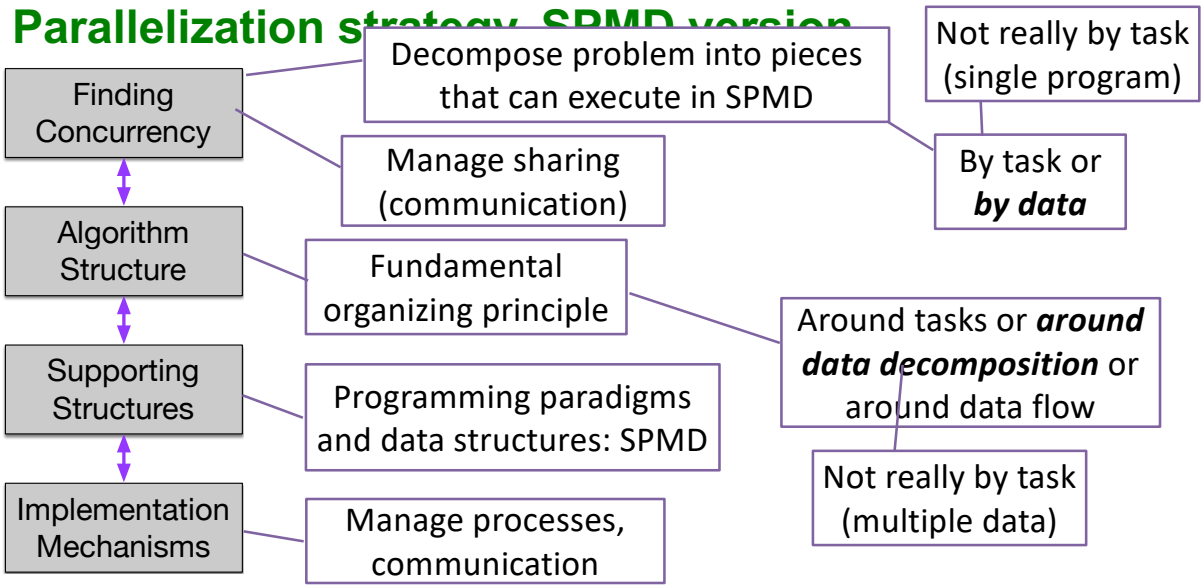
Parallelization Strategy

- How do we go from a problem want to solve
- And maybe know how to solve sequentially
- To a parallel program
- That scales



Timothy Mattson, Beverly Sanders, and Berna Massingill. 2004. *Patterns for Parallel Programming* (First ed.). Addison-Wesley Professional.

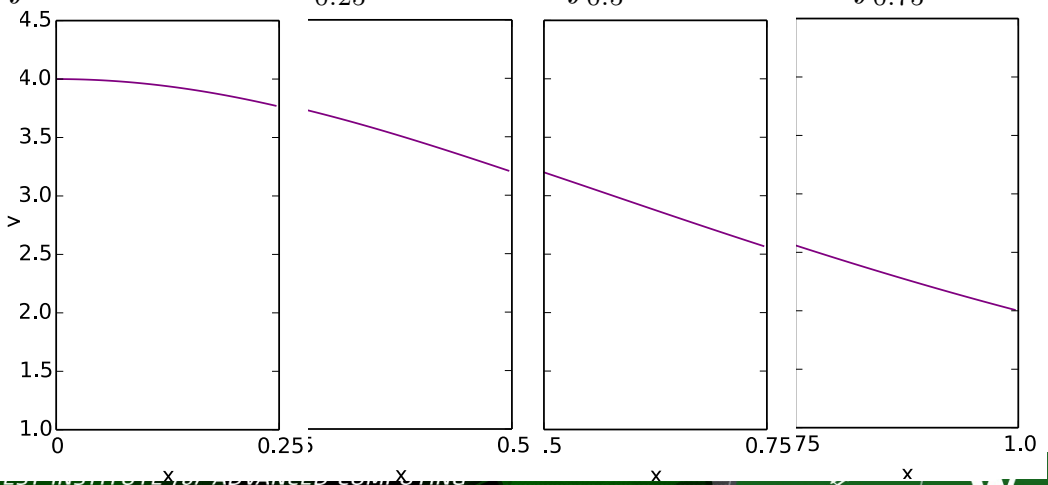
Parallelization strategy, SPMD version



Timothy Mattson, Beverly Sanders, and Berna Massingill. 2004. *Patterns for Parallel Programming* (First ed.). Addison-Wesley Professional.

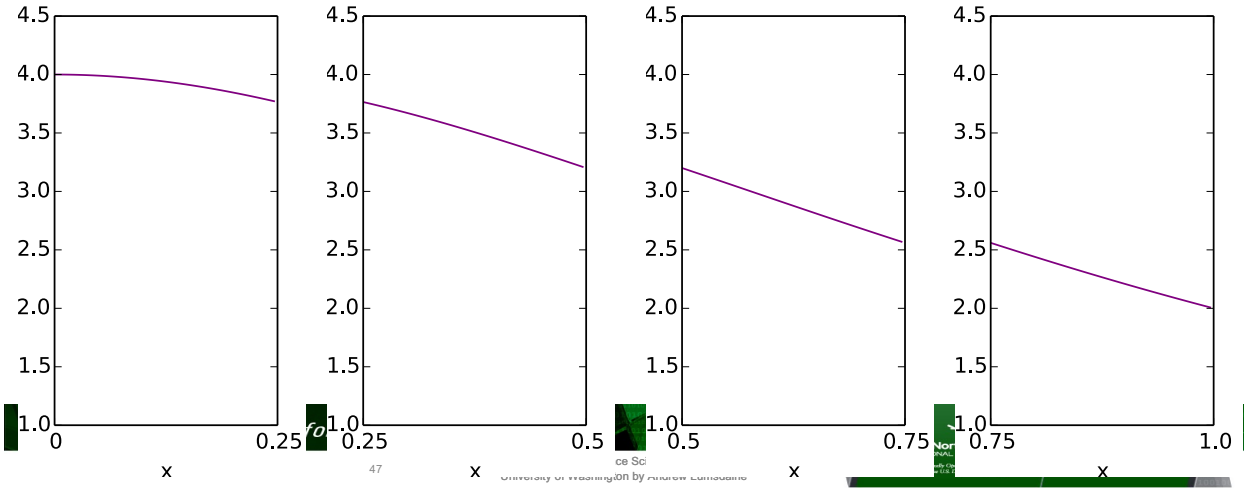
Finding Concurrency

$$\pi = \int_0^{0.25} \frac{4}{1+x^2}dx + \int_{0.25}^{0.5} \frac{4}{1+x^2}dx + \int_{0.5}^{0.75} \frac{4}{1+x^2}dx + \int_{0.75}^{1} \frac{4}{1+x^2}dx$$



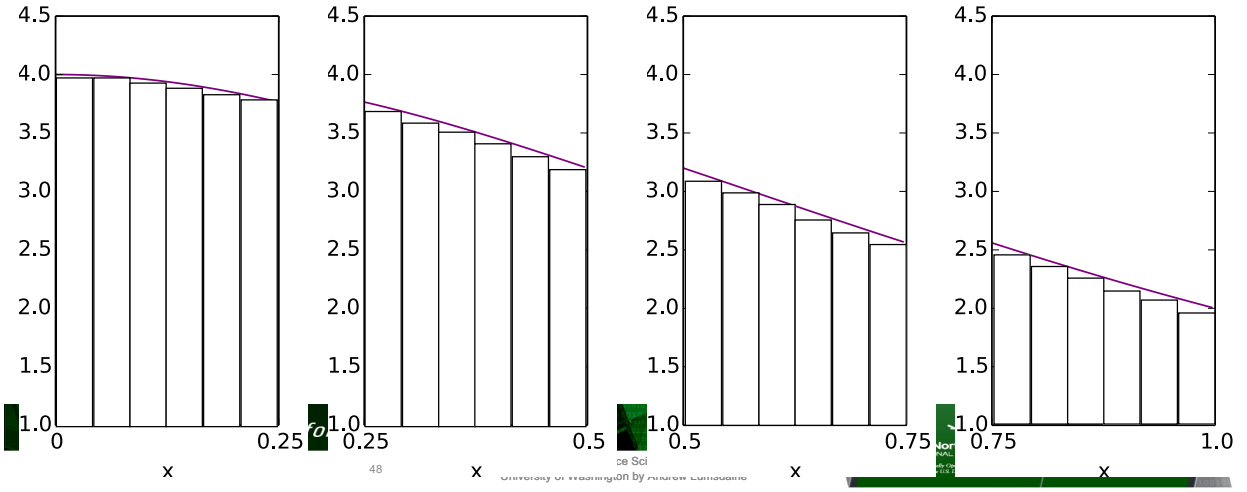
Finding Concurrency

$$\pi = \int_0^{0.25} \frac{4}{1+x^2}dx + \int_{0.25}^{0.5} \frac{4}{1+x^2}dx + \int_{0.5}^{0.75} \frac{4}{1+x^2}dx + \int_{0.75}^{1} \frac{4}{1+x^2}dx$$



