# AMATH 483/583 High Performance Scientific Computing

# Lecture 5: CPUs, hierarchical memory, matrices

Andrew Lumsdaine
Northwest Institute for Advanced Computing
Pacific Northwest National Laboratory
University of Washington
Seattle, WA

# Overview

- Classes, Vectors, const, overloading
- Tour of computer architecture
- Class Matrix
- Matrix matrix product

# SC'19 Student Cluster Competition Call-Out!

- Teams work with advisor and vendor to design and build a cutting-edge, commercially available cluster constrained by the 3000-watt power limit
- Cluster run a variety of HPC workflows, ranging from being limited by CPU performance to being memory bandwidth limited to I/O intensive
- Teams are comprised of six undergrad or high-school students plus advisor

https://sc19.supercomputing.org/program/studentssc/student-cluster-competition/

Team Meetings
Mondays 5:30PM-8:00PM

# C++ Core Guidelines related to classes

- C.1: Organize related data into structures (structs or classes)
- C.3: Represent the distinction between an interface and an implementation using a class
- C.4: Make a function a member only if it needs direct access to the representation of a class
- C.10: Prefer concrete types over class hierarchies
- C.11: Make concrete types regular

http://isocpp.github.io/CppCoreGuidelines/CppCoreGuidelines

# Anatomy of a C++ class

```
class Vector {
public:
  Vector(size_t M) : num_rows_(M), storage_(num_rows_) {}

  double& operator()(size_t i) { return storage_[i]; }

  size_t num_rows() const { return num_rows_; }

private:
  size_t              num_rows_;
  std::vector<double> storage_;
};
```

Declares interface

Hides definition

Public accessors

Private data

Maintain invariants

# Anatomy of a C++ class

```
class Vector {
public:
  Vector(size_t M) : num_rows_(M), storage_(num_rows_) {}

  double& operator()(size_t i) { return storage_[i]; }

  size_t num_rows() const { return num_rows_; }

private:
  size_t              num_rows_;
  std::vector<double> storage_;
};
```

Declares interface

Hides

Encapsulation

Public accessors

Private data

Maintain invariants

# The Vector class so Far

- Encapsulates vector data
- Member data for dimensions (rows) and for storing elements
- Member function to get number of rows
- Separate interface and implementation via public / private

- Three more things:
  - How to bring a Vector into being (“constructors”)
  - Function for getting vector data
  - Function for setting vector data

Also called function call operator

Revisit operator()

Can create ***function objects***

# Constructors

- The C++ compiler “knows” about built-in types
- When a variable of a built-in type is declared, the compiler just needs to allocate space for it
- C++ classes are user-defined
- Compiler can do its best (default constructor), but usually we need to do more to create a well-defined object

- For example, a well-defined vector should be given its (positive) dimension ***when it is created***. (And the data initialized.)

# Constructors

```
int x = 42;

Vector x;

Vector x(27);

std::cout << "x is " << x.num_rows() << " in length." << std::cout;
```

Built-in type, compiler allocates known amount of space

Default constructor is invoked when variable is declared with no arguments

Compiler creates x with ***default constructor***

In this case, the constructor that takes an integer argument

Compiler creates x by making a call to a specific constructor

Create a Vector x with 27 elements

Because that is how we defined the constructor

# Declaring Constructors

```
#include <vector>

class Vector {
public:
  Vector();
  Vector(size_t M);

  size_t num_rows() const  { r

private:
  size_t              num_rows_;
  std::vector<double> storage_;
};
```

A constructor is defined using the name of the class

And then the arguments

Can be ***overloaded*** (different functions distinguished by argument types)

Where have we already seen overloading?

# Defining Constructors

Vector.hpp

```cpp
#include <vector>

class Vector {
public:
  Vector();
  Vector(size_t M);

  size_t num_rows() const  { return num_rows; }

private:
  size_t              num_rows_;
  std::vector<double> storage_;
};
```

Vector.cpp

```cpp
#include "Vector.hpp"

Vector::Vector(size_t M) {
  num_rows_ = M;
  storage_ = std::vector<double>(num_rows);
}

Vector::Vector() {
  num_rows_ = 1;
  storage_ = std::vector<double>(num_rows_);
}
```

# Defining Constructors

Vector.hpp

```
#include <vector>

class Vector {
public:
  Vector() {
    num_rows_ = 1;
    storage_  = std::vector<double>(num_rows);
  }
  Vector(size_t M) {
    num_rows_ = M;
    storage_  = std::vector<double>(num_rows);
  }

  size_t num_rows() const { return num_rows; }

private:
  size_t              num_rows_;
  std::vector<double> storage_;
};
```

# Initialization

- We have said that variables should always be initialized
- Different syntaxes

```
int a = 42;

int b = int(42);

int c(42);

int d = { 42 };

std::vector<double> x = std::vector<double>(27);

std::vector<double> y(27);
```

c(42)

y(27)

# Defining Constructors

Vector.hpp

```
#include <vector>

class Vector {
public:
  Vector(size_t M) : num_rows_(M), storage_(num_rows_) {}

  size_t num_rows() const { return num_rows_; }

private:
  size_t              num_rows_;
  std::vector<double> storage_;
};
```

Note order of initialization

Initialization syntax
Introduce with :
Construct data members

Omit default constructor (why?)

Note order of declaration

# Defining Constructors

Vector.hpp

```cpp
#include <vector>

class Vector {
public:
  Vector(size_t M) : num_rows_(M), storage_(num_rows_) {}

  size_t num_rows() const { return num_rows_; }

private:
  size_t              num_rows_;
  std::vector<double> storage_;
};
```

Initialization

Primordial

Object doesn't yet exist

Object exists

# What Should operator() return?

```
class Vector {
public:
  double& operator()(size_t i);

private:
  size_t              num_rows_;
  std::vector<double> storage_;
};
```

Return a ***reference*** to internal member data

```
Vector x(5);
```

```
Vector x(5);

double foo = x(3);
x(2) = 0.0;
```

Can assign to internal data through the reference

Can read from internal data through the reference

# All Together

Vector.hpp

```
#include <vector>

class Vector {
public:
  Vector(size_t M) : num_rows_(M), storage_(num_rows_) {}

  double& operator()(size_t i) { return storage_[i]; }

  size_t num_rows() const { return num_rows_; }

private:
  size_t              num_rows_;
  std::vector<double> storage_;
};
```

# Reprise operator+()

```
#include <vector>

class Vector {
public:
  Vector operator+(const Vector& y);

private:
  size_t              num_rows_;
  std::vector<double> storage_;
};
```

# Reprise operator+()

C.4: Make a function a member only if it needs direct access to the representation of a class

```
#include <vector>

class Vector {
public:
  Vector operator+(const Vector& y) {
    Vector z(num_rows_);
    for (size_t i = 0; i < num_rows_; ++i) {
      z.storage_[i] = storage_[i] + y.storage[i];
    }
  }

private:
  size_t              num_rows_;
  std::vector<double> storage_;
};
```

Does this need to be a member?

Data for z

Data for “x”

Data for y

# All Together

Vector.hpp

```
#include <vector>

class Vector {
public:
  Vector(size_t M) : num_rows_(M), storage_(num_rows_) {}

  double& operator()(size_t i) { return storage_[i]; }

  size_t num_rows() const { return num_rows_; }

private:
  size_t              num_rows_;
  std::vector<double> storage_;
};
```

Amath583.cpp

```
#include "Vector.hpp"

Vector operator+(const Vector& x, const Vector& y) {
  Vector z(x.num_rows());
  for (size_t i = 0; i < z.num_rows(); ++i) {
    z(i) = x(i) + y(i);
  }
}
```

Can access via operator()

Don’t need access to internals

Return a Vector

Take args by const reference

Nicely symmetric

# All Together

Vector.hpp

```
#include <vector>

class Vector {
public:
  Vector(size_t M) : num_rows_(M), storage_(num_rows_) {}

  double& operator()(size_t i) { return storage_[i]; }

  size_t num_rows() const { return num_rows_; }

private:
  size_t              num_rows_;
  std::vector<double> storage_;
};
```

Amath583.hpp

```
#include "Vector.hpp"

Vector operator+(const Vector& x, const Vector& y);
```

Amath583.cpp

```
#include "Vector.hpp"
#include "amath583.hpp"

Vector operator+(const Vector& x, const Vector& y) {
  Vector z(x.num_rows());
  for (size_t i = 0; i < z.num_rows(); ++i) {
    z(i) = x(i) + y(i);
  }
}
```

# Not quite finished

```cpp
#include "Vector.hpp"

int main() {

  Vector x(100), y(100), z(100), w(100);

  z = x + y;

  return 0;
}
```

```
% c++ constness.cpp
constness.cpp:20:12: error: no matching function for call to object of type 'const Vector'
    z(i) = x(i) + y(i);
           ^
constness.cpp:7:11: note: candidate function not viable: 'this' argument has type
      'const Vector', but method is not marked const
  double& operator()(size_t i) { return storage_[i]; }
          ^
constness.cpp:20:19: error: no matching function for call to object of type 'const Vector'
    z(i) = x(i) + y(i);
                  ^
constness.cpp:7:11: note: candidate function not viable: 'this' argument has type
      'const Vector', but method is not marked const
  double& operator()(size_t i) { return storage_[i]; }
          ^
2 errors generated.
```

# Constness

Vector.hpp

```cpp
#include <vector>

class Vector {
public:
  Vector(size_t M) : num_rows_(M), storage_(num_rows_) {}

  double& operator()(size_t i) { return storage_[i]; }

  size_t num_rows() const { return num_rows_; }

private:
  size_t              num_rows_;
  std::vector<double> storage_;
};
```

"this" is not const

x and y are defined to be const

Amath583.hpp

```cpp
#in

Vector operator+(const Vector& x, const Vector& y);
```

Amath583.cpp

```cpp
#include "Vector.hpp"
#include "amath583.hpp"

Vector operator+(const Vector& x, const Vector& y) {
  Vector z(x.num_rows());
  for (size_t i = 0; i < z.num_rows(); ++i) {
    z(i) = x(i) + y(i);
  }
}
```

# Overloading

```
void foo(size_t i) {
  std::cout << "foo(size_t i)" << std::endl;
}

void foo(double d) {
  std::cout << "foo(double d)" << std::endl;
}
```

Takes a size_t

Takes a double

```
int main() {

  size_t a = 0;
  double b = 0.0;

  foo(a);
  foo(b);

  return 0;
}
```

```
% ./a.out
foo(size_t i)
foo(double d)
```

# Overloading

```
void foo(size_t i) {
  std::cout << "void foo(size_t i)" << std::endl;
}

size_t foo(size_t i) {
  std::cout << "size_t foo(size_t i)" << std::endl;
}
```

Returns void

Returns size_t

```
% c++ overload.cpp
overload.cpp:7:8: error: functions that differ only in their return type cannot be overloaded
size_t foo(size_t i) {
~~~~~~ ^
overload.cpp:3:6: note: previous definition is here
void foo(size_t i) {
~~~~ ^
```

```
int main() {

  size_t a = 0;
  size_t b = 0;

  foo(a);
  double c = foo(a);

  return 0;
}
```

Have to pick the function then call it

# No overloading on return values

```
size_t foo(size_t i) {
  std::cout << "size_t foo(size_t i)" << std::

  return i;
}


int main() {

  size_t a = 0;

  foo(a);
  size_t b = foo(a);
  double c = foo(a);

  return 0;
}
```

What happens to the return value is not the concern of the function

Ignore return value

Assign to size_t

Assign to double

# Constness

```
double parens(double& x, size_t i) {
  std::cout << "called non const parens" << std::endl;
  double y = x;
  // .. some things
  return y;
}
```

x is a ref

```
c++ const3.cpp
const3.cpp:27:14: error: no matching function for call to 'parens'
  double w = parens(z, 27);
             ^~~~~~
const3.cpp:13:8: note: candidate function not viable: 1st argument ('const double') would lose const
      qualifier
double parens(double& x, size_t i) {
       ^
```

```
int main() {

  double x = 5.0;
  double y = parens(x);

  const double z = 5.0;
  double w = parens(z);

  double a = parens(5.0);
  double b = parens(x + y);

  const double c = parens(x + y + z + 5.0);

  return 0;
}
```

```
const3.cpp:29:14: error: no matching function for call to 'parens'
  double a = parens(5.0, 27);
             ^~~~~~
const3.cpp:13:8: note: candidate function not viable: expects an l-value for 1st argument
double parens(double& x, size_t i) {
       ^
```

Not okay

```
const3.cpp:32:20: error: no matching function for call to 'parens'
  const double c = parens(x + y + 5.0, 27);
                   ^~~~~~
const3.cpp:13:8: note: candidate function not viable: expects an l-value for 1st argument
double parens(double& x, size_t i) {
       ^
```

Not okay

# Constness

```
double parens(const double& x, size_t i) {
  std::cout << "called const parens" << std::endl;
  double y = x;
  // .. some things
  return y;
}
```

x is a const ref

```
int main() {

  double x = 5.0;
  double y = parens(x);

  const double z = 5.0;
  double w = parens(z);

  double a = parens(5.0);
  double b = parens(x + y);

  const double c = parens(x + y + z + 5.0);

  return 0;
}
```

okay

okay

okay

okay

```
./a.out
called const parens
called const parens
called const parens
called const parens
called const parens
```

# Constness

x is a const ref

```
double parens(const double& x, size_t i) {
  std::cout << "called const parens" << std::endl;
  double y = x;
  // .. some things
  return y;
}
```

x is a ref

```
double parens(double& x, size_t i) {
  std::cout << "called non const parens" << std::endl;
  double y = x;
  // .. some things
  return y;
}
```

```
int main() {

  double x = 5.0;
  double y = parens(x);

  const double z = 5.0;
  double w = parens(z);

  double a = parens(5.0);
  double b = parens(x + y);

  const double c = parens(x + y + z + 5.0);

  return 0;
}
```

x is lvalue

z marked const

5.0 is an rvalue

x + y is an rvalue

```
./a.out
called non const parens
called const parens
called const parens
called const parens
called const parens
```

# Why not always pass const reference?

```
double parens(const double& x, size_t i) {
  std::cout << "called const parens" << std::endl;
  double y = x;
  // .. some things
  return x;
}
```

Return double

```
int main() {
  double y = 0.5;
  double p = 3.14;

  double x = 5.0;
  parens(x, 27) = p;

  const double z = 5.0;
  parens(z, 27) = p;

  parens(5.0, 27) = p;
  parens(x + y, 27) = p;

  return 0;
}
```

```
c++ const4.cpp
const4.cpp:23:17: error: expression is not assignable
  parens(x, 27) = p;
  ~~~~~~~~~~~~~ ^
const4.cpp:26:17: error: expression is not assignable
  parens(z, 27) = p;
  ~~~~~~~~~~~~~ ^
const4.cpp:28:19: error: expression is not assignable
  parens(5.0, 27) = p;
  ~~~~~~~~~~~~~~~ ^
const4.cpp:29:21: error: expression is not assignable
  parens(x + y, 27) = p;
  ~~~~~~~~~~~~~~~~~ ^
```

# Before

```
double parens(const double& x, size_t i) {
  std::cout << "called const parens" << std::endl;
  double y = x;
  // .. some things
  return x;
}
```

# After

```
double& parens(const double& x, size_t i) {
  std::cout << "called const parens" << std::endl;
  double y = x;
  // .. some things
  return x;
}
```

# Why not always pass const reference?

```
double& parens(const double& x, size_t i) {
  std::cout << "called const parens" << std::endl;
  double y = x;
  // .. some things
  return x;
}
```

But x is const

Return ref to double

Can't return const

```
int main() {
  double y = 0.5;
  double p = 3.14;

  double x = 5.0;
  parens(x, 27) = p;

  const double z = 5.0;
  parens(z, 27) = p;

  parens(5.0, 27) = p;
  parens(x + y, 27) = p;

  return 0;
}
```

```
c++ const5.cpp
const5.cpp:9:10: error: binding value of type 'const double' to reference to type 'double' drops
      'const' qualifier
  return x;
         ^
```

# Before

```
double& parens(const double& x, size_t i) {
  std::cout << "called const parens" << std::endl;
  double y = x;
  // .. some things
  return x;
}
```

# After

```
const double& parens(const double& x, size_t i) {
  std::cout << "called const parens" << std::endl;
  double y = x;
  // .. some things
  return x;
}
```

# Why not always pass const reference?

```
const double& parens(const double& x, size_t i) {
  std::cout << "called const parens" << std::endl;
  double y = x;
  // .. some things
  return x;
}
```

```
int main() {
  double y = 0.5;
  double p = 3.14;

  double x = 5.0;
  parens(x, 27) = p;

  const double z = 5.0;
  parens(z, 27) = p;

  parens(5.0, 27) = p;
  parens(x + y, 27) = p;

  return 0;
}
```

```
c++ const5.cpp
const5.cpp:26:17: error: cannot assign to return value because function 'parens' returns a const value
  parens(x, 27) = p;
  ~~~~~~~~~~~~~ ^
const5.cpp:5:7: note: function 'parens' which returns const-qualified type 'const double &' declared
      here
const double& parens(const double& x, size_t i) {
      ^~~~~~~
const5.cpp:29:17: error: cannot assign to return value because function 'parens' returns a const value
  parens(z, 27) = p;
  ~~~~~~~~~~~~~ ^
const5.cpp:5:7: note: function 'parens' which returns const-qualified type 'const double &' declared
      here
const double& parens(const double& x, size_t i) {
      ^~~~~~~
const5.cpp:31:19: error: cannot assign to return value because function 'parens' returns a const value
  parens(5.0, 27) = p;
  ~~~~~~~~~~~~~~~ ^
const5.cpp:5:7: note: function 'parens' which returns const-qualified type 'const double &' declared
      here
const double& parens(const double& x, size_t i) {
      ^~~~~~~
const5.cpp:32:21: error: cannot assign to return value because function 'parens' returns a const value
  parens(x + y, 27) = p;
  ~~~~~~~~~~~~~~~~~ ^
const5.cpp:5:7: note: function 'parens' which returns const-qualified type 'const double &' declared
      here
const double& parens(const double& x, size_t i) {
      ^~~~~~~
```

# Before

```
double& parens(const double& x, size_t i) {
  std::cout << "called const parens" << std::endl;
  double y = x;
  // .. some things
  return x;
}
```

# After

```
double& parens(double& x, size_t i) {
  std::cout << "called const parens" << std::endl;
  double y = x;
  // .. some things
  return x;
}
```

# How about no const at all?

```
double& parens(double& x, size_t i) {
  std::cout << "called const parens" << std::endl;
  double y = x;
  // .. some things
  return x;
}
```

```
int main() {
  double y = 0.5;
  double p = 3.14;

  double x = 5.0;
  parens(x, 27) = p;

  const double z = 5.0;
  parens(z, 27) = p;

  parens(5.0, 27) = p;
  parens(x + y, 27) = p;

  return 0;
}
```

```
c++ const5.cpp
const5.cpp:30:3: error: no matching function for call to 'parens'
  parens(z, 27) = p;
  ^~~~~~
const5.cpp:14:9: note: candidate function not viable: 1st argument ('const double') would lose const
      qualifier
double& parens(double& x, size_t i) {
        ^
const5.cpp:32:3: error: no matching function for call to 'parens'
  parens(5.0, 27) = p;
  ^~~~~~
const5.cpp:14:9: note: candidate function not viable: expects an l-value for 1st argument
double& parens(double& x, size_t i) {
        ^
const5.cpp:33:3: error: no matching function for call to 'parens'
  parens(x + y, 27) = p;
  ^~~~~~
const5.cpp:14:9: note: candidate function not viable: expects an l-value for 1st argument
double& parens(double& x, size_t i) {
        ^
```

# How about no const at all?

```
int main() {
  double y = 0.5;
  double p = 3.14;

  double x = 5.0;
  parens(x, 27) = p;

  const double z = 5.0;
  parens(z, 27) = p;

  parens(5.0, 27) = p;
  parens(x + y, 27) = p;

  return 0;
}
```

This makes sense

This *should* be an error

This *should* be an error

This *should* be an error

# More sensible

```
int main() {
  double y = 0.5;
  double p = 3.14;

  double x = 5.0;
  parens(x, 27) = p;

  const double z = 5.0;
  double q = parens(z, 27);

  double r = parens(5.0, 27);
  double s = parens(x + y, 27);

  return 0;
}
```

This makes sense

This makes sense

This makes sense

This makes sense

# More sensible

```
double& parens(double& x, size_t i) {
  std::cout << "called non const parens" << std::endl;
  double y = x;
  // .. some things
  return x;
}
```

```
int main() {
  double y = 0.5;
  double p = 3.14;

  double x = 5.0;
  parens(x, 27) = p;

  const double z = 5.0;
  double q = parens(z, 27);

  double r = parens(5.0, 27);
  double s = parens(x + y, 27);

  return 0;
}
```

```
c++ const6.cpp
const6.cpp:30:14: error: no matching function for call to 'parens'
  double q = parens(z, 27);
             ^~~~~~
const6.cpp:14:9: note: candidate function not viable: 1st argument ('const double') would lose const
      qualifier
double& parens(double& x, size_t i) {
        ^
const6.cpp:32:14: error: no matching function for call to 'parens'
  double r = parens(5.0, 27);
             ^~~~~~
const6.cpp:14:9: note: candidate function not viable: expects an l-value for 1st argument
double& parens(double& x, size_t i) {
        ^
const6.cpp:33:14: error: no matching function for call to 'parens'
  double s = parens(x + y, 27);
             ^~~~~~
const6.cpp:14:9: note: candidate function not viable: expects an l-value for 1st argument
double& parens(double& x, size_t i) {
        ^
```

Oops, need to be const

Going in circles?

