Matrix Formulation

- Lexicographically order unknowns (note some will be boundary values)

$$\phi_{ij} \Rightarrow x_k \quad \text{where} \quad k = iN^2 + j$$

2D to 1D conversion

- Formulate as a matrix problem:

N^2 Linear equations

Each grid point gives us one equation

$$\frac{x_{i+1} + x_{i-1} + x_{i+N} + x_{i-N} - 4x_i}{h^2} = 0$$

$$\frac{1}{h^2}\begin{bmatrix} 4 & -1 & \cdots & -1 & & \\ -1 & \ddots & \ddots & \ddots & \ddots & \\ \vdots & \ddots & \ddots & \ddots & \ddots & -1 \\ -1 & \ddots & \ddots & \ddots & \ddots & \vdots \\ & \ddots & \ddots & \ddots & \ddots & -1 \\ & & -1 & \cdots & -1 & 4 \end{bmatrix} \begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ \vdots \\ \\ \end{bmatrix} = \begin{bmatrix} b_0 \\ b_1 \\ b_2 \\ \vdots \\ \\ \end{bmatrix}$$

Matrix representation