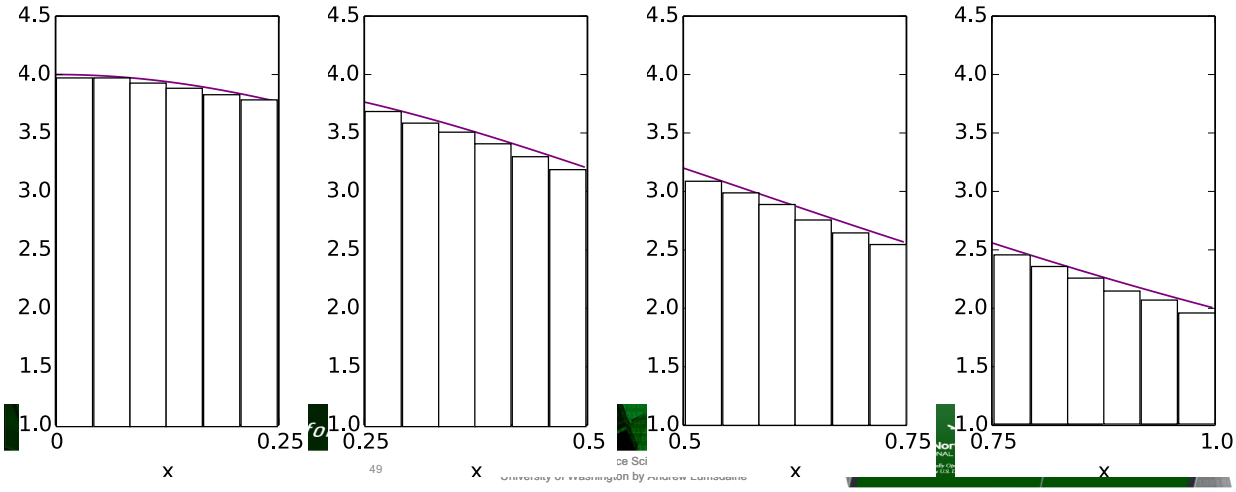
Finding Concurrency

$$\pi = \int_0^{0.25} \frac{4}{1+x^2}dx + \int_{0.25}^{0.5} \frac{4}{1+x^2}dx + \int_{0.5}^{0.75} \frac{4}{1+x^2}dx + \int_{0.75}^{1} \frac{4}{1+x^2}dx$$



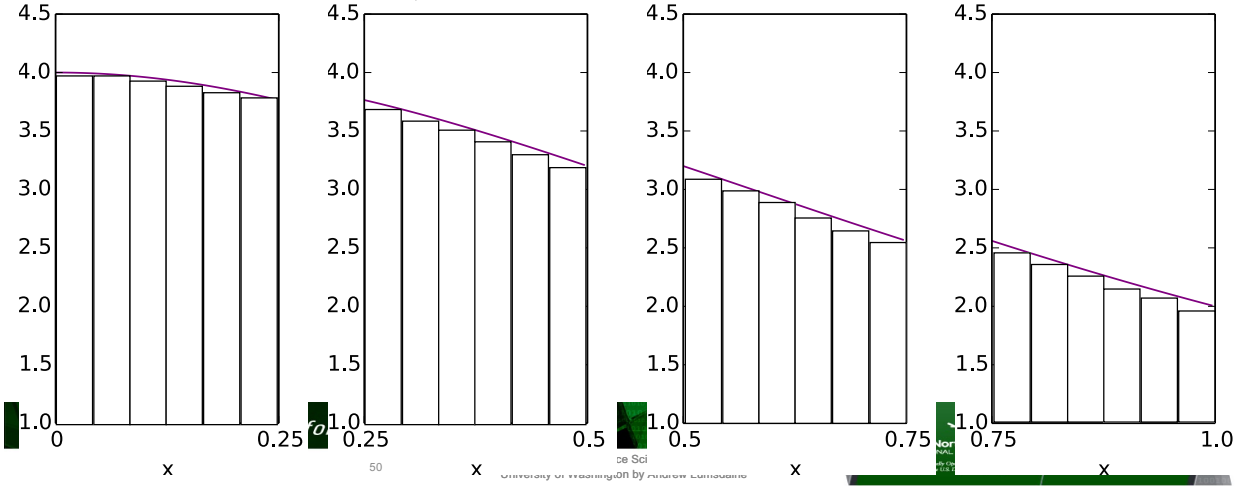
Finding Concurrency

$$\pi \approx h\sum_{i=0}^{N/4-1}\frac{4}{1+(ih)^2}+h\sum_{i=N/4}^{N/2-1}\frac{4}{1+(ih)^2}+h\sum_{i=N/2}^{3N/4-1}\frac{4}{1+(ih)^2}+h\sum_{i=3N/4}^{3N-1}\frac{4}{1+(ih)^2}$$



Finding Concurrency

$$h\sum_{i=0}^{N/4-1}\frac{4}{1+(ih)^2}\quad h\sum_{i=N/4}^{N/2-1}\frac{4}{1+(ih)^2}\quad h\sum_{i=N/2}^{3N/4-1}\frac{4}{1+(ih)^2}\quad h\sum_{i=3N/4}^{N-1}\frac{4}{1+(ih)^2}$$



Finding Concurrency

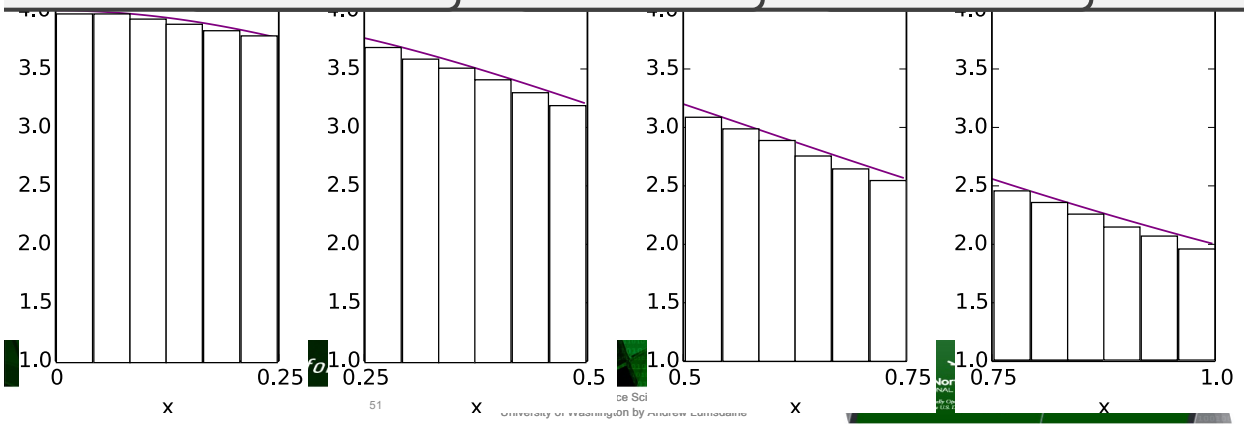
```
for (int i = begin; i < end; ++i) {
  pi += h * 4.0 / (1 + i*h*i*h);
}
```

```
int i = 0; i < N/4; ++i) {
+= h * 4.0 / (1 + i*h*i*h);
```

```
4; i < N/2; ++i) {
/ (1 + i*h*i*h);
```

```
I/2; i < 3*N/4; ++i) {
0 / (1 + i*h*i*h);
```

```
N/4; i < N
/ (1 + i*h
```