# More sensible

```
const double& parens(const double& x, size_t i) {
  std::cout << "called non const parens" << std::endl;
  double y = x;
  // .. some things
  return x;
}
```

```
int main() {
  double y = 0.5;
  double p = 3.14;

  double x = 5.0;
  parens(x, 27) = p;

  const double z = 5.0;
  double q = parens(z, 27);

  double r = parens(5.0, 27);
  double s = parens(x + y, 27);

  return 0;
}
```

```
c++ const6.cpp
const6.cpp:27:17: error: cannot assign to return value because function 'parens' returns a const value
  parens(x, 27) = p;
  ~~~~~~~~~~~~~ ^
const6.cpp:6:7: note: function 'parens' which returns const-qualified type 'const double &' declared
      here
const double& parens(const double& x, size_t i) {
      ^~~~~~~
```

Oops, need to be non const

Going in circles?

# Overloading to the rescue

```
const double& parens(const double& x, siz
  std::cout << "called non const parens" <
  double y = x;
  // .. some things
  return x;
}
```

```
double& parens(double& x, size_t i) {
  std::cout << "called non const parens" << std::endl;
  double y = x;
  // .. some things
  return x;
}
```

const

const

Not const

Not const

```
int main() {
  double y = 0.5;
  double p = 3.14;

  double x = 5.0;
  parens(x, 27) = p;

  const double z = 5.0;
  double q = parens(z, 27);

  double r = parens(5.0, 27);
  double s = parens(x + y, 27);

  return 0;
}
```

```
./a.out
called non const parens
called const parens
called const parens
called const parens
```

# What does this have to do with operator()

```
const double& parens(const double& x, siz
  std::cout << "called non const parens" <
  double y = x;
  // .. some things
  return x;
}
```

```
double& parens(double& x, size_t i) {
  std::cout << "called non const parens" << std::endl;
  double y = x;
  // .. some things
  return x;
}
```

const

const

Not const

Not const

```
class Vector {
public:
  Vector(size_t M) : num_rows_(M), storage_(num_rows_) {}

  double& operator()(size_t i) { return storage_[i]; }

private:
  size_t              num_rows_;
  std::vector<double> storage_;
};
```

Where is the const or non-const thing to overload on?

# What does this have to do with operator()

```
const double& parens(const double& x, siz
  std::cout << "called non const parens" 
  double y = x;
  // .. some things
  return x;
}
```

```
double& parens(double& x, size_t i) {
  std::cout << "called non const parens" << std::endl;
  double y = x;
  // .. some things
  return x;
}
```

const

const

Not const

Not const

```
class Vector {
public:
  Vector(size_t M) : num_rows_(M), storage_(num_rows_) {}

        double& operator()(size_t i) { return storage_[i]; }
  const double& operator()(size_t i) { return storage_[i]; }

private:
  si            um_rows_;
  st            torage_;
};
```

Only differing by return type

Where is the const or non-const thing to overload on?

# There is a secret argument

```
const double& parens(const double& x, siz
  std::cout << "called non const parens" <
  double y = x;
  // .. some things
  return x;
}
```

```
double& parens(double& x, size_t i) {
  std::cout << "called non const parens" << std::endl;
  double y = x;
  // .. some things
  return x;
}
```

const

const

Not const

Not const

```
class Vector
public:
  Vector(size_t M) : num_rows_(M), storage_(num_rows_) {}

        double& operator()(size_t i) { return storage_[i]; }
  const double& operator()(size_t i) { return storage_[i]; }

                   num_rows_;
  std::vector<double> storage_;
};
```

Called “this”

There is a secret argument

There is a secret argument

# There is a secret argument

```
const double& parens(const double& x, siz
  std::cout << "called non const parens" <
  double y = x;
  // .. some things
  return x;
}
```

```
double& parens(double& x, size_t i) {
  std::cout << "called non const parens" << std::endl;
  double y = x;
  // .. some things
  return x;
}
```

const

const

Not const

Not const

```
class Vector {
public:
  Vector(size_t M) : num_rows_(M), storage_(num_rows_) {}

        double& operator()(Vector *this, size_t i) { return storage_[i]; }
  const double& operator()(Vector *this, size_t i) { return storage_[i]; }

private:
  size_t              num_rows_;
  std::vector<double> storage_;
};
```

How would we fix our const problem?

# Before

```
class Vector {
public:
  Vector(size_t M) : num_rows_(M), storage_(num_rows_) {}

        double& operator()(Vector *this, size_t i) { return storage_[i]; }
  const double& operator()(Vector *this, size_t i) { return storage_[i]; }

private:
  size_t              num_rows_;
  std::vector<double> storage_;
};
```

# After

```
class Vector {
public:
  Vector(size_t M) : num_rows_(M), storage_(num_rows_) {}

        double& operator()(Vector *this, size_t i) { return storage_[i]; }
  const double& operator()(const Vector *this, size_t i) { return storage_[i]; }

private:
  size_t              num_rows_;
  std::vector<double> storage_;
};
```

const “this”

# After After

```
class Vector {
public:
  Vector(size_t M) : num_rows_(M), storage_(num_rows_) {}

        double& operator()(size_t i)       { return storage_[i]; }
  const double& operator()(size_t i) const { return storage_[i]; }

private:
  size_t              num_rows_;
  std::vector<double> storage_;
};
```

const “this”

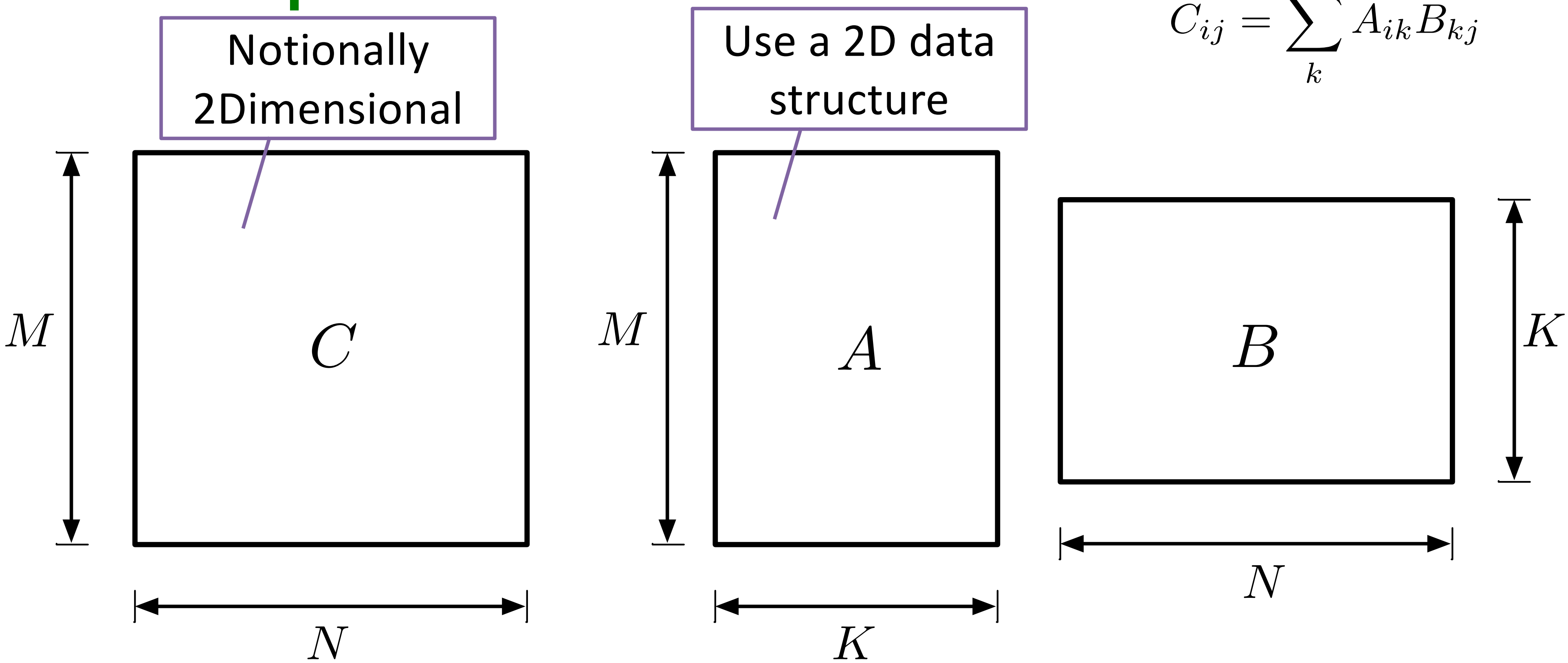
# Matrix Representation

- Two issues
  - Interface (what is the abstraction we want to present?)
  - Implementation (how is the abstraction realized?)
- Sometimes there are tradeoffs
  - Evaluate relative to end user
  - In HPC – performance is most important
  - Elsewhere – safety, ease of use, standards compliance, etc

```
Matrix A(M,K), B(K,N), C(M,N);
...
for (int i = 0; i < N; ++i)
  for (int j = 0; j < N; ++j)
    for (int k = 0; k < N; ++k)
      C(i,j) += A(i,k) * B(k,j)
```

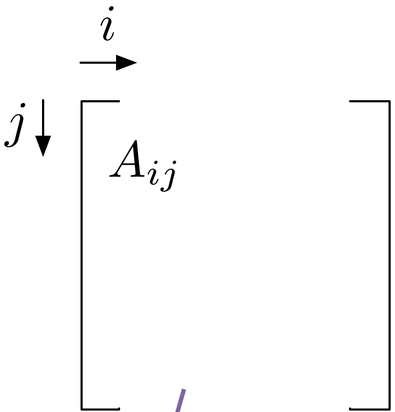
# Matrix Representation

$$C_{ij} = \sum_k A_{ik} B_{kj}$$

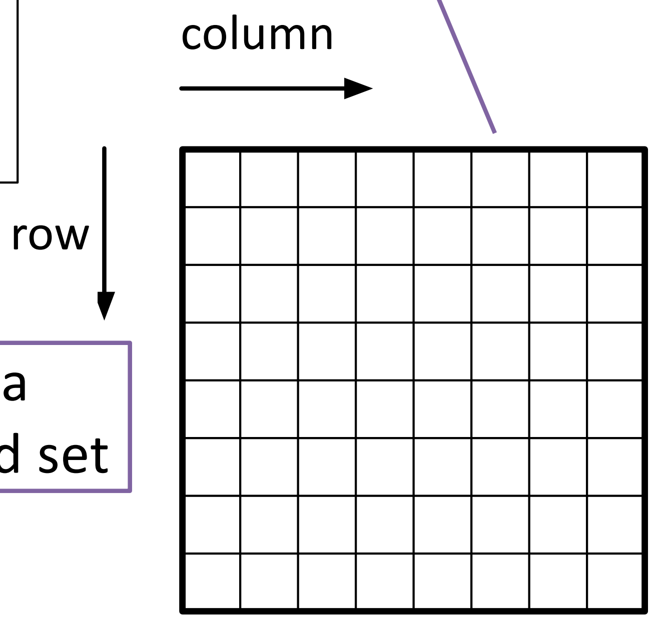


# Matrix Representation

$$C_{ij} = \sum_k A_{ik} B_{kj}$$




Use a doubly indexed data structure



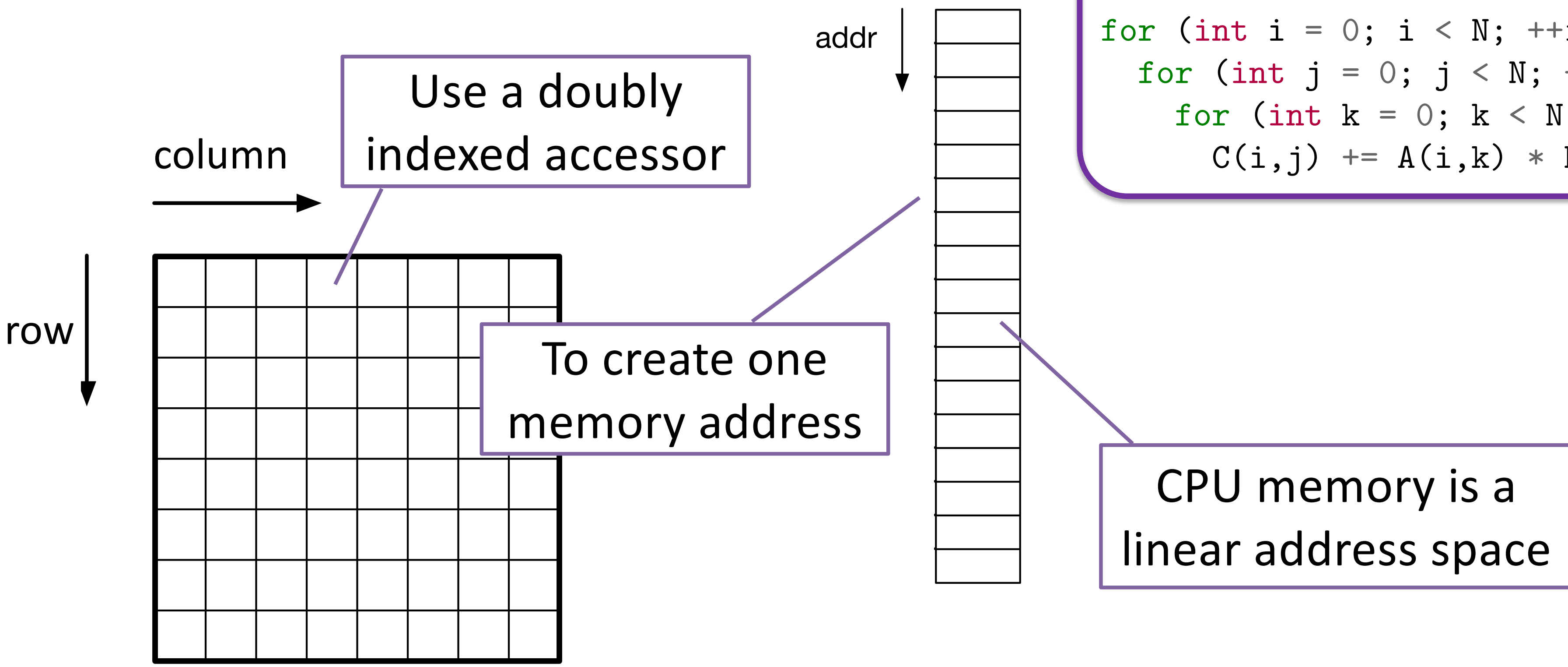

```
Matrix A(M,K), B(K,N), C(M,N);
...
for (int i = 0; i < N; ++i)
  for (int j = 0; j < N; ++j)
    for (int k = 0; k < N; ++k)
      C(i,j) += A(i,k) * B(k,j)
```

A matrix is a doubly indexed set

Use a doubly indexed accessor

# Matrix Representation

```
Matrix A(M,K), B(K,N), C(M,N);
...
for (int i = 0; i < N; ++i)
  for (int j = 0; j < N; ++j)
    for (int k = 0; k < N; ++k)
      C(i,j) += A(i,k) * B(k,j)
```



# Matrix Representation

- To translate double index to single address

```
double **storage_;                                    storage_[i][j]

std::vector<std::vector<double> > storage_;     storage_[i][j]

std::vector<double> storage_;                        storage_[k]
```

Use inner pointer to get data element

Array of arrays

Lookup inner pointer from outer pointer

Use inner vector to get data element

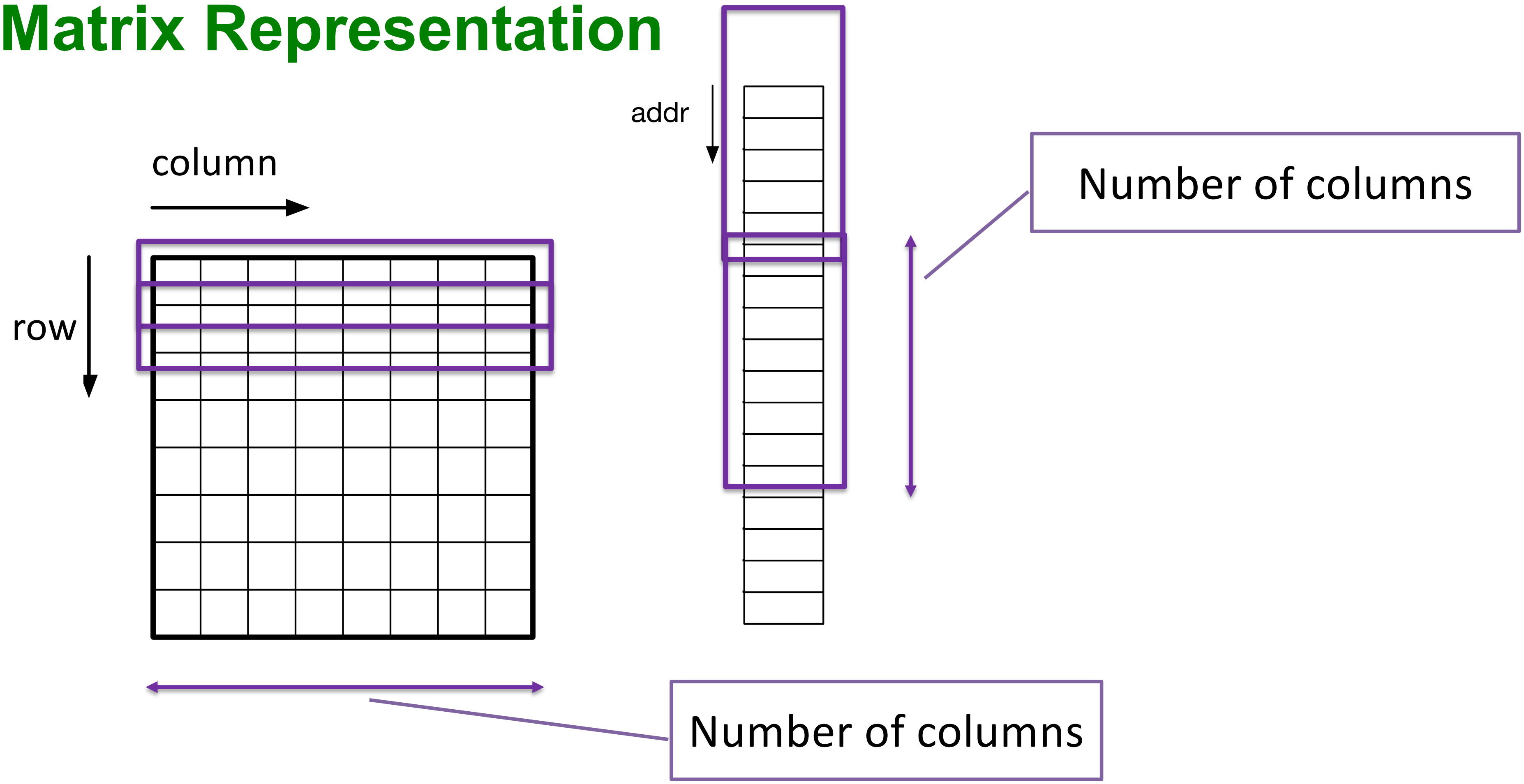
Vector of vectors

Lookup inner vector from outer vector

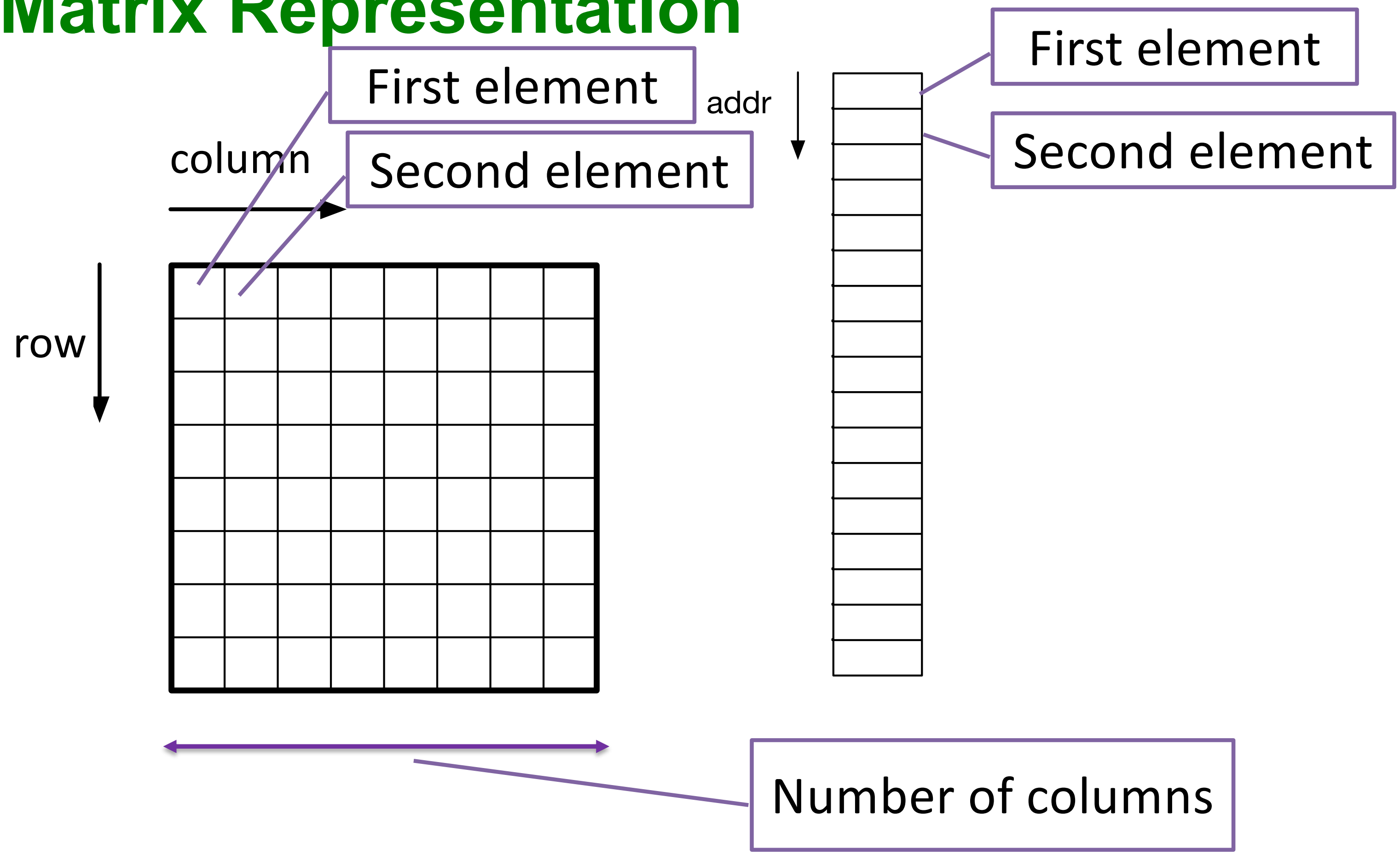
Use single vector to get data element

Need to compute this

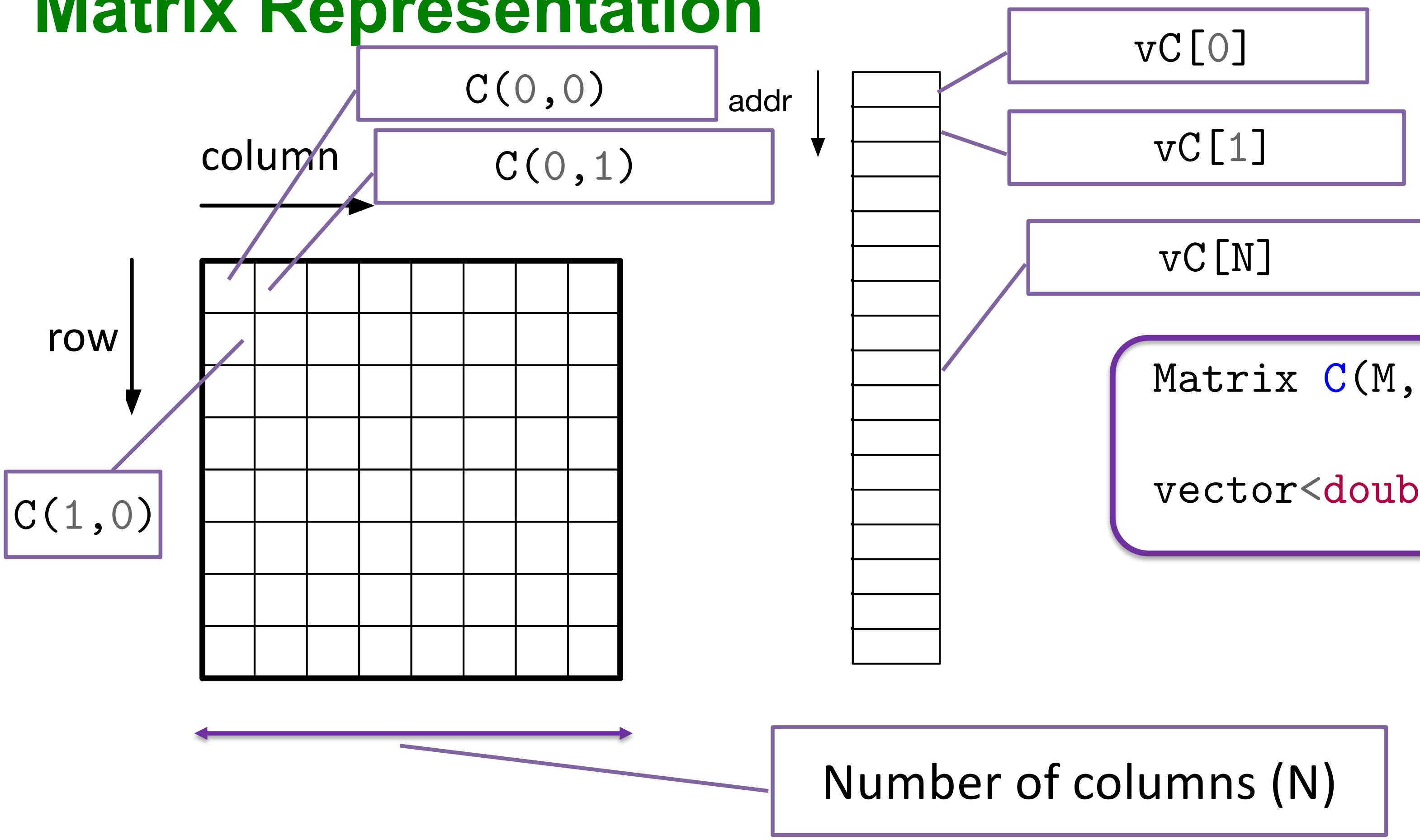
# Matrix Representation



# Matrix Representation



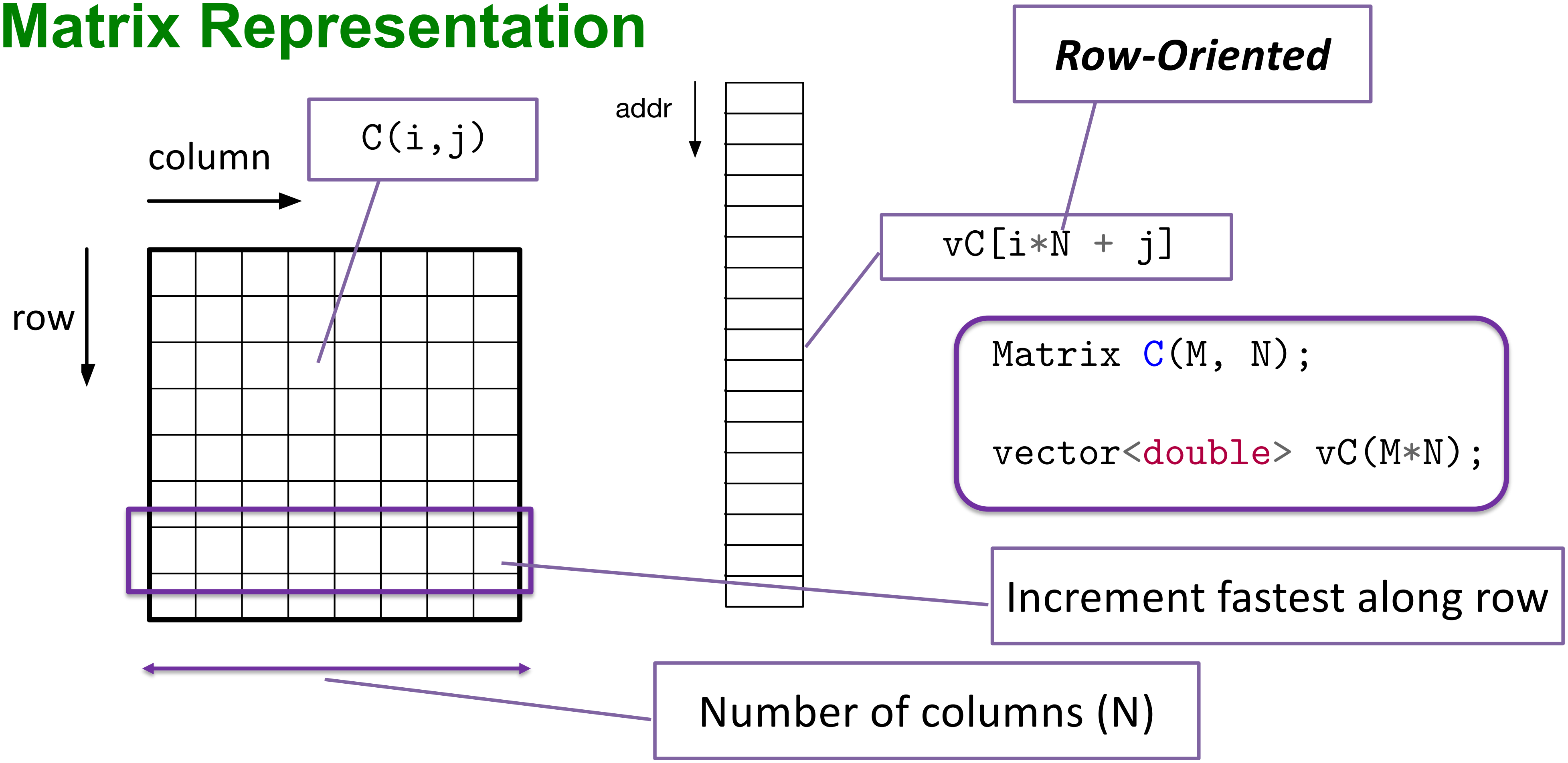
# Matrix Representation




```
Matrix C(M, N);

vector<double> vC(M*N);
```

# Matrix Representation

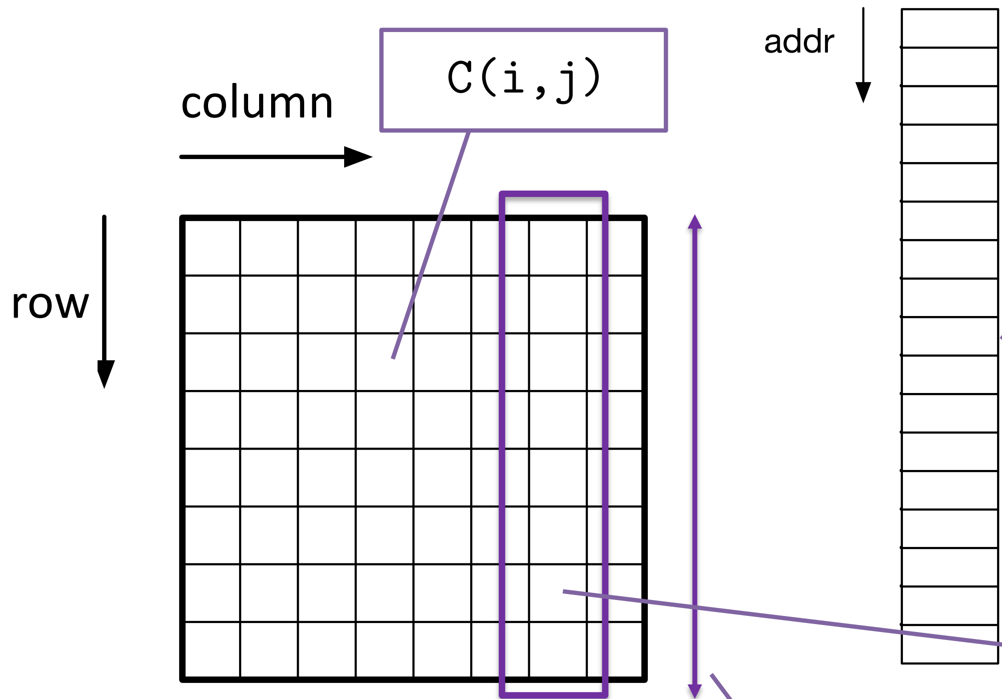



```
Matrix C(M, N);

vector<double> vC(M*N);
```

# Matrix Representation

***Column-Oriented***




vC[j*M + i]

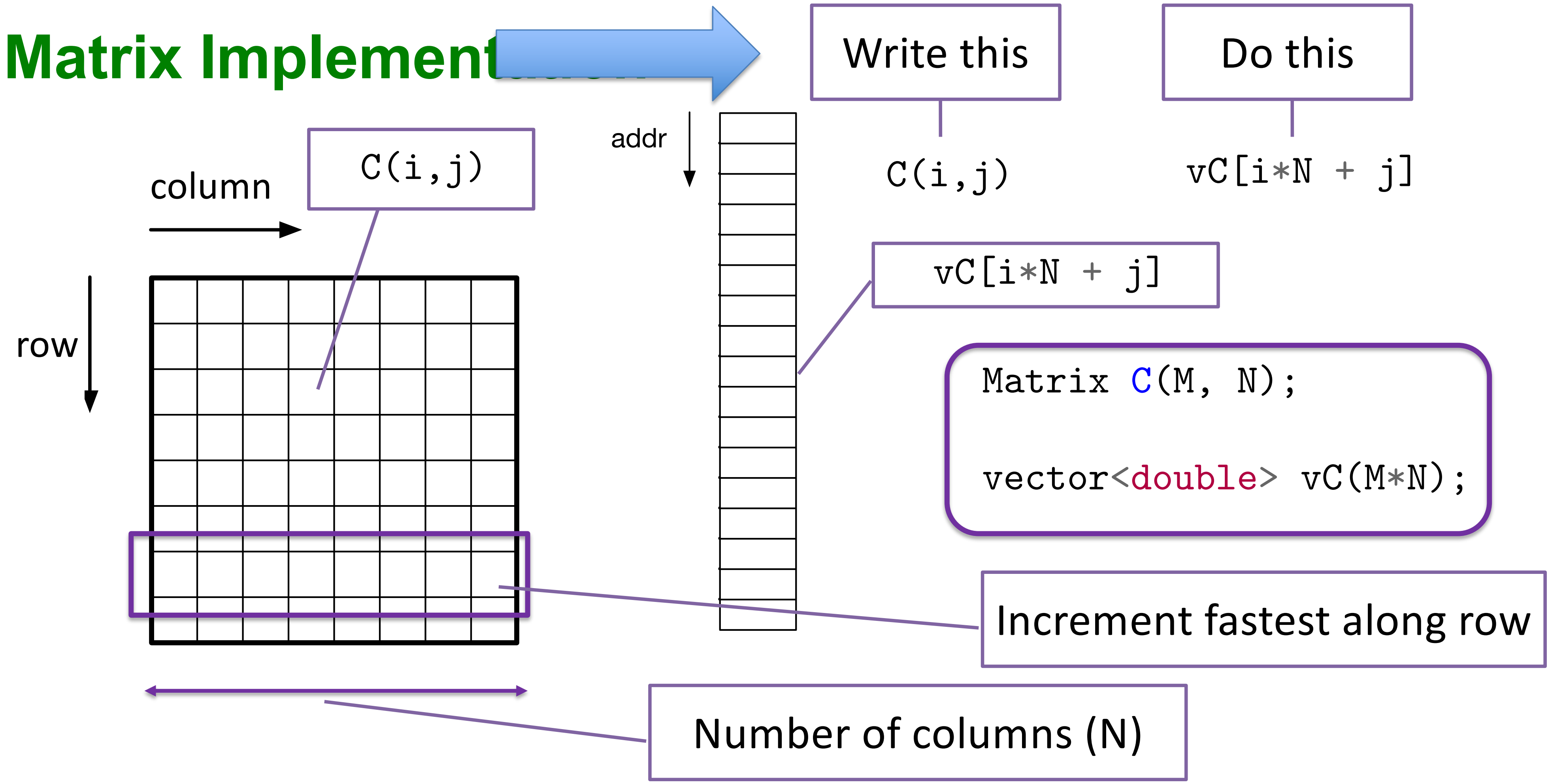
```
Matrix C(M, N);

vector<double> vC(M*N);
```

Increment fastest along column

Number of rows (M)

# Matrix Implementation

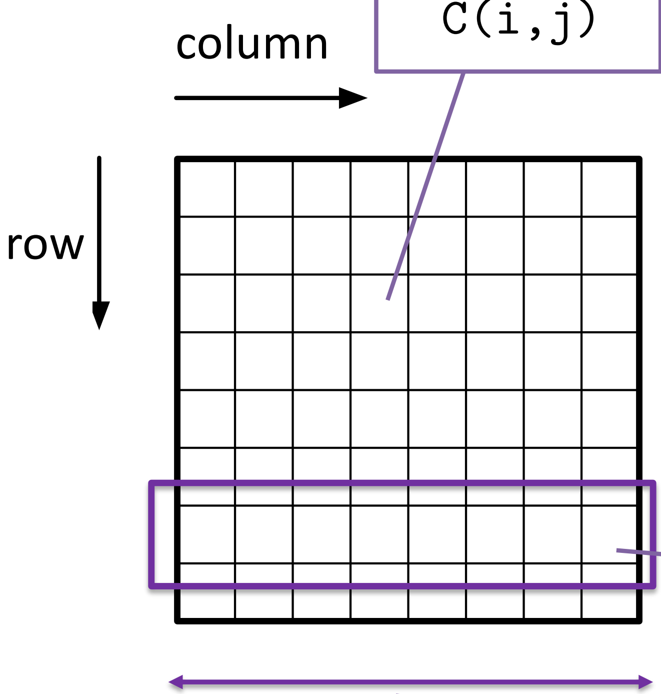


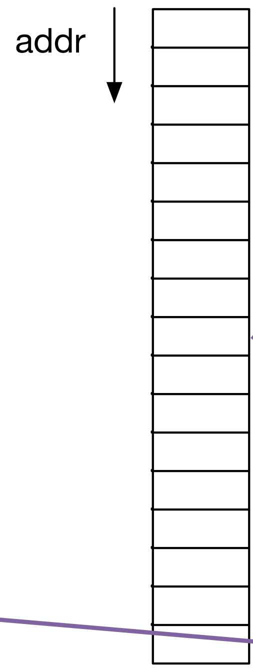
# Matrix Implementation

Fortran did this transparently

C(i,j) vC[i*N + j]

C(i,j)




vC[i*N + j]