Finding Concurrency

$$h\sum_{i=0}^{N/4-1}\frac{4}{1+(ih)^2}$$

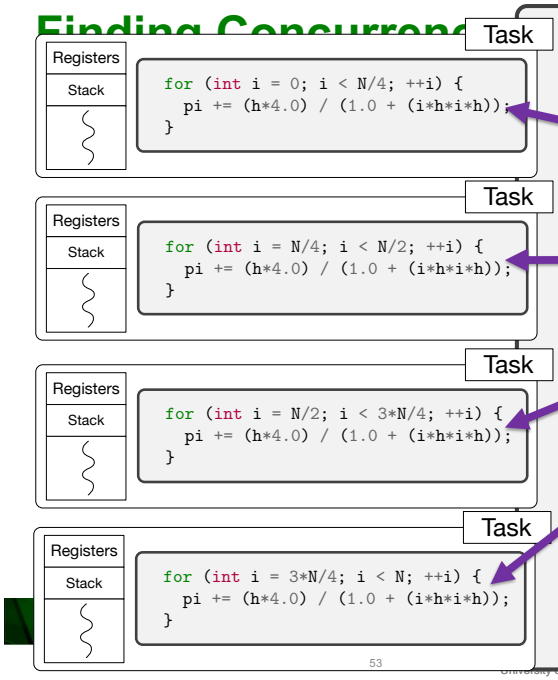
$$h\sum_{i=N/4}^{N/2-1}\frac{4}{1+(ih)^2}$$

$$h\sum_{i=N/2}^{3N/4-1}\frac{4}{1+(ih)^2}$$

$$h\sum_{i=3N/4}^{N-1}\frac{4}{1+(ih)^2}$$

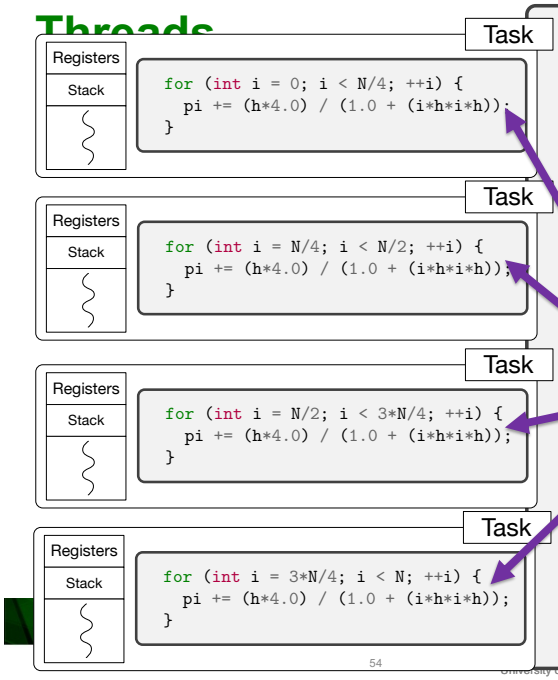
```
int main() {
  double pi = 0.0;  int N = 1024*1024;

  for (int i = 0; i < N/4; ++i)
    pi += (h*4.0) / (1.0 + (i*h*i*h));

  for (int i = N/4; i < N/2; ++i)
    pi += (h*4.0) / (1.0 + (i*h*i*h));

  for (int i = N/2; i < 3*N/4; ++i)
    pi += (h*4.0) / (1.0 + (i*h*i*h));

  for (int i = 3*N/4; i < N; ++i)
    pi += (h*4.0) / (1.0 + (i*h*i*h));

  std::cout << "pi ~ " << pi << std::endl;
  return 0;
}
```

Finding Concurrency




```
int main() {
  double pi = 0.0;  int N = 1024*1024;

  for (int i = 0; i < N/4; ++i)
    pi += (h*4.0) / (1.0 + (i*h*i*h));

  for (int i = N/4; i < N/2; ++i)
    pi += (h*4.0) / (1.0 + (i*h*i*h));

  for (int i = N/2; i < 3*N/4; ++i)
    pi += (h*4.0) / (1.0 + (i*h*i*h));

  for (int i = 3*N/4; i < N; ++i)
    pi += (h*4.0) / (1.0 + (i*h*i*h));

  std::cout << "pi ~ " << pi << std::endl;
  return 0;
}
```


Threads




```
double pi = 0.0;

void pi_helper(int begin, int end, double h) {
  for (int i = begin; i < end; ++i)
    pi += (h*4.0) / (1.0 + (i*h*i*h));
}

int main(int argc, char* argv[]) {
  int N = 1024 * 1024; double h = 1.0/ (double)N;

  std::thread t0(pi_helper, 0,     N/4,   h);
  std::thread t1(pi_helper, N/4,   N/2,   h);
  std::thread t2(pi_helper, N/2,   3*N/4, h);
  std::thread t3(pi_helper, 3*N/4, N,     h);

  t0.join();  t1.join();  t2.join();  t3.join();

  std::cout << "pi is ~ " << pi << std::endl;

  return 0;
}
```

Finding Concurrency

$$h\sum_{i=0}^{N/4-1}\frac{4}{1+(ih)^2}$$

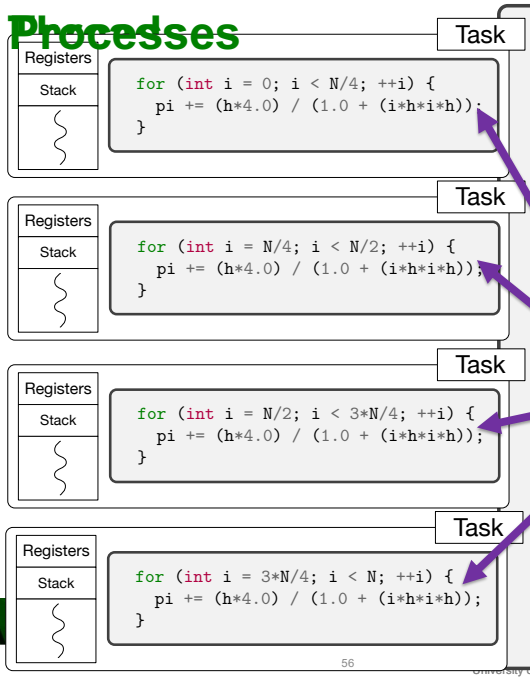
$$h\sum_{i=N/4}^{N/2-1}\frac{4}{1+(ih)^2}$$

$$h\sum_{i=N/2}^{3N/4-1}\frac{4}{1+(ih)^2}$$

$$h\sum_{i=3N/4}^{N-1}\frac{4}{1+(ih)^2}$$

```
int main() {
  double pi = 0.0;  int N = 1024*1024;

  for (int i = 0; i < N/4; ++i)
    pi += (h*4.0) / (1.0 + (i*h*i*h));

  for (int i = N/4; i < N/2; ++i)
    pi += (h*4.0) / (1.0 + (i*h*i*h));

  for (int i = N/2; i < 3*N/4; ++i)
    pi += (h*4.0) / (1.0 + (i*h*i*h));

  for (int i = 3*N/4; i < N; ++i)
    pi += (h*4.0) / (1.0 + (i*h*i*h));

  std::cout << "pi ~ " << pi << std::endl;
  return 0;
}
```


Processes




```
double pi = 0.0;

void pi_helper(int begin, int end, double h) {
  for (int i = begin; i < end; ++i)
    pi += (h*4.0) / (1.0 + (i*h*i*h));
}

int main(int argc, char* argv[]) {
  int N = 1024 * 1024; double h = 1.0/ (double)N;

  std::thread t0(pi_helper, 0,     N/4,   h);
  std::thread t1(pi_helper, N/4,   N/2,   h);
  std::thread t2(pi_helper, N/2,   3*N/4, h);
  std::thread t3(pi_helper, 3*N/4, N,     h);

  t0.join();  t1.join();  t2.join();  t3.join();

  std::cout << "pi is ~ " << pi << std::endl;

  return 0;
}
```

Processes

Process

```
int main() {
  double pi = 0.0; double h = 1./(double) N;
  for (size_t i = 0; i < N/4; ++i)
    pi += (h * 4.0) / (1.0 + (i * h * i * h));
  std::cout << "pi is ~ " <<  pi << std
```

Process

```
int main() {
  double pi = 0.0; double h = 1./(double) N;
  for (size_t i = N/4; i < N/2; ++i)
    pi += (h * 4.0) / (1.0 + (i * h * i * h));
  std::cout << "pi is ~ " <<  pi << std
```

Process

```
int main() {
  double pi = 0.0; double h = 1./(double) N;
  for (size_t i = N/2; i < 3*N/4; ++i)
    pi += (h * 4.0) / (1.0 + (i * h * i * h));
  std::cout << "pi is ~ " <<  pi << std
```