```
Matrix C(M, N);

vector<double> vC(M*N);
```

Increment fastest along row

Number of columns (N)

# Matrix in C++

- Two dimensional accessor `C(i,j)`
- One dimensional access `vC[i*N + j]`

- Simultaneously (and safely) need matrix with
  - (i,j) two dimensional accessor
  - Transparent translation to one dimensional accessor

- Preprocessor? `#define C(i,j) vC[i*N+j]`

Only works for C and vC

Where does N come from

# Matrix in C++

- Two dimensional accessor `C(i,j)`
- One dimensional access `vC[i*N + j]`

- A `Matrix` needs to
  - Have its "own" `vector<double> vC`
  - Have its "own" `N`
  - Have a doubly indexed accessor

# Class Matrix

```cpp
#include <vector>

class Matrix {
public:
  Matrix(size_type M, size_type N)
      : num_rows_(M), num_cols_(N), storage_(num_rows_ * num_cols_) {}
  Matrix(size_type M, size_type N, double init)
      : num_rows_(M), num_cols_(N), storage_(num_rows_ * num_cols_, init) {}

  double &operator()(size_type i, size_type j) {
    return storage_[i * num_cols_ + j];
  }
  const double &operator()(size_type i, size_type j) const {
    return storage_[i * num_cols_ + j];
  }

  size_type num_rows() const { return num_rows_; }
  size_type num_cols() const { return num_cols_; }

private:
  size_type num_rows_, num_cols_;
  std::vector<double> storage_;
};
```

# Class Matrix

```
class Matrix {
public:
  Matrix(size_type M, size_type N)
      : num_rows_(M), num_cols_(N), storage_(num_rows_ * num_cols_) {}

  Matrix(size_type M, size_type N, double init)
      : num_rows_(M), num_cols_(N), storage_(num_rows_ * num_cols_, init) {}

private:
  size_type num_rows_, num_cols_;
  std::vector<double> storage_;
};
```

# Class Matrix

```
class Matrix {
public:
  double &operator()(size_type i, size_type j) {
    return storage_[i * num_cols_ + j];
  }




  size_type num_rows() const { return num_rows_; }
  size_type num_Cols() const { return num_cols_; }

private:
  size_type num_rows_, num_cols_;
  std::vector<double> storage_;
};
```

# Class Matrix

```
class Matrix {
public:
  double &operator()(size_type i, size_type j) {
    return storage_[i * num_cols_ + j];
  }
  const double &operator()(size_type i, size_type j) const {
    return storage_[i * num_cols_ + j];
  }

  size_type num_rows() const { return num_rows_; }
  size_type num_Cols() const { return num_cols_; }

private:
  size_type num_rows_, num_cols_;
  std::vector<double> storage_;
};
```

# Class Matrix

```
class Matrix {
public:
  double &operator()(size_type i, size_type j) {
    return storage_[i * num_cols_ + j];
  }
  const double &operator()(size_type i, size_type j) const {
    return storage_[i * num_cols_ + j];
  }

  size_type num_rows() const { return num_rows_; }
  size_type num_Cols() const { return num_cols_; }

private:
  size_type num_rows_, num_cols_;
  std::vector<double> storage_;
};
```

How would we write the other orientation?

What is the orientation?

Does it matter which?

# Before

```
class Matrix {
public:
  double &operator()(size_type i, size_type j) {
    return storage_[i * num_cols_ + j];
  }
  const double &operator()(size_type i, size_type j) const {
    return storage_[i * num_cols_ + j];
  }

  size_type num_rows() const { return num_rows_; }
  size_type num_Cols() const { return num_cols_; }

private:
  size_type num_rows_, num_cols_;
  std::vector<double> storage_;
};
```

# After

```
class Matrix {
public:
  double &operator()(size_type i, size_type j) {
    return storage_[j * num_rows_ + i];
  }
  const double &operator()(size_type i, size_type j) const {
    return storage_[j * num_rows_ + i];
  }

  size_type num_rows() const { return num_rows_; }
  size_type num_Cols() const { return num_cols_; }

private:
  size_type num_rows_, num_cols_;
  std::vector<double> storage_;
};
```

# Finally

```
#include <vector>

class Vector {
public:
  Vector(size_t M) : num_rows_(M), storage_(num_rows_) {}

        double& operator()(size_t i)       { return storage_[i]; }
  const double& operator()(size_t i) const { return storage_[i]; }

  size_t num_rows() { return num_rows_; }

private:
  size_t              num_rows_;
  std::vector<double> storage_;
};
```

# Example: Matrix-Matrix Product

- For matrices $\mathbf{A} \in \mathbb{R}^{M \times K}$ and $\mathbf{B} \in \mathbb{R}^{K \times N}$, compute $\mathbf{C} \in \mathbb{R}^{M \times N}$

  $\mathbf{C} \leftarrow \mathbf{A} \times \mathbf{B}$ Defined according to $C_{ij} = \sum_k A_{ik} B_{kj}$

- Workhorse computational kernel (underlying LINPACK, HPL)
- Compute-intensive: $O(N^3)$ work with $O(N^2)$ data
- Basic algorithm in C++

Locality! (Data reuse)

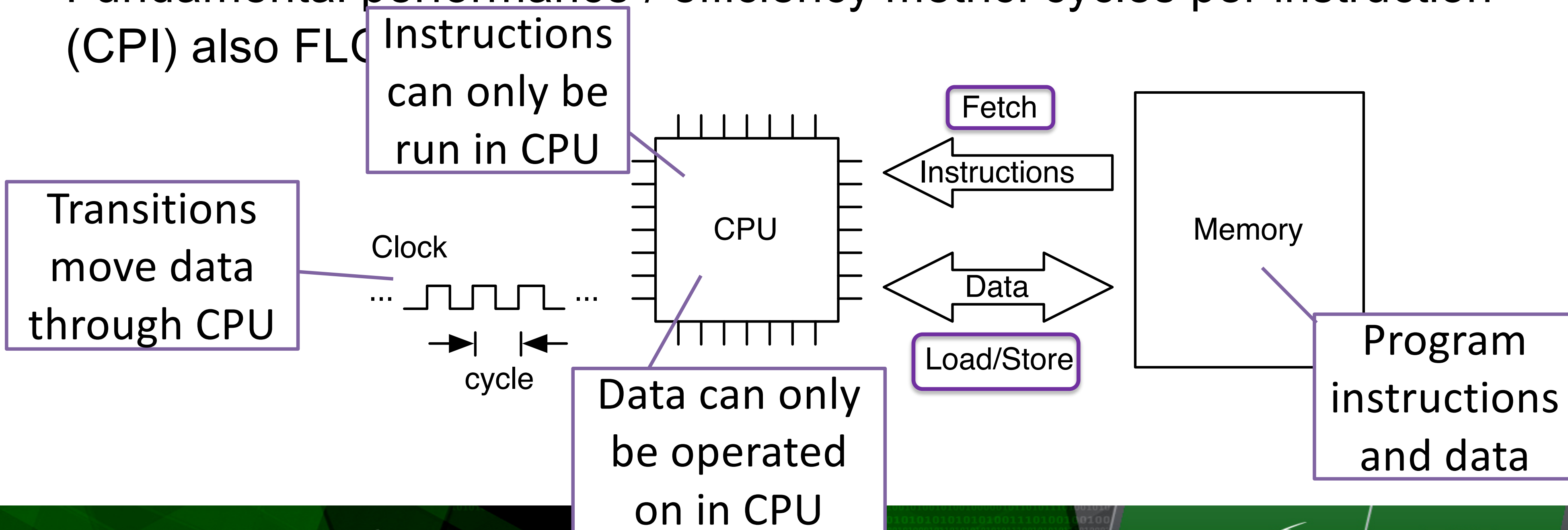
Every element is accessed N times