Process

```
int main() {
  double pi = 0.0; double h = 1./(double) N;
  for (size_t i = 3*N/4; i < N; ++i)
    pi += (h * 4.0) / (1.0 + (i * h * i * h));
  std::cout << "pi is ~ " <<  pi << std::endl

  return 0;
}
```

```
double pi = 0.0;

void pi_helper(int begin, int end, double h) {
  for (int i = begin; i < end; ++i)
    pi += (h*4.0) / (1.0 + (i*h*i*h));
}

int main(int argc, char* argv[]) {
  int N = 1024 * 1024; double h = 1.0/ (double)N;

  std::cout << "pi is ~ " << pi << std::endl;

  return 0;
}
```


Communicating sequential processes / SPMD

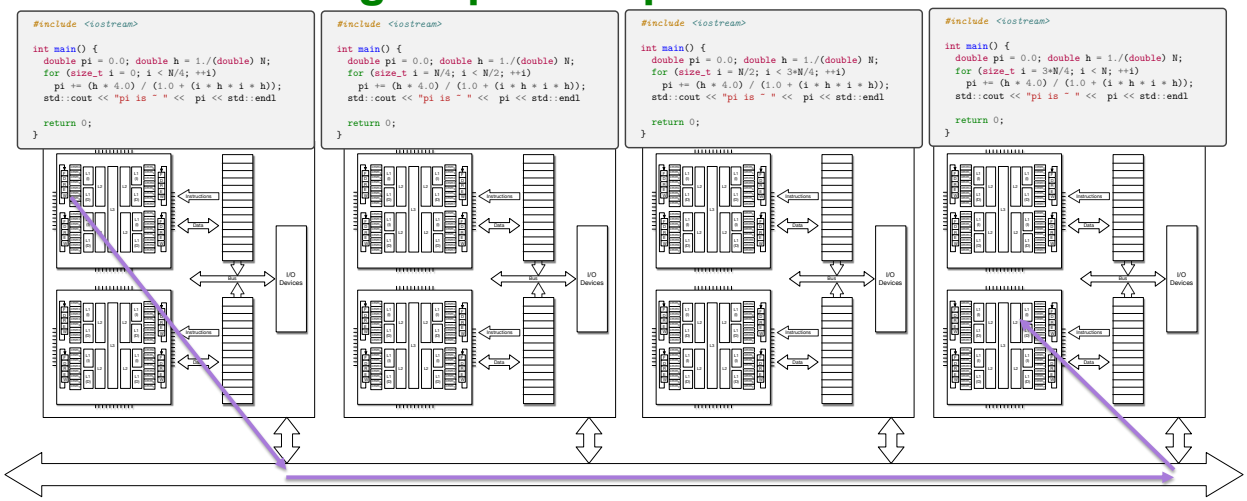
```
#include <iostream>

int main() {
  double pi = 0.0; double h = 1./(double) N;
  for (size_t i = 0; i < N/4; ++i)
    pi += (h * 4.0) / (1.0 + (i * h * i * h));
  std::cout << "pi is ~ " <<  pi << std::endl

  return 0;
}
```

```
#include <iostream>

int main() {
  double pi = 0.0; double h = 1./(double) N;
  for (size_t i = N/4; i < N/2; ++i)
    pi += (h * 4.0) / (1.0 + (i * h * i * h));
  std::cout << "pi is ~ " <<  pi << std::endl

  return 0;
}
```

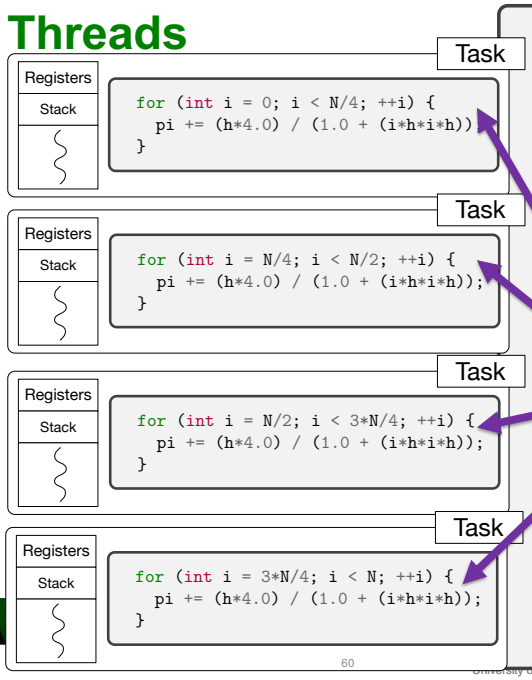
```
#include <iostream>

int main() {
  double pi = 0.0; double h = 1./(double) N;
  for (size_t i = N/2; i < 3*N/4; ++i)
    pi += (h * 4.0) / (1.0 + (i * h * i * h));
  std::cout << "pi is ~ " <<  pi << std::endl

  return 0;
}
```

```
#include <iostream>

int main() {
  double pi = 0.0; double h = 1./(double) N;
  for (size_t i = 3*N/4; i < N; ++i)
    pi += (h * 4.0) / (1.0 + (i * h * i * h));
  std::cout << "pi is ~ " <<  pi << std::endl

  return 0;
}
```