```
Matrix A(M,K), B(K,N), C(M,N);
...
for (int i = 0; i < N; ++i)
  for (int j = 0; j < N; ++j)
    for (int k = 0; k < N; ++k)
      C(i,j) += A(i,k) * B(k,j)
```

Maximize locality

# Microprocessors

- Basic operation: read and execute program instructions stored in memory
- Fundamental performance / efficiency metric: cycles per instruction (CPI) also FL

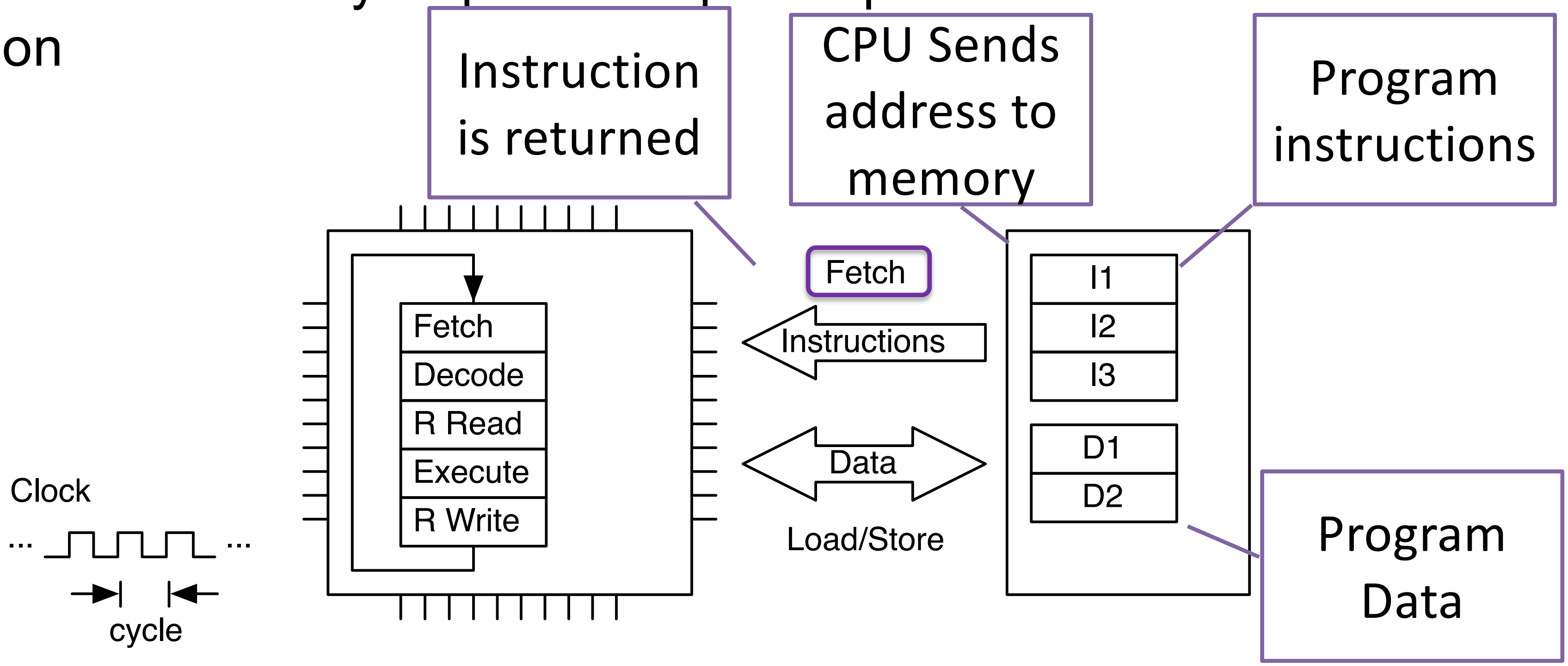


# Performance-Oriented Architecture Features

- Execution Pipeline
  - Stages of functionality to process issued instructions
  - Hazards are conflicts with continued execution
  - Forwarding supports closely associated operations exhibiting precedence constraints
- Out of Order Execution
  - Uses reservation stations
  - Hides some core latencies and provide fine grain asynchronous operation supporting concurrency
- Branch Prediction
  - Permits computation to proceed at a conditional branch point prior to resolving predicate value
  - Overlaps follow-on computation with predicate resolution
  - Requires roll-back or equivalent to correct false guesses
  - Sometimes follows both paths, and several deep

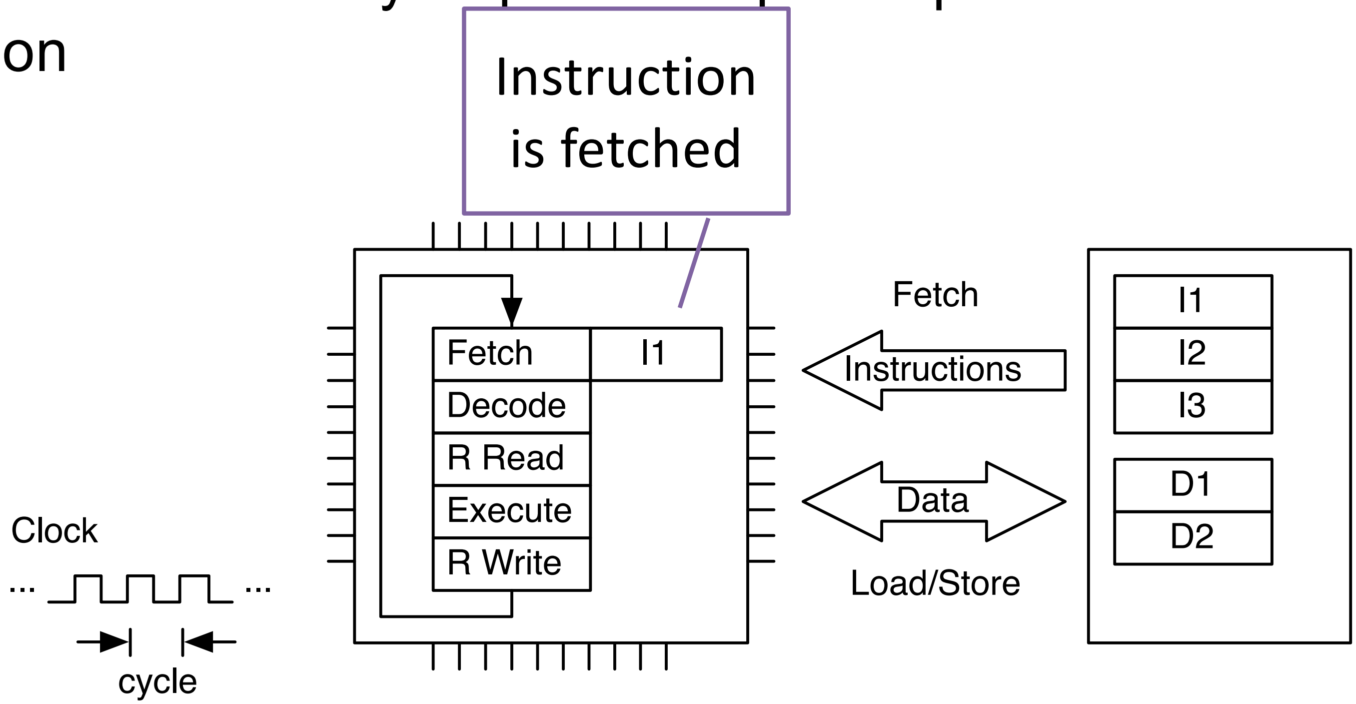
# Processor Core Instruction Handling

- A single instruction may require multiple steps from fetch to completion



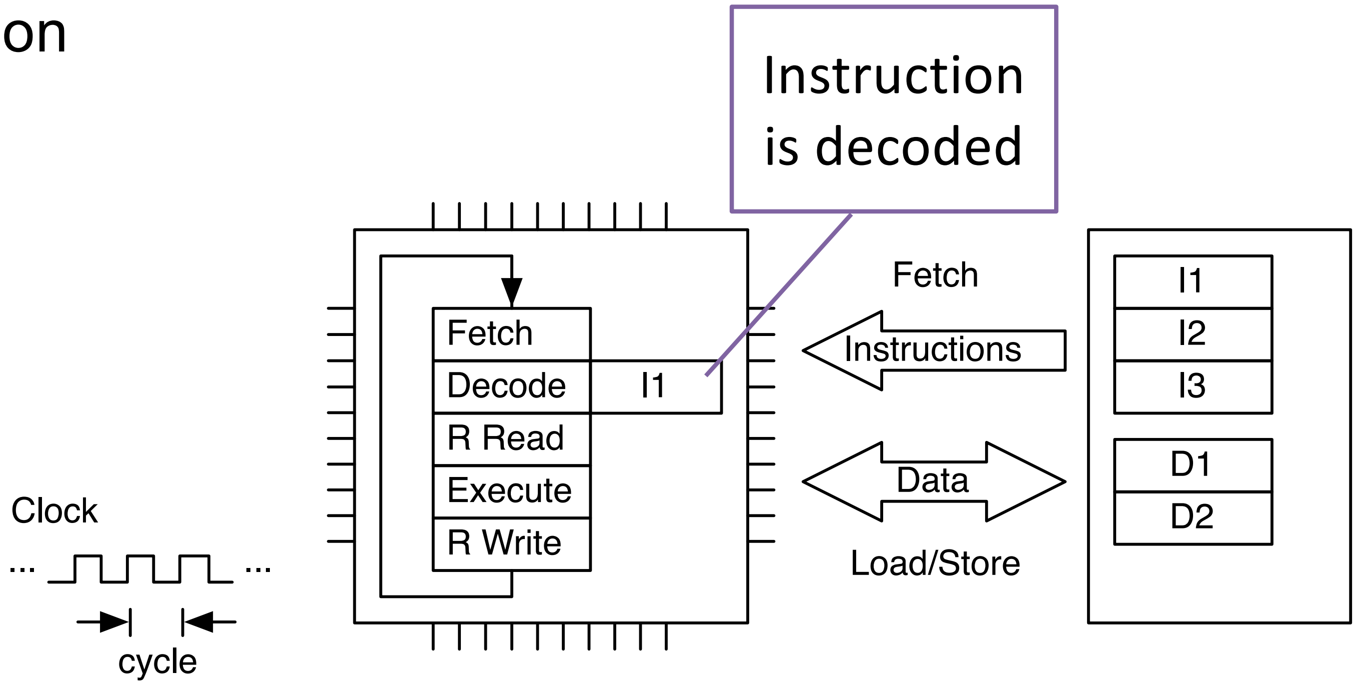
# Processor Core Instruction Handling

- A single instruction may require multiple steps from fetch to completion



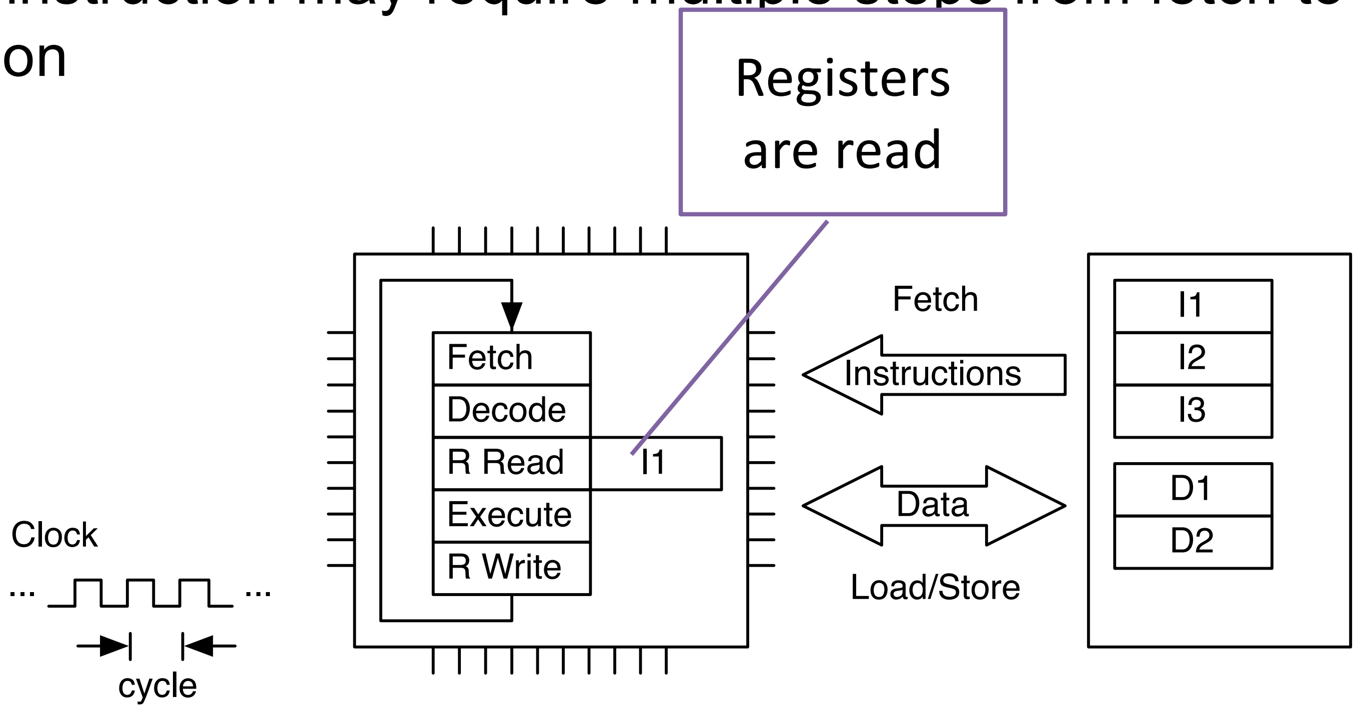
# Processor Core Instruction Handling

- A single instruction may require multiple steps from fetch to completion



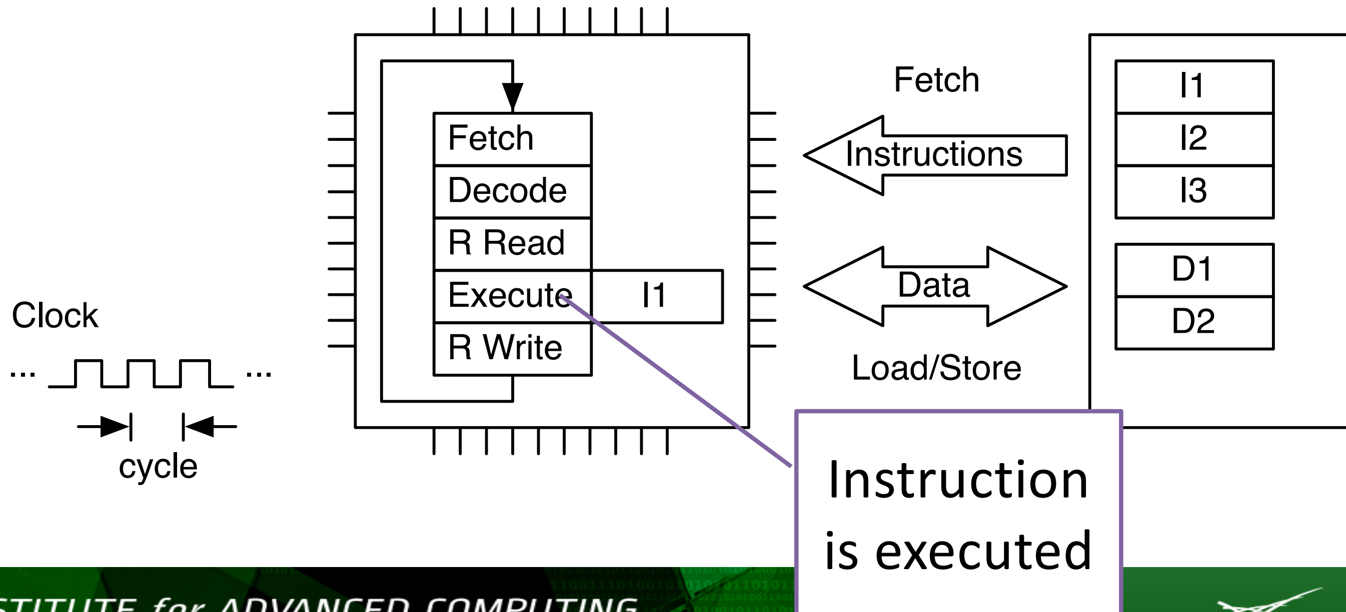
# Processor Core Instruction Handling

- A single instruction may require multiple steps from fetch to completion



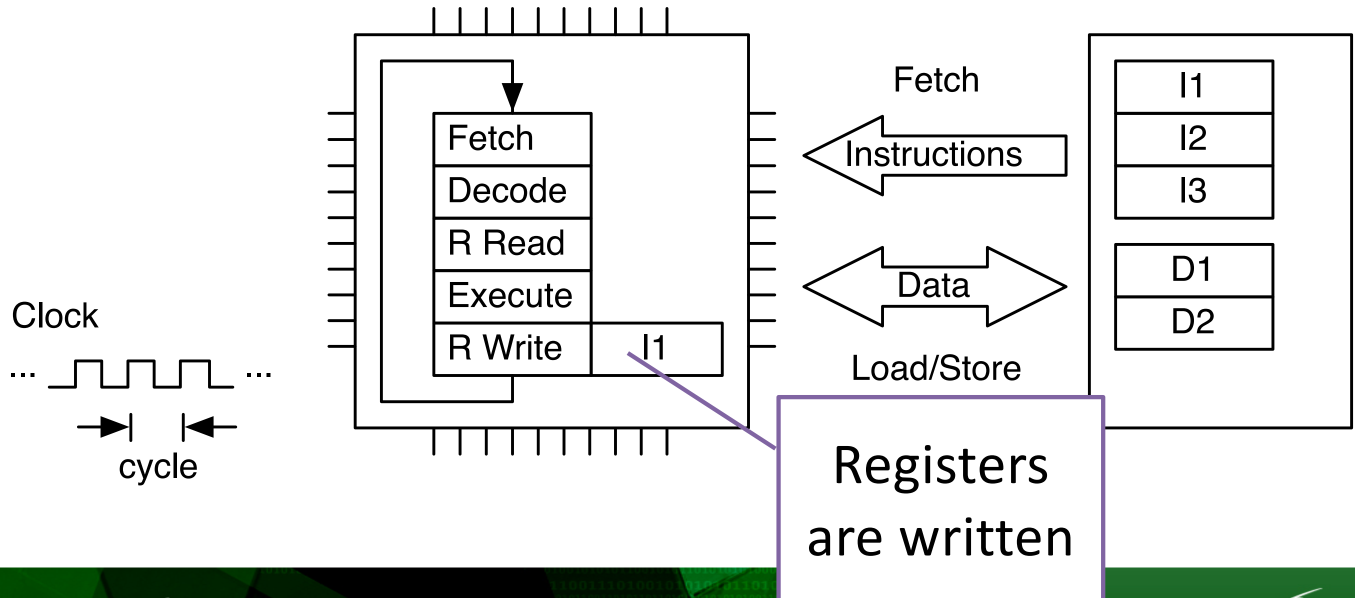
# Processor Core Instruction Handling

- A single instruction may require multiple steps from fetch to completion



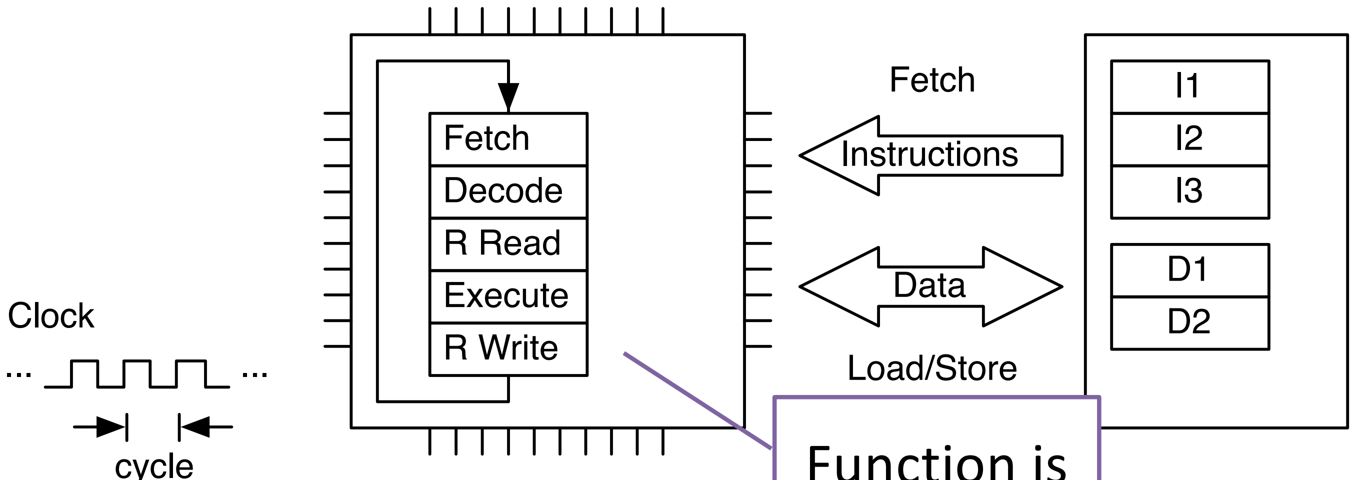
# Processor Core Instruction Handling

- A single instruction may require multiple steps from fetch to completion



# Processor Core Instruction Handling

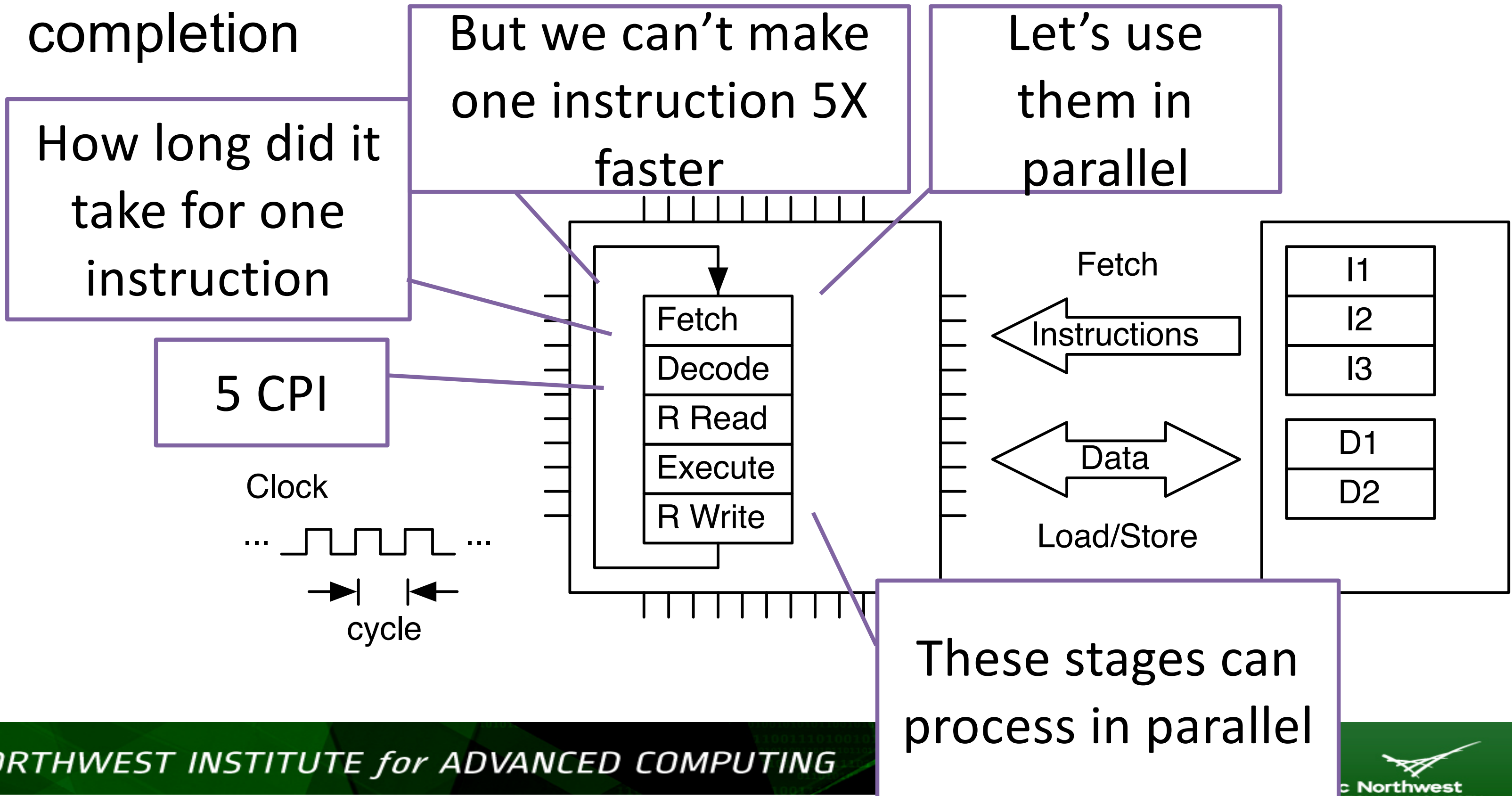
- A single instruction may require multiple steps from fetch to completion




Function is retired

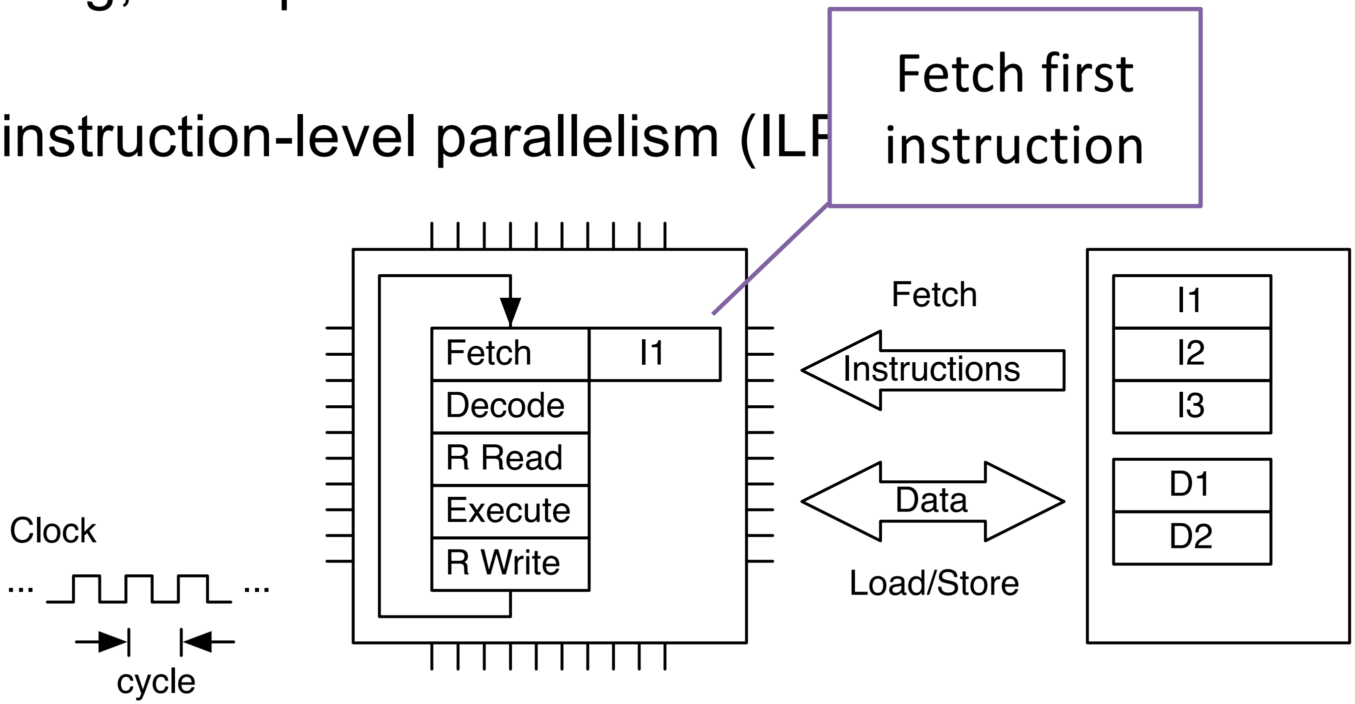
# Processor Core Instruction Handling

- A single instruction may require multiple steps from fetch to completion



# Processor Core Instruction Handling

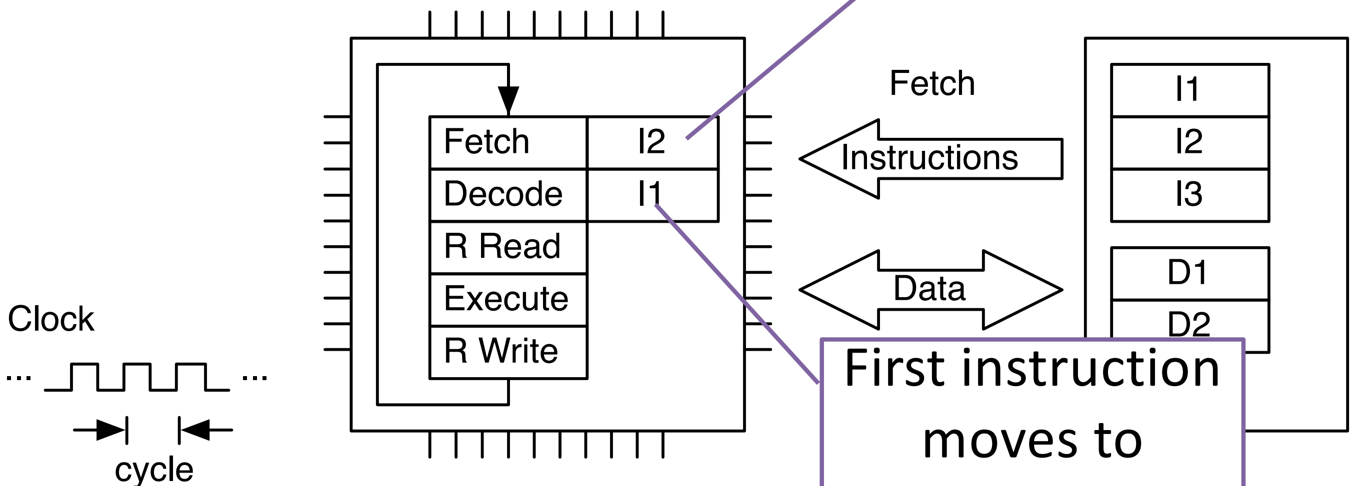
- By pipelining, multiple instructions can be executed at each clock cycle
- Form of instruction-level parallelism (ILP)



# Processor Core Instruction Handling

- By pipelining, multiple instructions can be executed at each clock cycle
- Form of instruction-level parallelism (ILP)

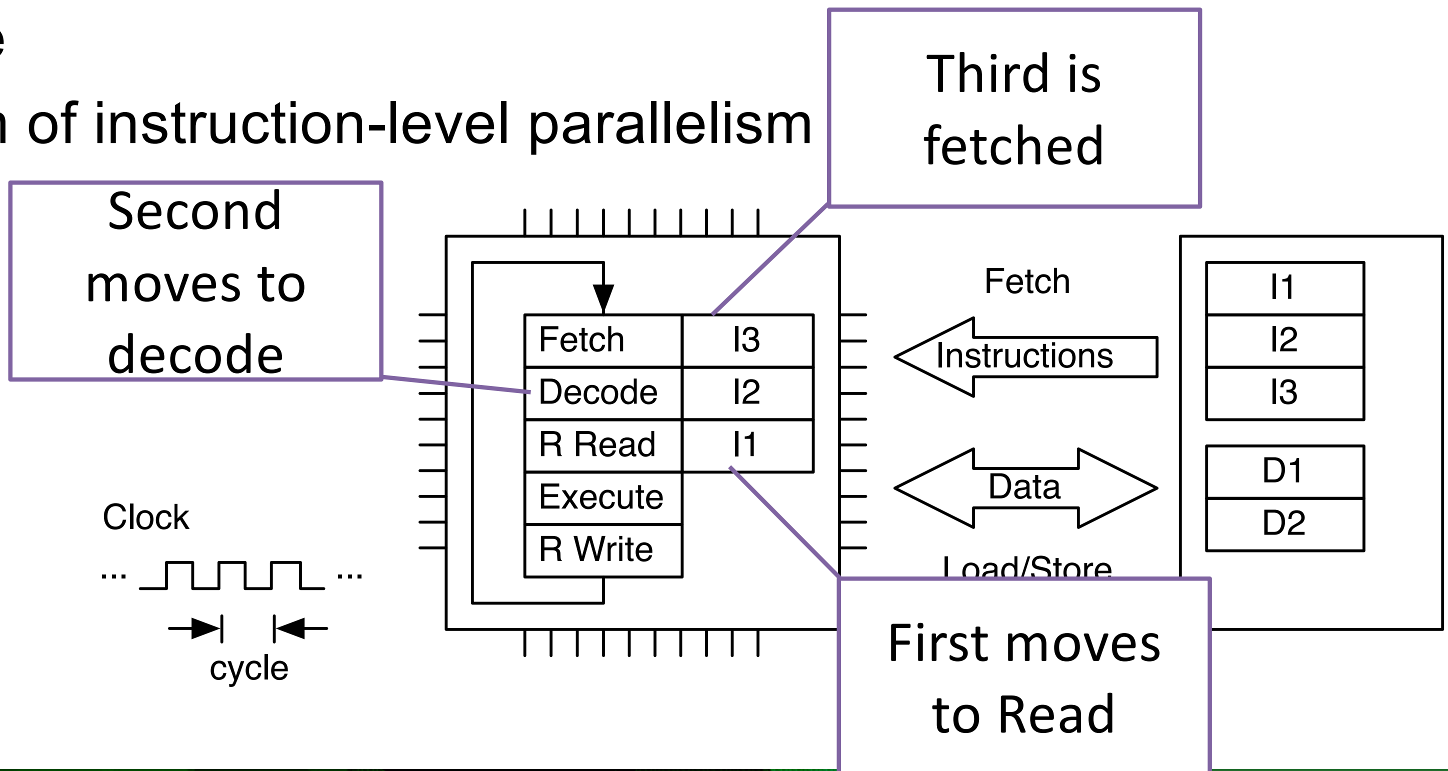
When first instruction is in decode, fetch second




First instruction moves to decode

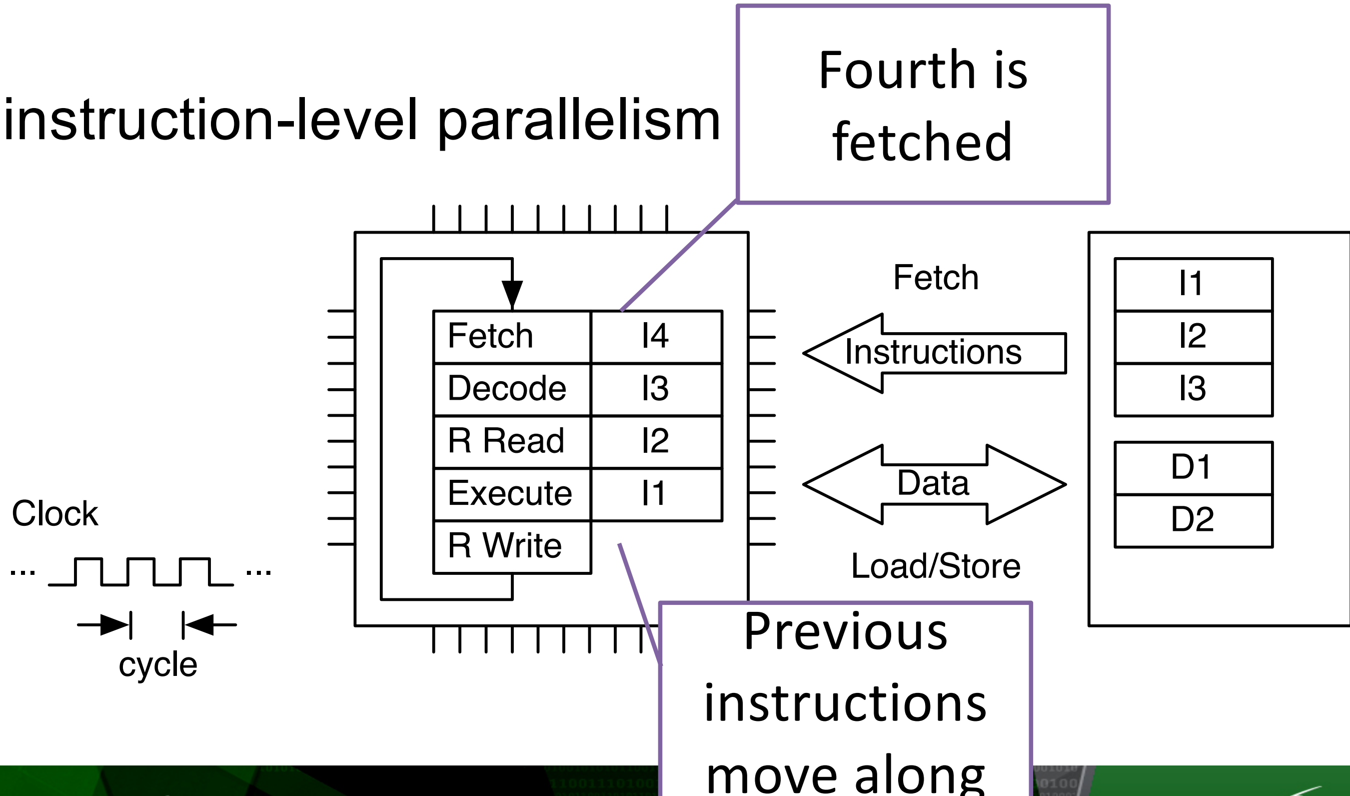
# Processor Core Instruction Handling

- By pipelining, multiple instructions can be executed at each clock cycle
- Form of instruction-level parallelism



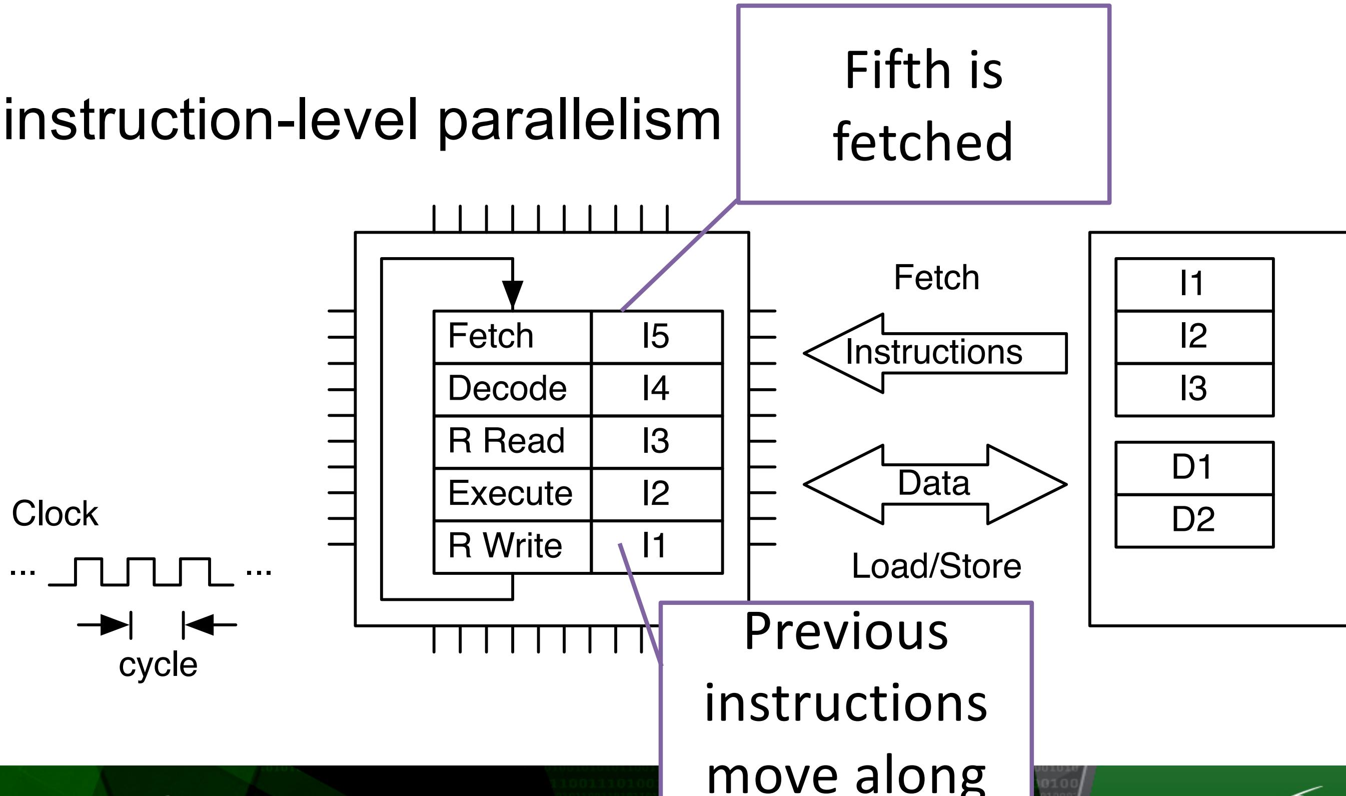
# Processor Core Instruction Handling

- By pipelining, multiple instructions can be executed at each clock cycle
- Form of instruction-level parallelism



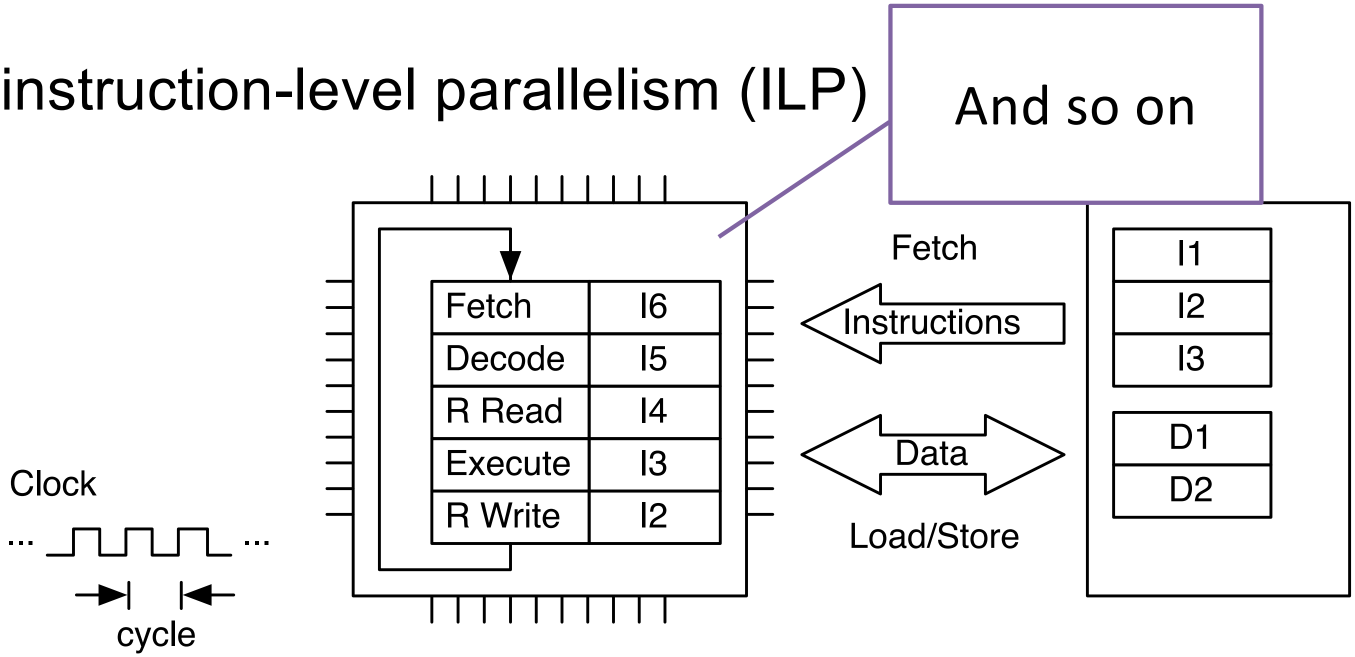
# Processor Core Instruction Handling

- By pipelining, multiple instructions can be executed at each clock cycle
- Form of instruction-level parallelism



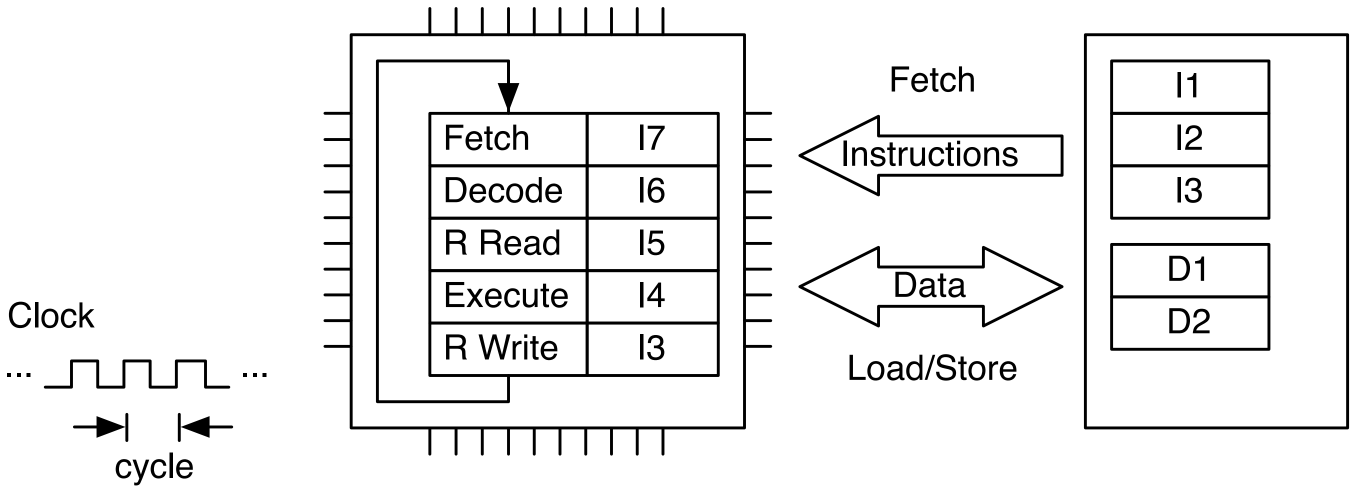
# Processor Core Instruction Handling

- By pipelining, multiple instructions can be executed at each clock cycle