Communicating sequential processes / SPMD

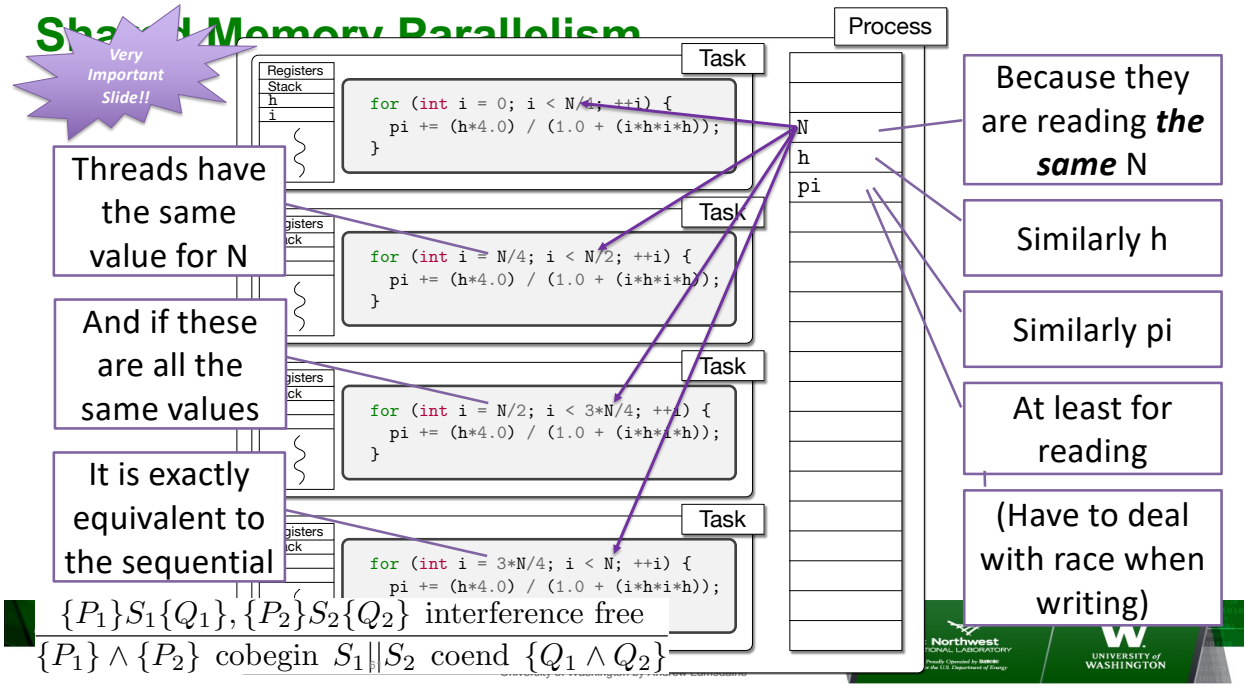


Threads



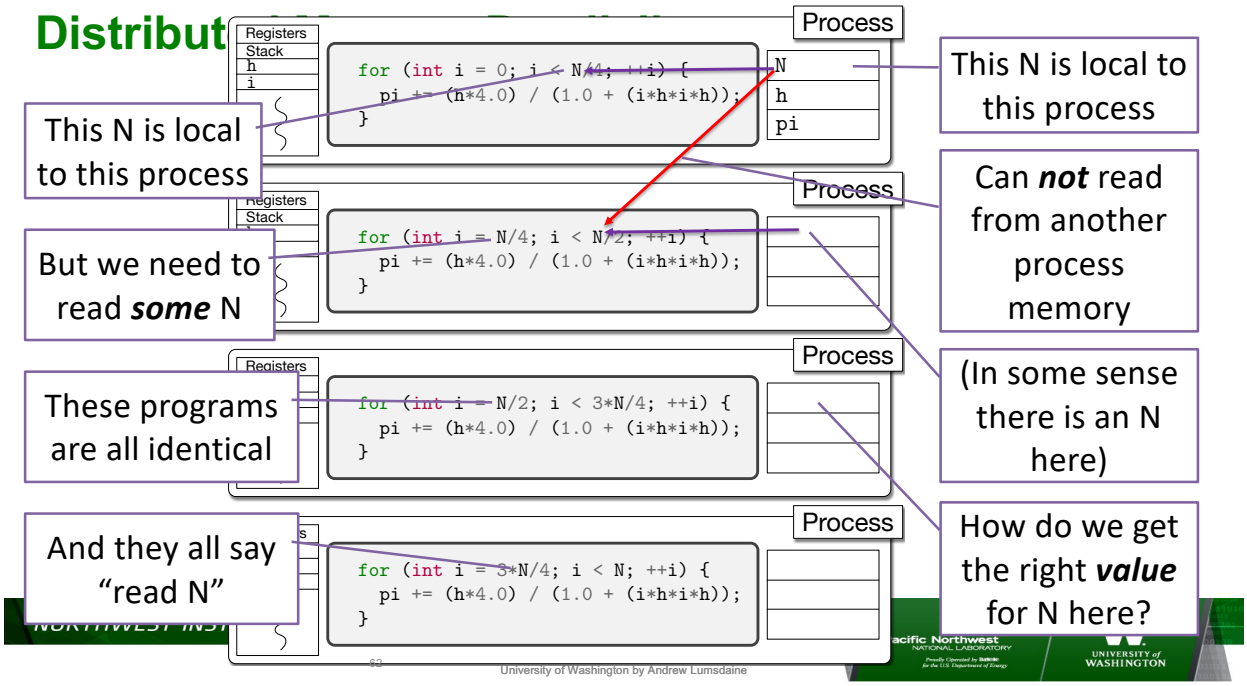

```
double pi = 0.0;

void pi_helper(int begin, int end, double h) {
  for (int i = begin; i < end; ++i)
    pi += (h*4.0) / (1.0 + (i*h*i*h));
}

int main(int argc, char* argv[]) {
  int N = 1024 * 1024; double h = 1.0/ (double)N;

  std::thread t0(pi_helper, 0,     N/4,   h);
  std::thread t1(pi_helper, N/4,   N/2,   h);
  std::thread t2(pi_helper, N/2,   3*N/4, h);
  std::thread t3(pi_helper, 3*N/4, N,     h);

  t0.join();  t1.join();  t2.join();  t3.join();

  std::cout << "pi is ~ " << pi << std::endl;

  return 0;
}
```

Shared Memory Parallelism

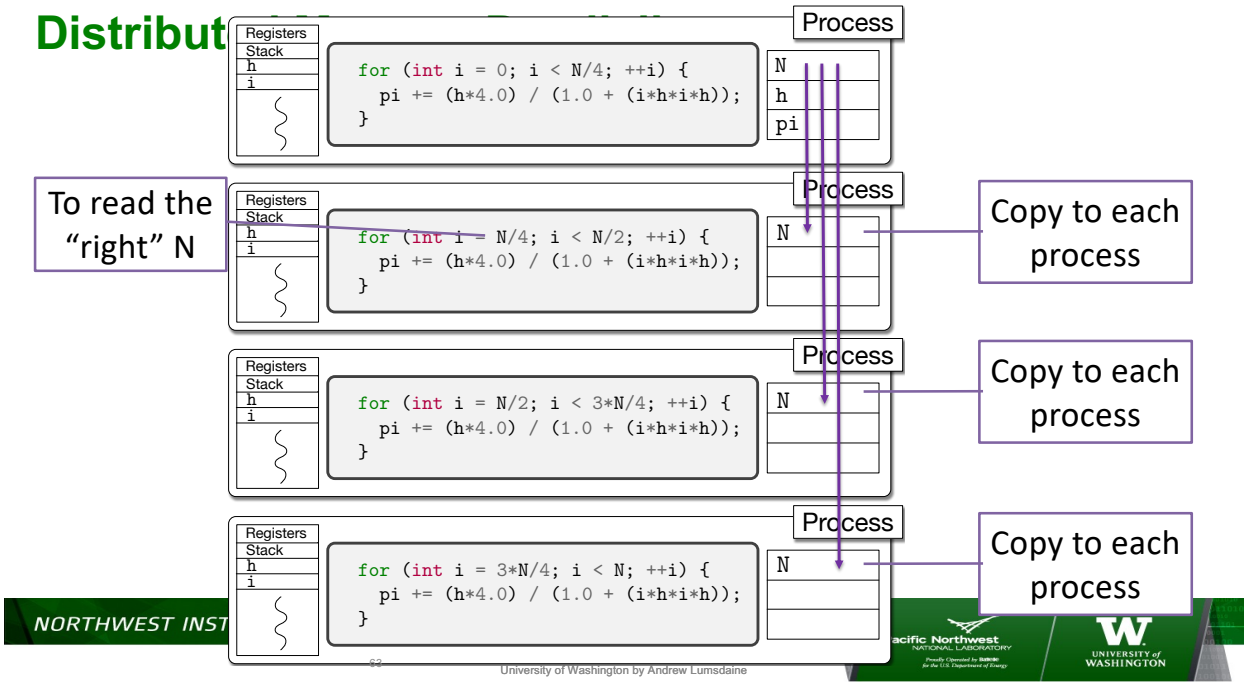


$$\frac{\{P_1\}S_1\{Q_1\}, \{P_2\}S_2\{Q_2\} \text{ interference free}}{\{P_1\} \wedge \{P_2\} \text{ cobegin } S_1 || S_2 \text{ coend } \{Q_1 \wedge Q_2\}}$$

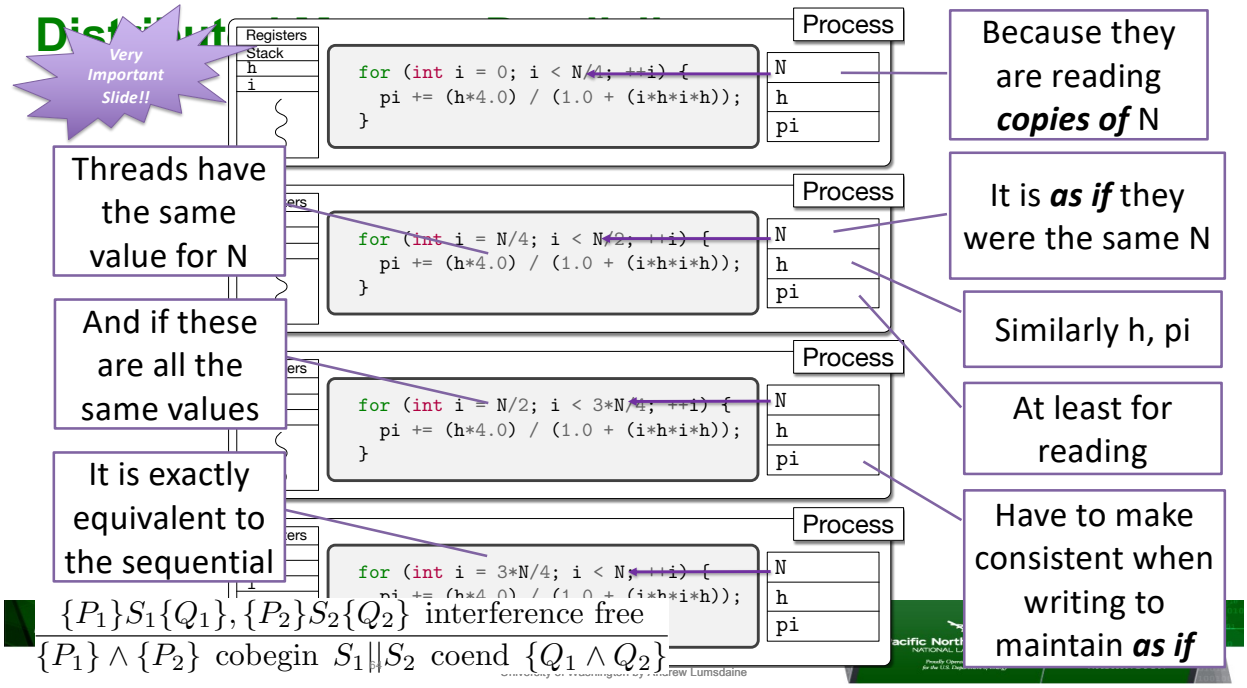
Distributed Memory Parallelism



Distribut



Distribut



$$\frac{\{P_1\}S_1\{Q_1\}, \{P_2\}S_2\{Q_2\} \text{ interference free}}{\{P_1\} \wedge \{P_2\} \text{ cobegin } S_1 || S_2 \text{ coend } \{Q_1 \wedge Q_2\}}$$