- Form of instruction-level parallelism (ILP)



# Processor Core Instruction Handling

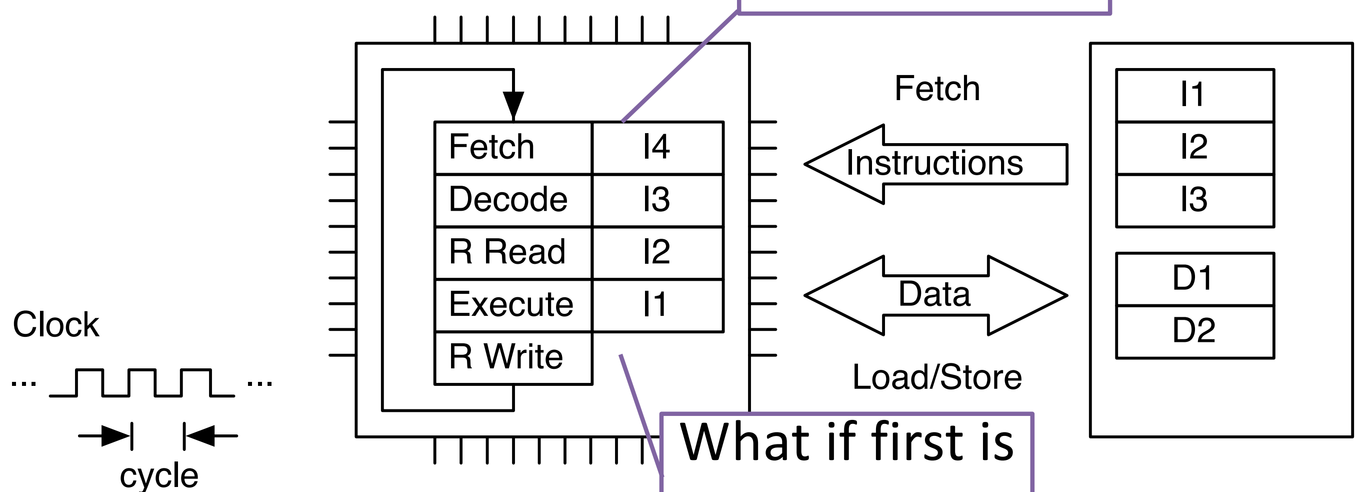
- By pipelining, multiple instructions can be executed at each clock cycle
- Form of instruction-level parallelism (ILP)



# Pipeline Stall

- By pipelining, multiple instructions can be executed at each clock cycle
- Form of instruction-level parallelism

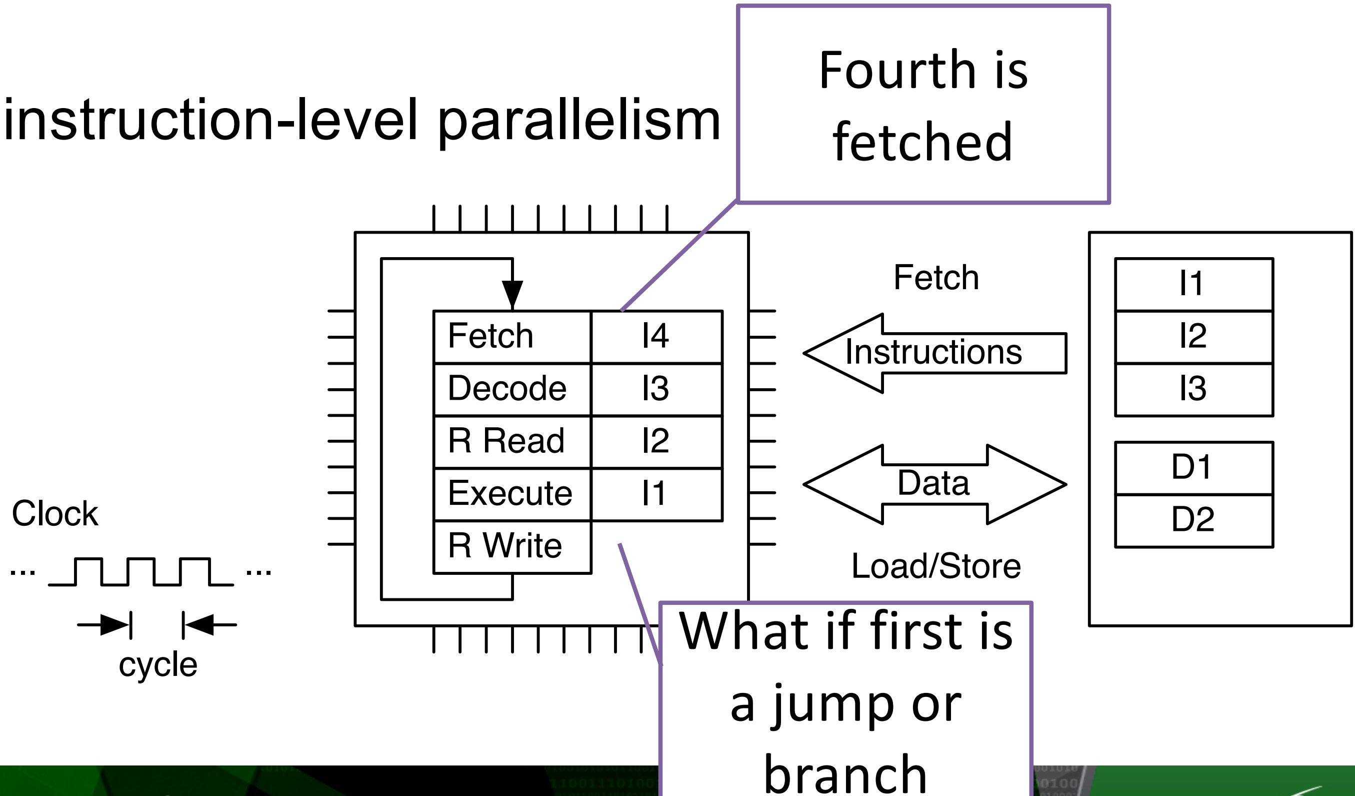
Fourth is fetched




What if first is a jump or branch

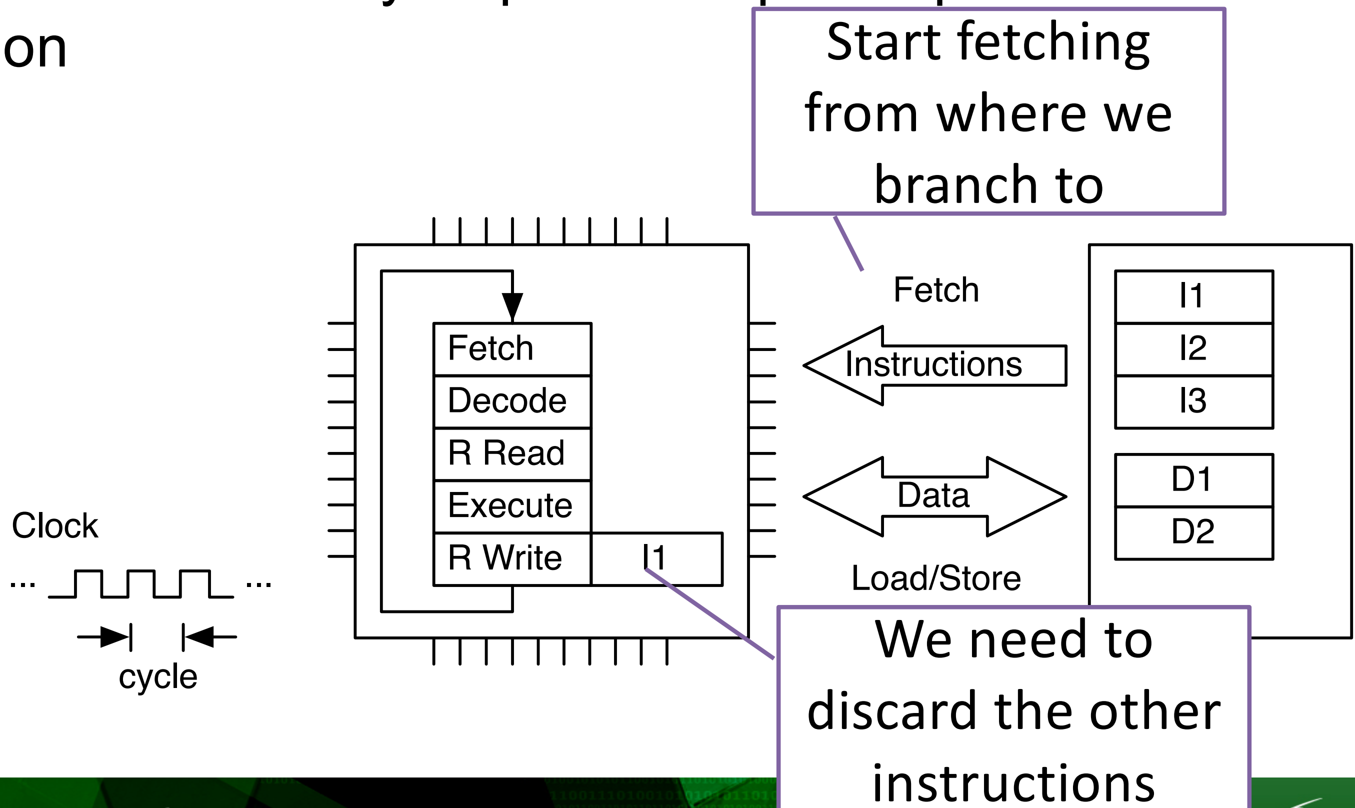
# Pipeline Stall

- By pipelining, multiple instructions can be executed at each clock cycle
- Form of instruction-level parallelism



# Pipeline Stall

- A single instruction may require multiple steps from fetch to completion

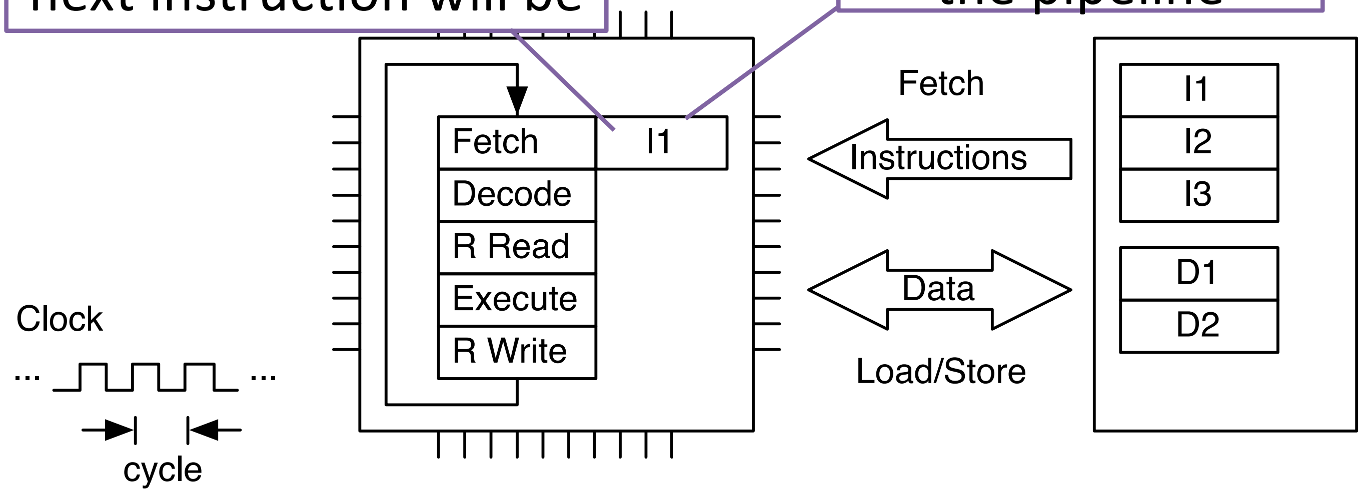


# Branch Prediction

- Load the instructions we think will be branched to

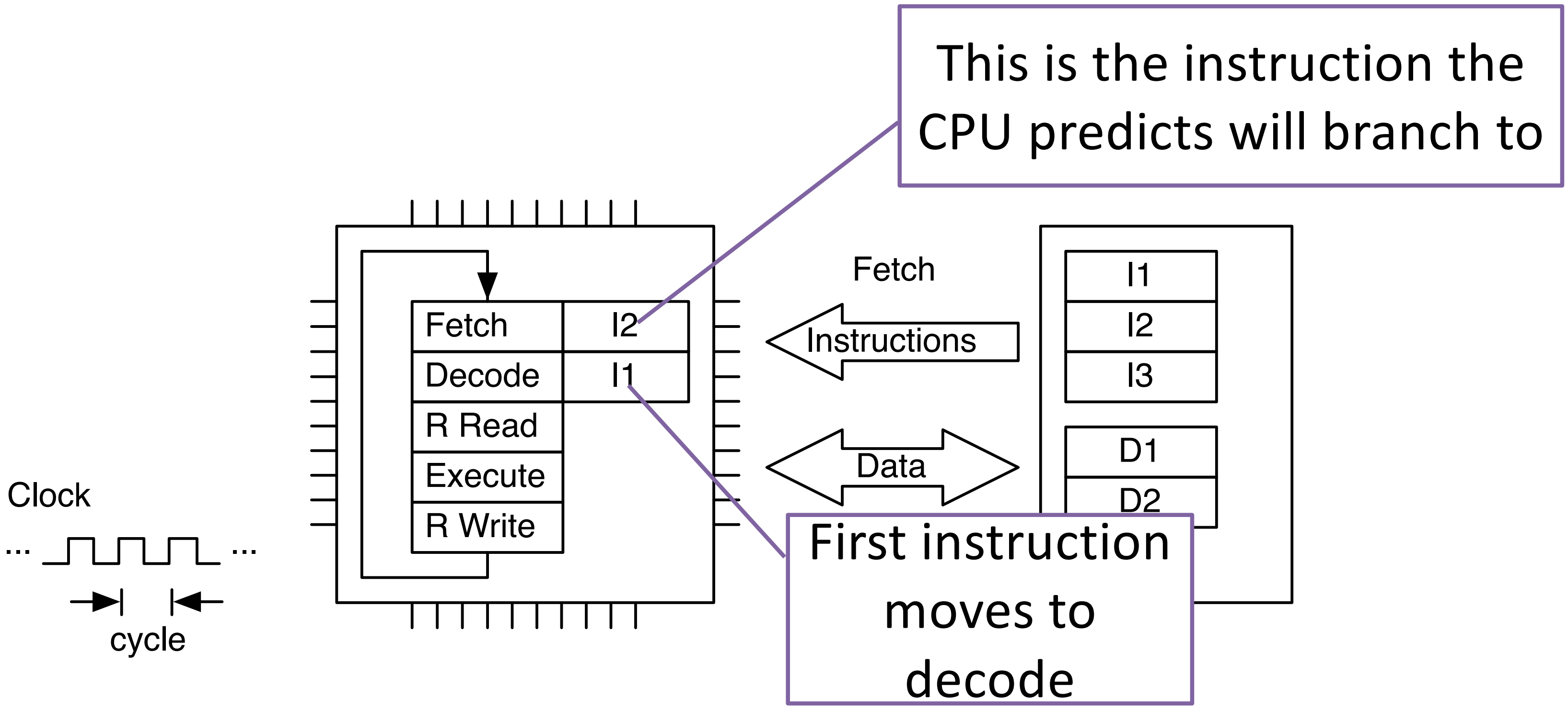
There are two possibilities for what next instruction will be

When a branch instruction enters the pipeline



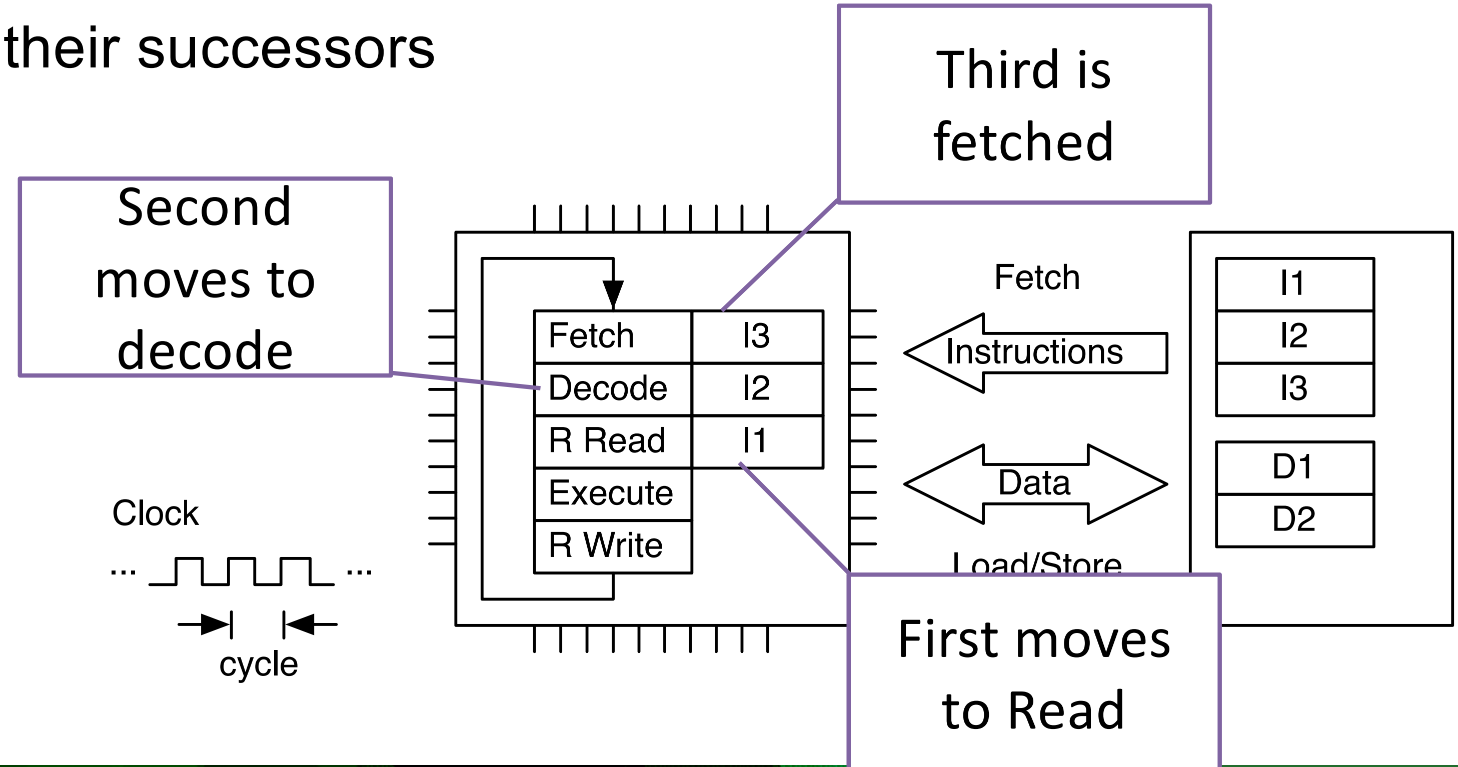
# Branch Prediction

- Load the instructions we think will be branched to



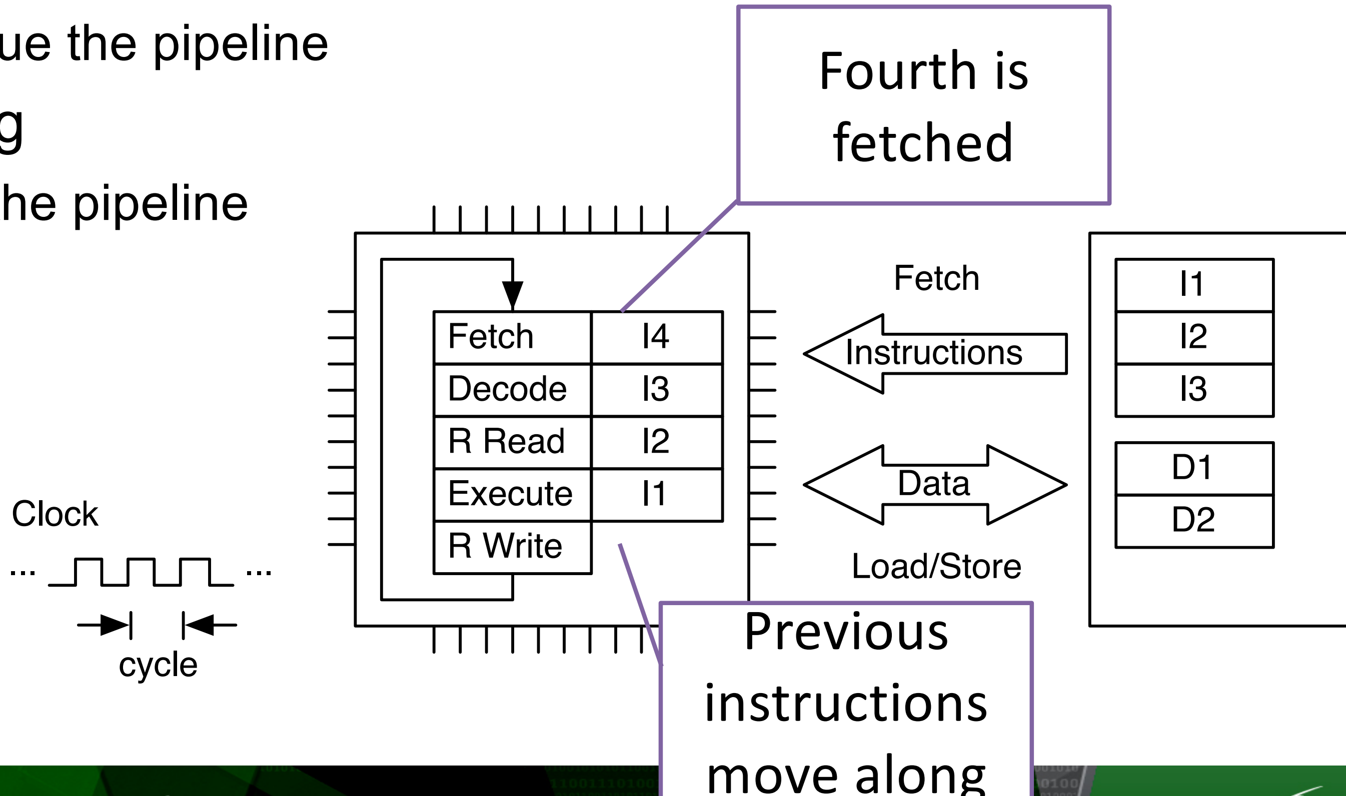
# Branch Prediction

- Load the instructions we think will be branched to
- And their successors



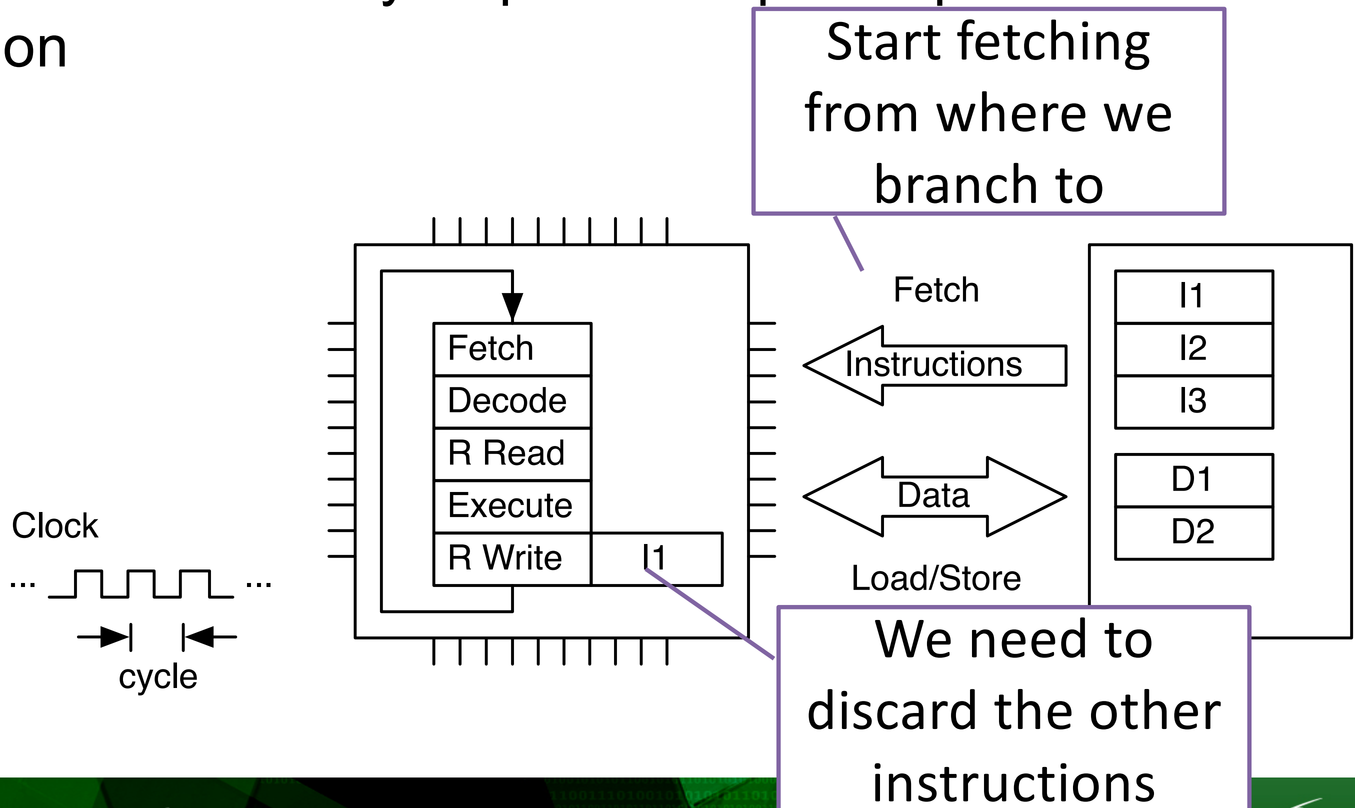
# Instruction Pipelining

- When instruction is executed we were either right
  - Continue the pipeline
- Or wrong
  - Flush the pipeline



# Pipeline Stall from Mis-Predict

- A single instruction may require multiple steps from fetch to completion



# Performance-Oriented Architecture Features

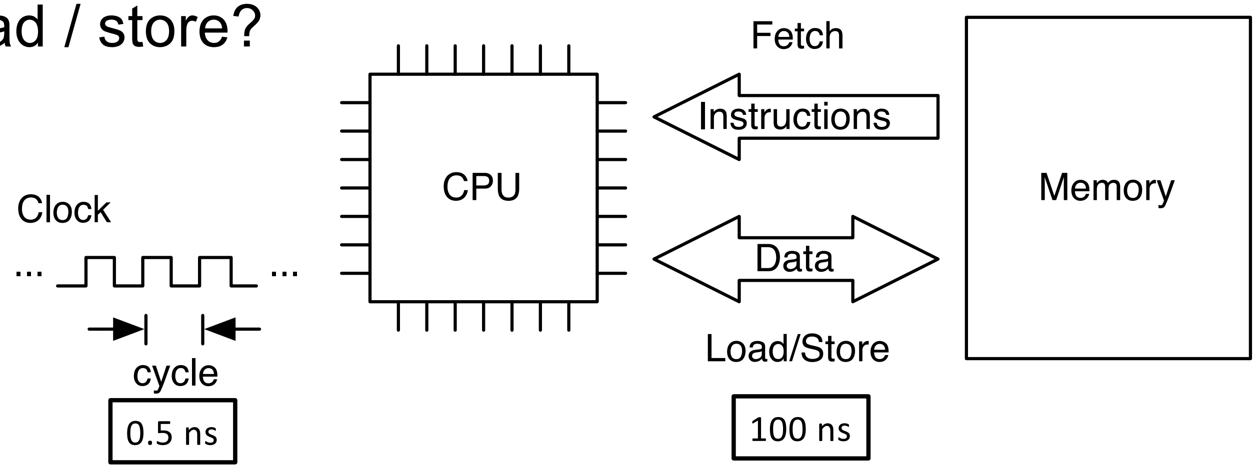
- Execution Pipeline
  - Stages of functionality to process issued instructions
  - Hazards are conflicts with continued execution