SPMD? Single program multiple data?

```
#include <iostream>

int main() {
  double pi = 0.0; double h = 1./(double) N;
  for (size_t i = 0; i < N/4; ++i)
    pi += (h * 4.0) / (1.0 + (i * h * i * h));
  std::cout << "pi is ~ " <<  pi << std::endl

  return 0;
}
```

Multiple data (different limits)

Multiple program (limits hard-coded)

```
#include <iostream>

int main() {
  double pi = 0.0; double h = 1./(double) N;
  for (size_t i = N/2; i < 3*N/4; ++i)
    pi += (h * 4.0) / (1.0 + (i * h * i * h));
  std::cout << "pi is ~ " <<  pi << std::endl

  return 0;
}
```


Single Program Multiple Data (SPMD)

```
#include <iostream>

int main() {
  double pi = 0.0; double h = 1./(double) N;
  for (size_t i = begin; i < end; ++i)
    pi += (h * 4.0) / (1.0 + (i * h * i * h));
  std::cout << "pi is ~ " <<  pi << std::endl

  return 0;
}
```

Multiple data (different limits)

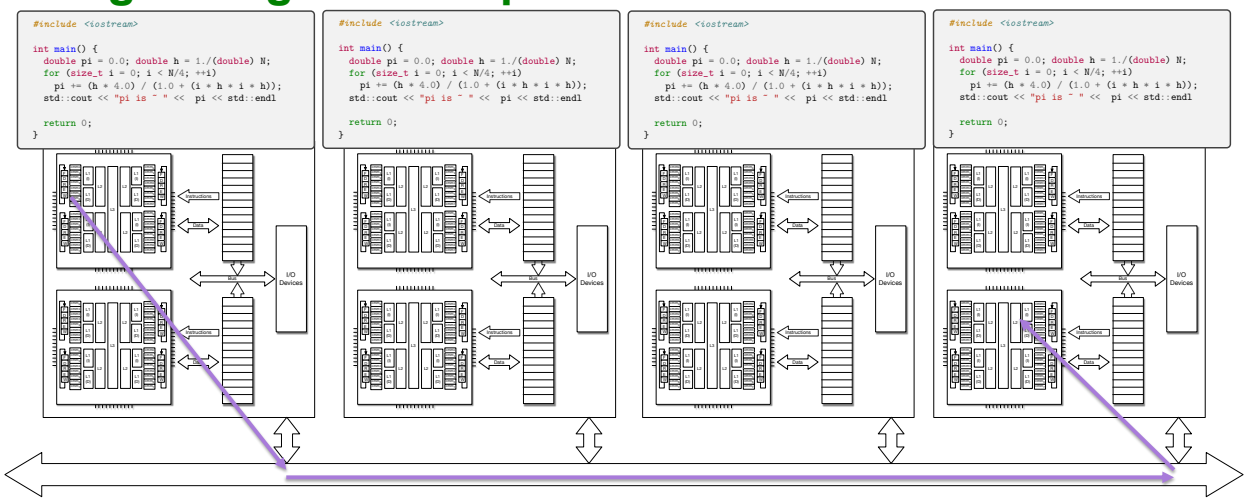
Different, provided each process has a different begin, end

But this is now exactly the same program

Single program

```
#include <iostream>

int main() {
  double pi = 0.0; double h = 1./(double) N;
  for (size_t i = begin; i < end; ++i)
    pi += (h * 4.0) / (1.0 + (i * h * i * h));
  std::cout << "pi is ~ " <<  pi << std::endl

  return 0;
}
```

Single Program Multiple Data



Single Program Multiple Data (SPMD)

```
#include <iostream>

int main() {
  double pi = 0.0; double h = 1./(double) N;
  for (size_t i = begin; i < end; ++i)
    pi += (h * 4.0) / (1.0 + (i * h * i * h));
  std::cout << "pi is ~ " << pi << std::endl

  return 0;
}
```

How do we set N, begin, end?

How do we do these two things?

We need exactly the same N everywhere

```
#include <iostream>

int main() {
  double pi = 0.0; double h = 1./(double) N;
  for (size_t i = begin; i < end; ++i)
    pi += (h * 4.0) / (1.0 + (i * h * i * h));
  std::cout << "pi is ~ " << pi << std::endl

  return 0;
}
```

These are exactly the same program

Each program computes same thing

But a different begin and end everywhere

Single Program Multiple Data

```
int main(size_t argc, char* argv[]) {
  size_t N    = atol(argv[1]);
  double h    = 1.0 / (double) N;
  double pi   = 0.0;

  for (size_t i = begin; i < end; ++i)
    pi_i += (h * 4.0) / (1.0 + (i * h * i

  std::cout << "pi is ~ " << pi << std::en

  return 0;
}
```

We can get N from the command line

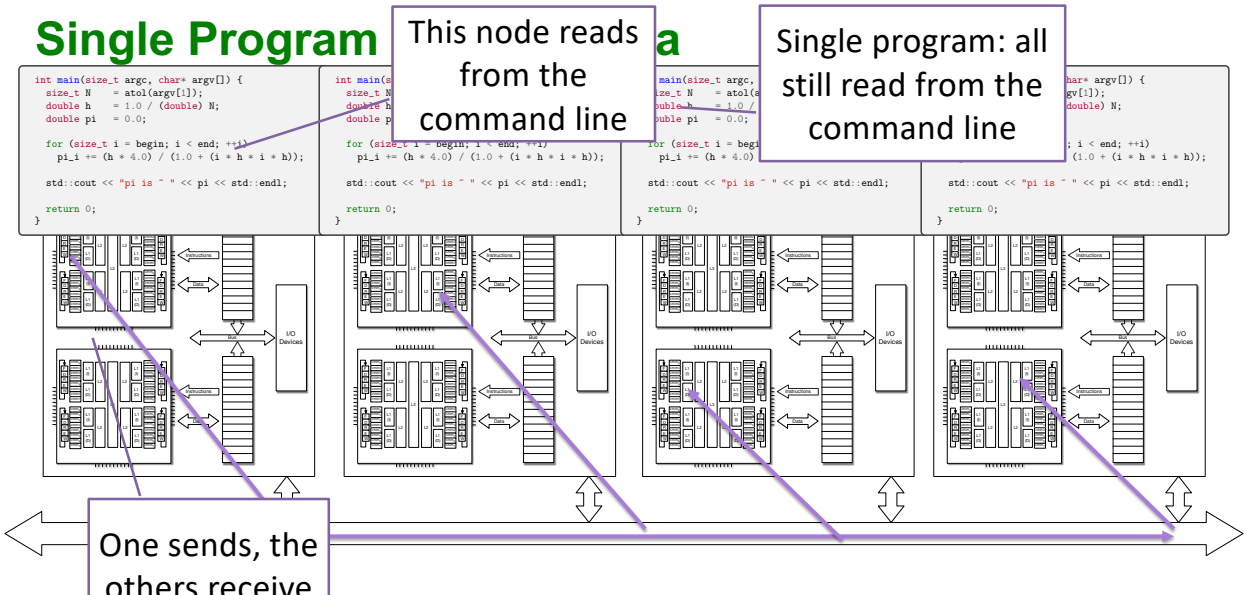
From every node? That's a lot of typing

Better to get it at just one node and send it around

Single Program a

This node reads from the command line

Single program: all still read from the command line



Single Program Multiple Data

```
int main(size_t argc, char* argv[]) {
  size_t N    = atol(argv[1]);
  double h    = 1.0 / (double) N;
  double pi   = 0.0;

  for (size_t i = begin; i < end; ++i)
    pi_i += (h * 4.0) / (1.0 + (i * h * i

  std::cout << "pi is ~ " << pi << std::e

  return 0;
```

How do we get the same program to different things

While keeping them the same?

Hint: multiple data

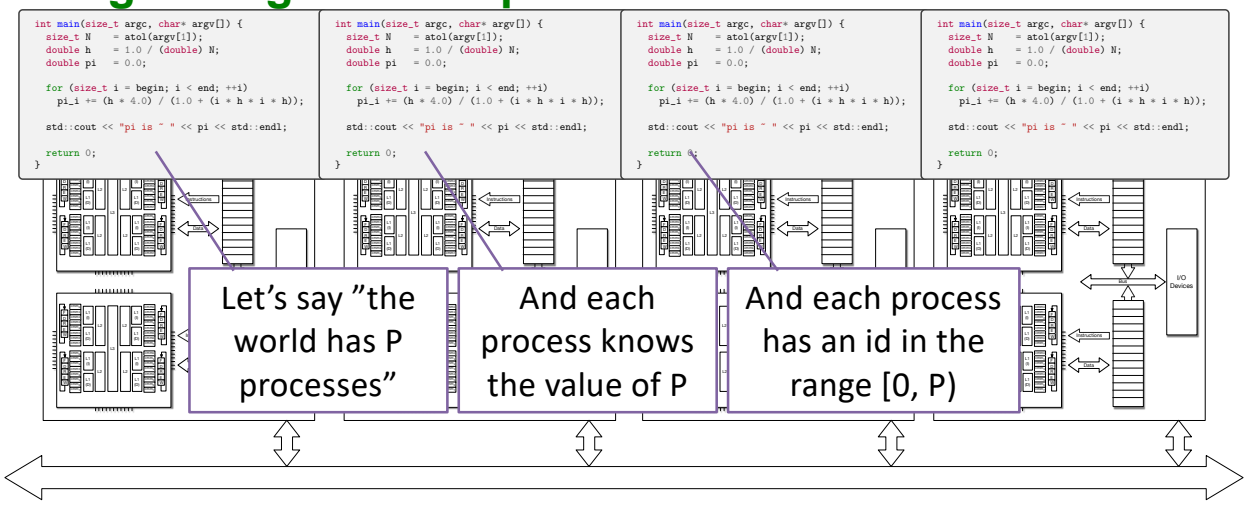
With, say, an if statement

Where have we already seen identical functions that need to distinguish themselves?

fork()

How did they distinguish each other?

Single Program Multiple Data



A better name than MIMD or SPMD

Distinguished replicated processes, distributed data

(DRPDD)

Pronounced “drop dee”

Single Program

```
int main(size_t argc, char* argv[]) {
  size_t partitions = magically_get_P();
  size_t my_id      = magically_get_id();

  size_t N          = atol(argv[1]);
  size_t block_size = N / partitions;
  size_t begin      = block_size * my_id;
  size_t end        = block_size * (my_id + 1);
  double h          = 1.0 / (double) N;

  for (size_t i = begin; i < end; ++i)
    pi += (h * 4.0) / (1.0 + (i * h * i * h));

  std::cout << "pi is ~ " << pi << std::endl;

  return 0;
}
```

Magically get P

Magically get id

Oops

Oops

This distinguishes the processes

Distinguished Replicated Process

```
int main(size_t argc, char* argv[]) {
  size_t partitions = magically_get_P();
  size_t my_id      = magically_get_id();

  size_t N          = -1;
  if (0 == my_id) {
    N = atol(argv[1]);
  }
  size_t block_size = N / partitions;
  size_t begin      = block_size * my_id;
  size_t end        = block_size * (my_id + 1);
  double h          = 1.0 / (double) N;

  for (size_t i = begin; i < end; ++i)
    pi += (h * 4.0) / (1.0 + (i * h * i * h));

  if (0 == my_id) {
    std::cout << "pi is ~ " << pi << std::endl;
  }

  return 0;
}
```

No and no.

Only one node reads N

Is that going to be correct?

Compute begin and end

Is that going to be correct?

Only one node prints pi

Distinguished Replicated Process

```
int main(size_t argc, char* argv[]) {
  size_t partitions = magically_get_P();
  size_t my_id      = magically_get_id();

  size_t N          = -1;
  if (0 == my_id) {
    N = atol(argv[1]);
  }
  size_t block_size = N / partitions;
  size_t begin      = block_size * my_id;
  size_t end        = block_size * (my_id + 1);
  double h          = 1.0 / (double) N;

  for (size_t i = begin; i < end; ++i)
    pi += (h * 4.0) / (1.0 + (i * h * i * h));

  if (0 == my_id) {
    std::cout << "pi is ~ " << pi << std::endl;
  }

  return 0;
}
```

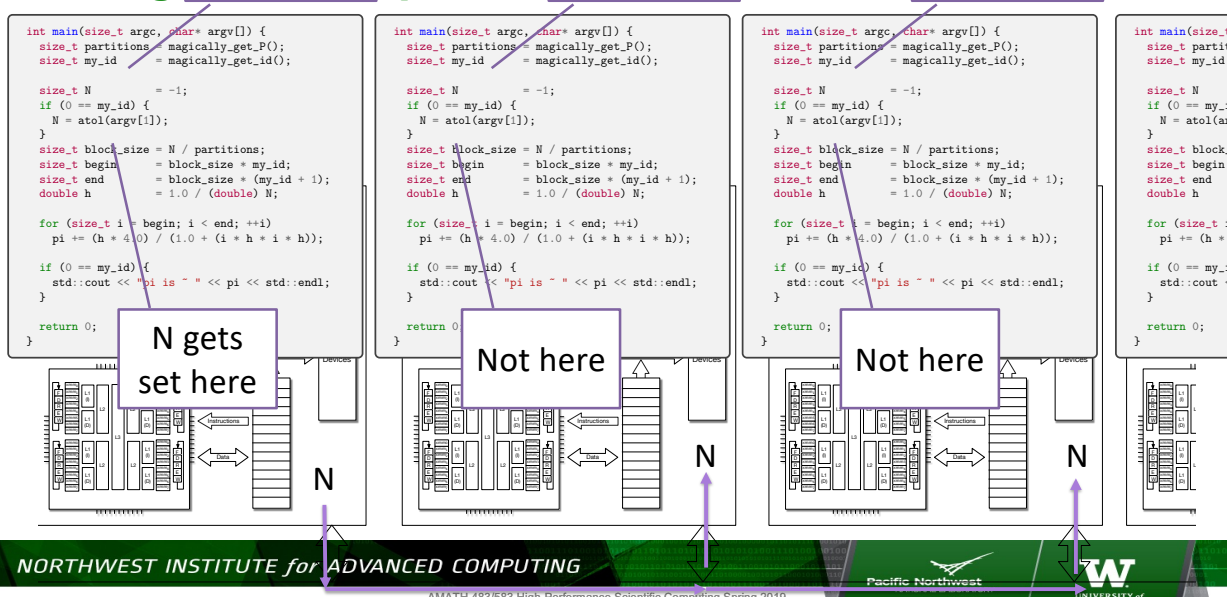
What is this value?

Distingu eplicate ses

my_id == 0

my_id == 1

my_id == 2



Finally

Get our id and number of other nodes

id 0 gets N

This pattern is ubiquitous

id shares N

Everyone computes their own partial

id 0 collects all partials, adds them, and prints

```
int main(size_t argc, char* argv[]) {
  size_t partitions = magically_get_P();
  size_t my_id      = magically_get_id();

  size_t N          = -1;
  if (0 == my_id) {
    N = atol(argv[1]);
  }
  N                 = magically_share(N);
  size_t block_size = N / partitions;
  size_t begin      = block_size * my_id;
  size_t end        = block_size * (my_id + 1);
  double h          = 1.0 / (double) N;

  for (size_t i = begin; i < end; ++i)
    pi += (h * 4.0) / (1.0 + (i * h * i * h));

  if (0 == my_id) {
    pi              = magically_combine(pi);
    std::cout << "pi is ~ " << pi << std::endl;
  }

  return 0;
}
```

MPI

Get our id and number of other nodes

id 0 gets N

This pattern is ubiquitous

id shares N

Everyone has same N

Everyone computes their own partial

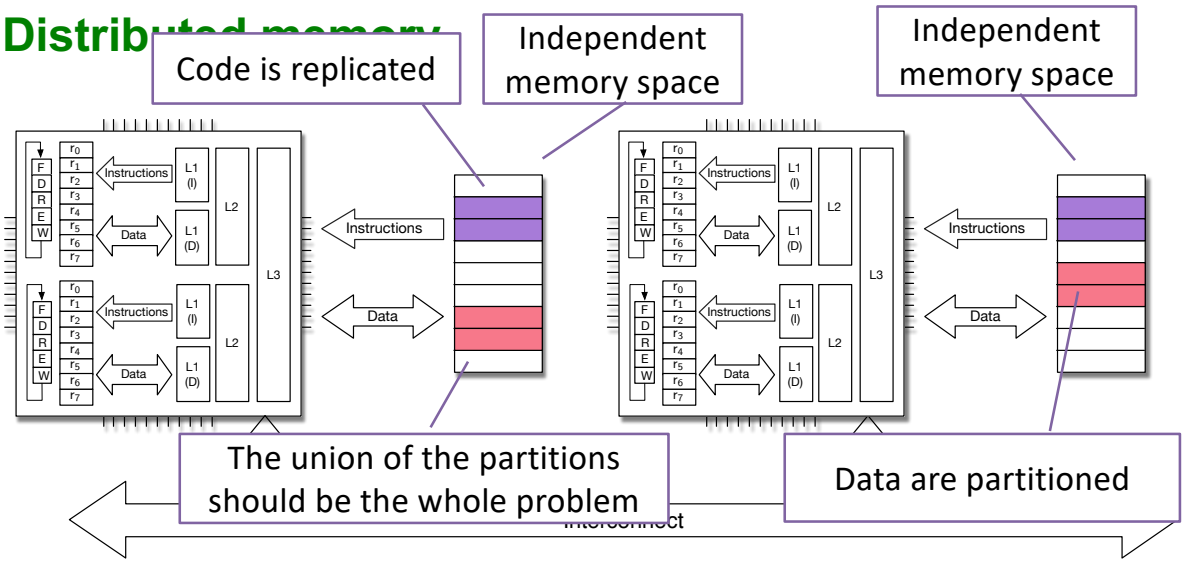
id 0 collects all partials, adds them, and prints

id 0 is root

```
int main(int argc, char* argv[]) {
  size_t intervals = 1024 * 1024;

  MPI::Init();

  int myrank = MPI::COMM_WORLD.Get_rank();
  int mysize = MPI::COMM_WORLD.Get_size();

  if (0 == myrank) {
    if (argc >= 2) intervals = std::atol(argv[1]);
  }

  MPI::COMM_WORLD.Bcast(&intervals, 1, MPI::UNSIGNED_LONG, 0);

  size_t blocksize = intervals / mysize;
  size_t begin     = blocksize * myrank;
  size_t end       = blocksize * (myrank + 1);
  double h         = 1.0 / ((double)intervals);

  double pi        = 0.0;
  for (size_t i = begin; i < end; ++i) {
    pi += 4.0 / (1.0 + (i * h * i * h));

  MPI::COMM_WORLD.Reduce(&mypi, &pi, 1, MPI::DOUBLE, MPI::SUM, 0);

  if (0 == myrank) {
    std::cout << "pi is approximately " << pi << std::endl;
  }

  MPI::Finalize();

  return 0;
}
```


The Message Passing Interface (MPI)

Distrib



Finding Concurrency

$$h\sum_{i=0}^{N/4-1}\frac{4}{1+(ih)^2}$$

$$h\sum_{i=N/4}^{N/2-1}\frac{4}{1+(ih)^2}$$

$$h\sum_{i=N/2}^{3N/4-1}\frac{4}{1+(ih)^2}$$

$$h\sum_{i=3N/4}^{N-1}\frac{4}{1+(ih)^2}$$

```
int main() {
  double pi = 0.0;  int N = 1024*1024;

  for (int i = 0; i < N/4; ++i)
    pi += (h*4.0) / (1.0 + (i*h*i*h));

  for (int i = N/4; i < N/2; ++i)
    pi += (h*4.0) / (1.0 + (i*h*i*h));

  for (int i = N/2; i < 3*N/4; ++i)
    pi += (h*4.0) / (1.0 + (i*h*i*h));

  for (int i = 3*N/4; i < N; ++i)
    pi += (h*4.0) / (1.0 + (i*h*i*h));

  std::cout << "pi ~ " << pi << std::endl;
  return 0;
}
```

Processes

```
int main() {
  double pi = 0.0; double h = 1./(double) N;
  for (size_t i = 0; i < N/4; ++i)
    pi += (h * 4.0) / (1.0 + (i * h * i * h));
  std::cout << "pi is ~ " <<  pi << std
```

```
int main() {
  double pi = 0.0; double h = 1./(double) N;
  for (size_t i = N/4; i < N/2; ++i)
    pi += (h * 4.0) / (1.0 + (i * h * i * h));
  std::cout << "pi is ~ " <<  pi << std
```

```
int main() {
  double pi = 0.0; double h = 1./(double) N;
  for (size_t i = N/2; i < 3*N/4; ++i)
    pi += (h * 4.0) / (1.0 + (i * h * i * h));
  std::cout << "pi is ~ " <<  pi << std
```

```
int main() {
  double pi = 0.0; double h = 1./(double) N;
  for (size_t i = 3*N/4; i < N; ++i)
    pi += (h * 4.0) / (1.0 + (i * h * i * h));
  std::cout << "pi is ~ " <<  pi << std::endl

  return 0;
}
```

Process

Process

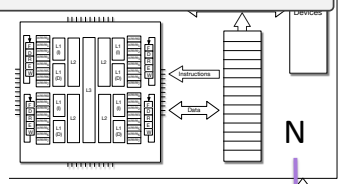
Process

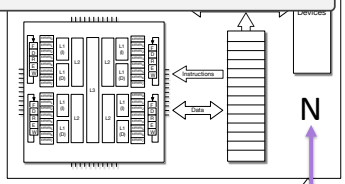
Process

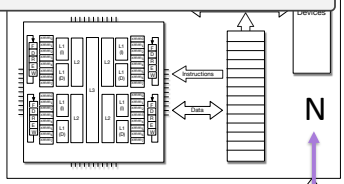
```
double pi = 0.0;

void pi_helper(int begin, int end, double h) {
  for (int i = begin; i < end; ++i)
    pi += (h*4.0) / (1.0 + (i*h*i*h));
}

int main(int argc, char* argv[]) {
  int N = 1024 * 1024; double h = 1.0/ (double)N;

  std::cout << "pi is ~ " << pi << std::endl;

  return 0;
}
```

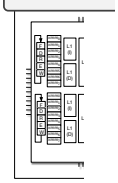

Distinguished Replicated Processes

```
int main(size_t argc, char* argv[]) {
  size_t partitions = magically_get_P();
  size_t my_id      = magically_get_id();

  size_t N          = -1;
  if (0 == my_id) {
    N = atol(argv[1]);
  }
  size_t block_size = N / partitions;
  size_t begin      = block_size * my_id;
  size_t end        = block_size * (my_id + 1);
  double h          = 1.0 / (double) N;

  for (size_t i = begin; i < end; ++i)
    pi += (h * 4.0) / (1.0 + (i * h * i * h));

  if (0 == my_id) {
    std::cout << "pi is ~ " << pi << std::endl;
  }

  return 0;
}
```









MPI

Get our id and number of other nodes

id 0 gets N

This pattern is ubiquitous

id shares N

Everyone has same N

Everyone computes their own partial

id 0 collects all partials, adds them, and prints

id 0 is root

```
int main(int argc, char* argv[]) {
  size_t intervals = 1024 * 1024;

  MPI::Init();

  int myrank = MPI::COMM_WORLD.Get_rank();
  int mysize = MPI::COMM_WORLD.Get_size();

  if (0 == myrank) {
    if (argc >= 2) intervals = std::atol(argv[1]);
  }

  MPI::COMM_WORLD.Bcast(&intervals, 1, MPI::UNSIGNED_LONG, 0);

  size_t blocksize = intervals / mysize;
  size_t begin     = blocksize * myrank;
  size_t end       = blocksize * (myrank + 1);
  double h         = 1.0 / ((double)intervals);

  double pi        = 0.0;
  for (size_t i = begin; i < end; ++i) {
    pi += 4.0 / (1.0 + (i * h * i * h));

  MPI::COMM_WORLD.Reduce(&mypi, &pi, 1, MPI::DOUBLE, MPI::SUM, 0);

  if (0 == myrank) {
    std::cout << "pi is approximately " << pi << std::endl;
  }

  MPI::Finalize();

  return 0;
}
```


The Message Passing Interface (MPI)

Thank You!

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