  - Forwarding supports closely associated operations exhibiting precedence constraints
- Out of Order Execution
  - Uses reservation stations
  - Hides some core latencies and provide fine grain asynchronous operation supporting concurrency
- Branch Prediction
  - Permits computation to proceed at a conditional branch point prior to resolving predicate value
  - Overlaps follow-on computation with predicate resolution
  - Requires roll-back or equivalent to correct false guesses
  - Sometimes follows both paths, and several deep

# Memory Access

- What are typical costs for accessing memory?
- What is typical clock cycle time?
- How many clock cycles to fetch an instruction? 200
- How many clock cycles to execute load / store instruction? 400
- CPI for load / store?

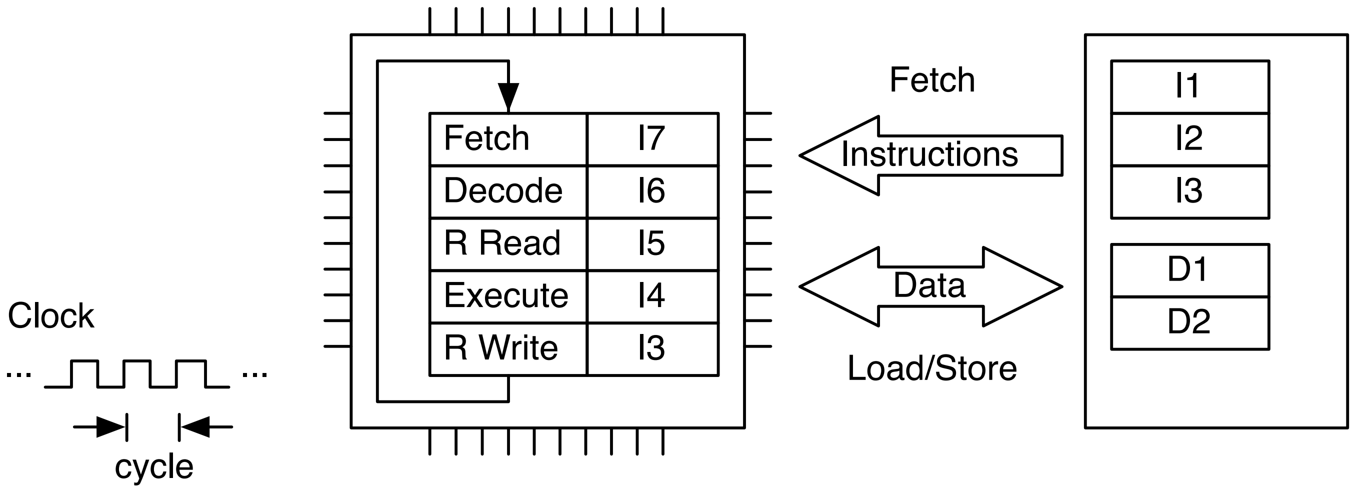
600

The next one may be cheaper



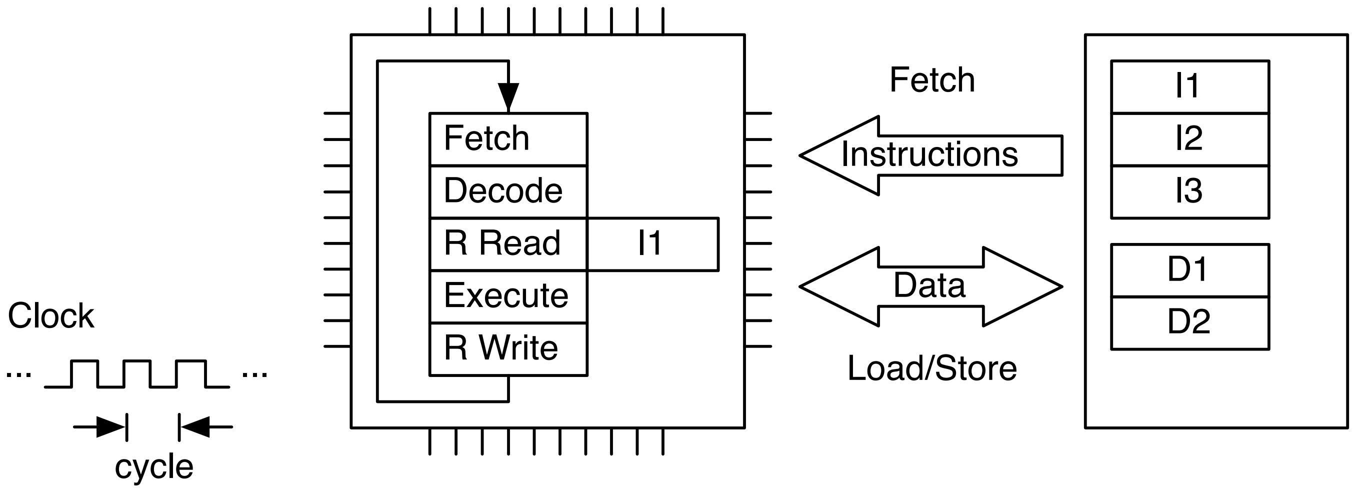
# Memory Access Costs

- Access to main memory has huge impact on performance



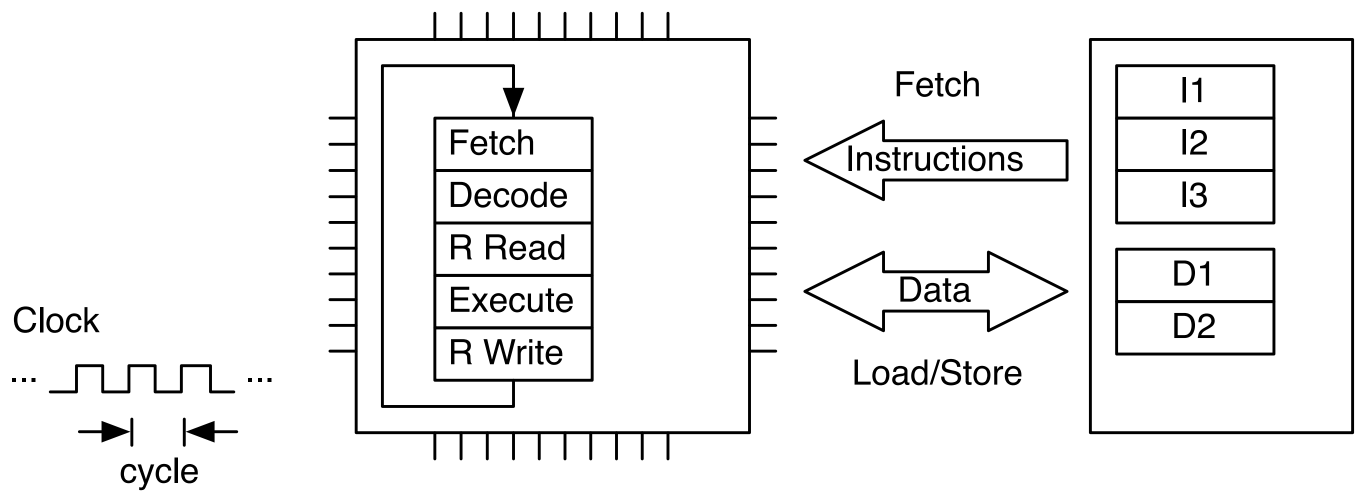
# Memory Access Costs

- Access to main memory has huge impact on performance
- ***Latency***: How long does the first access to data take
- ***Bandwidth***: How much data can we continuously fetch

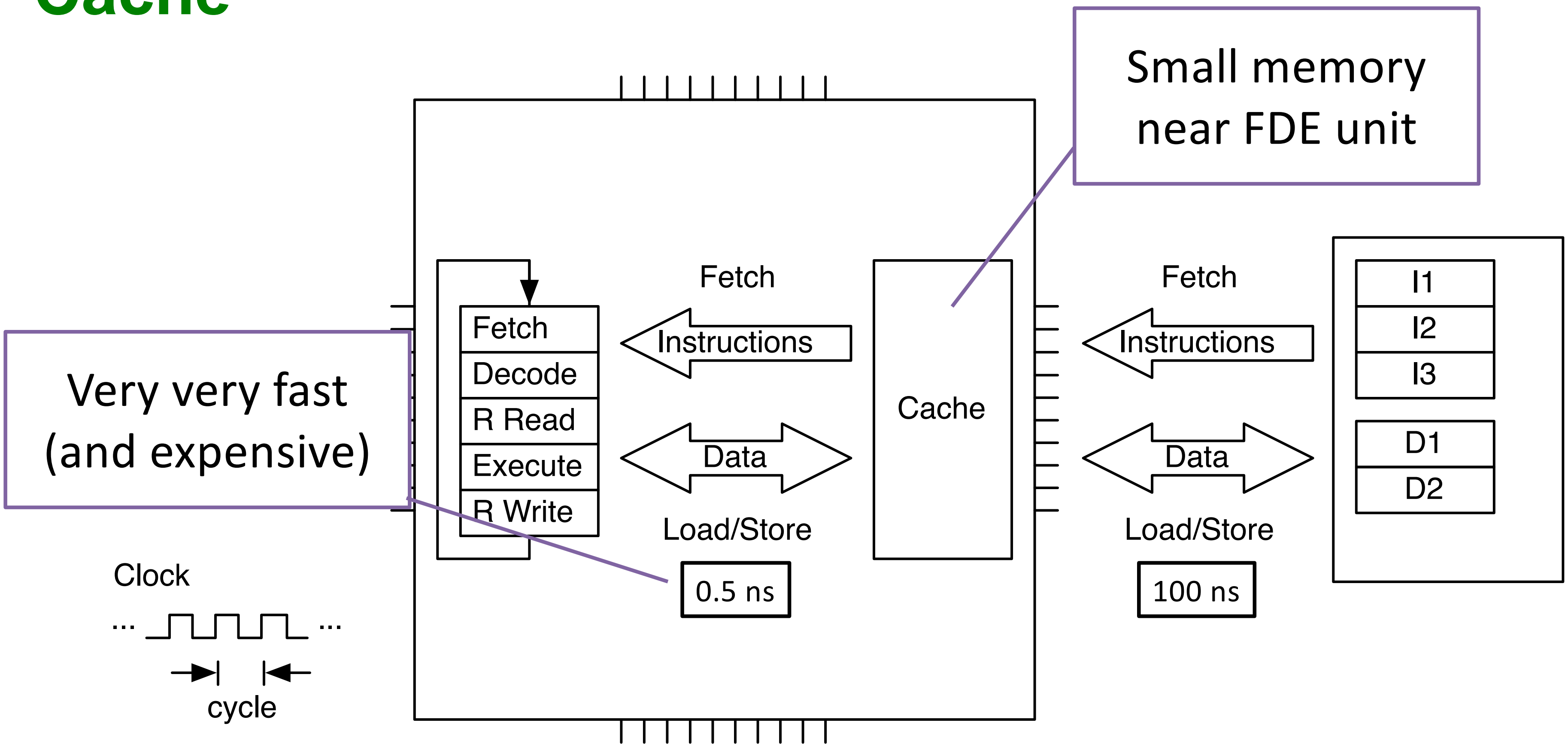


# Memory Access Costs

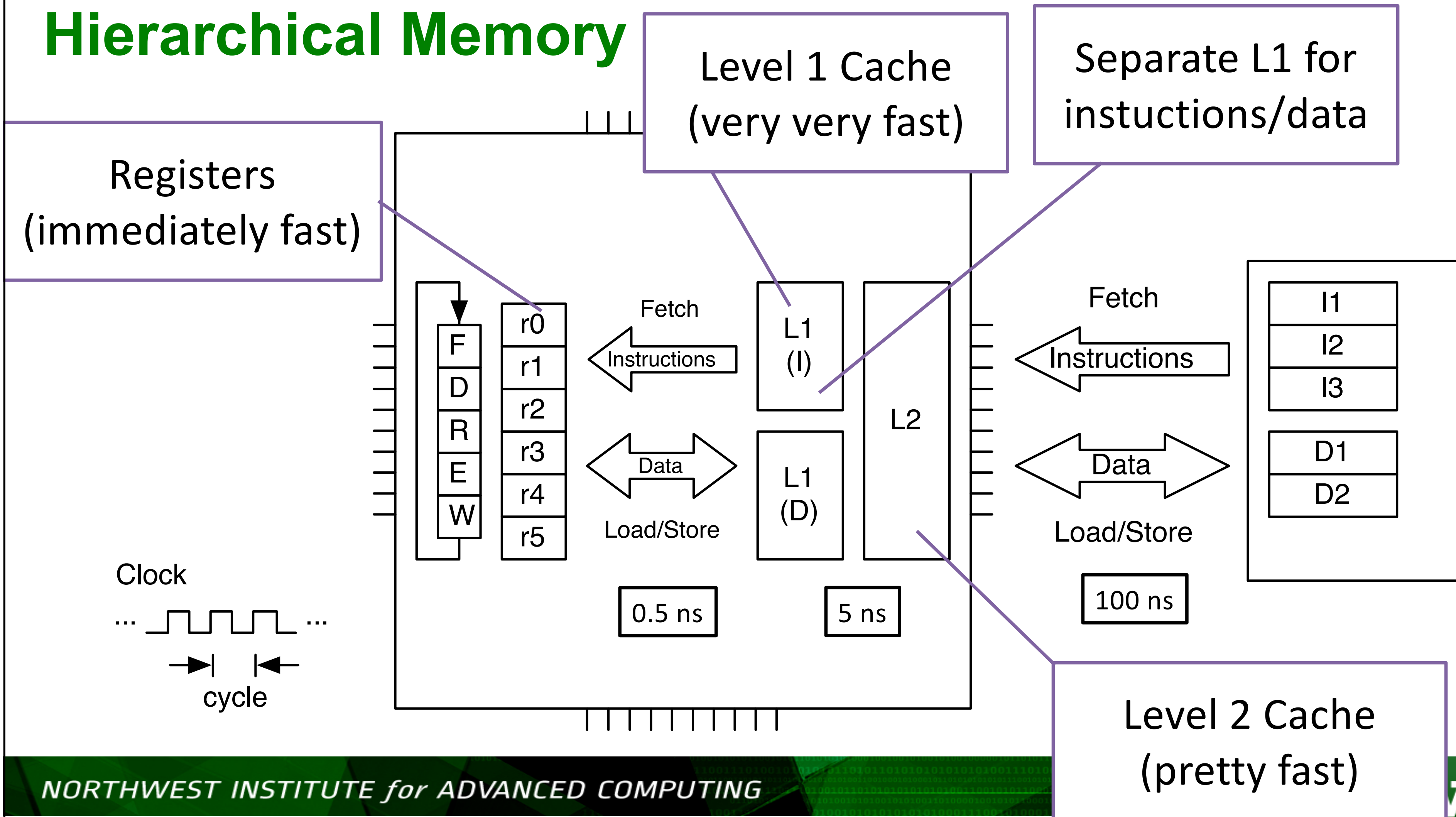
- Access to main memory has huge impact on performance (600X)
- Processor would be idle almost all the time



# Cache


Small memory
near FDE unit
Very very fast
(and expensive)
Fetch
Decode
R Read
Execute
R Write
Fetch
Instructions
Data
Load/Store
0.5 ns
Cache
Fetch
Instructions
Data
Load/Store
100 ns
I1
I2
I3
D1
D2
Clock
... ...
cycle

# Hierarchical Memory



# Hierarchical Memory

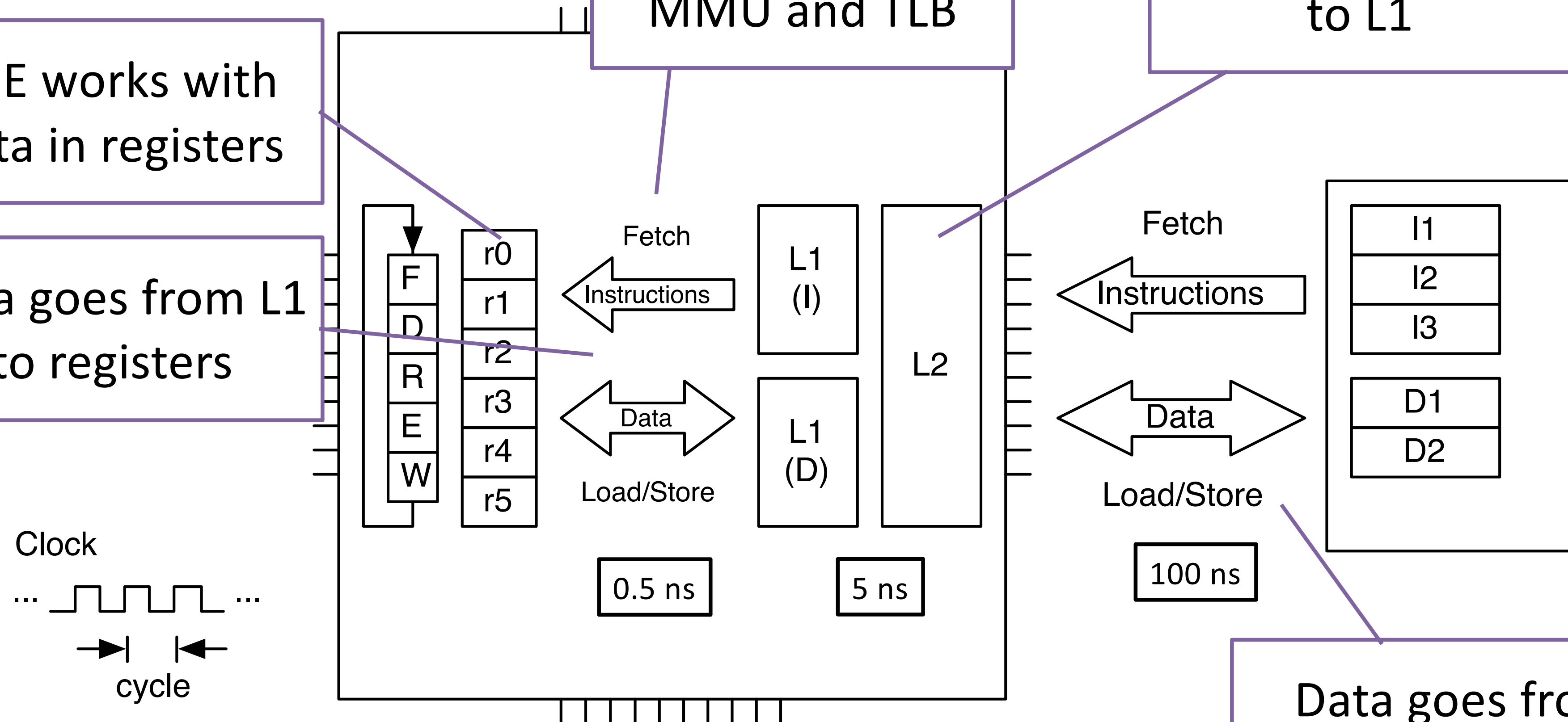
There is also an MMU and TLB

Data goes from L2 to L1

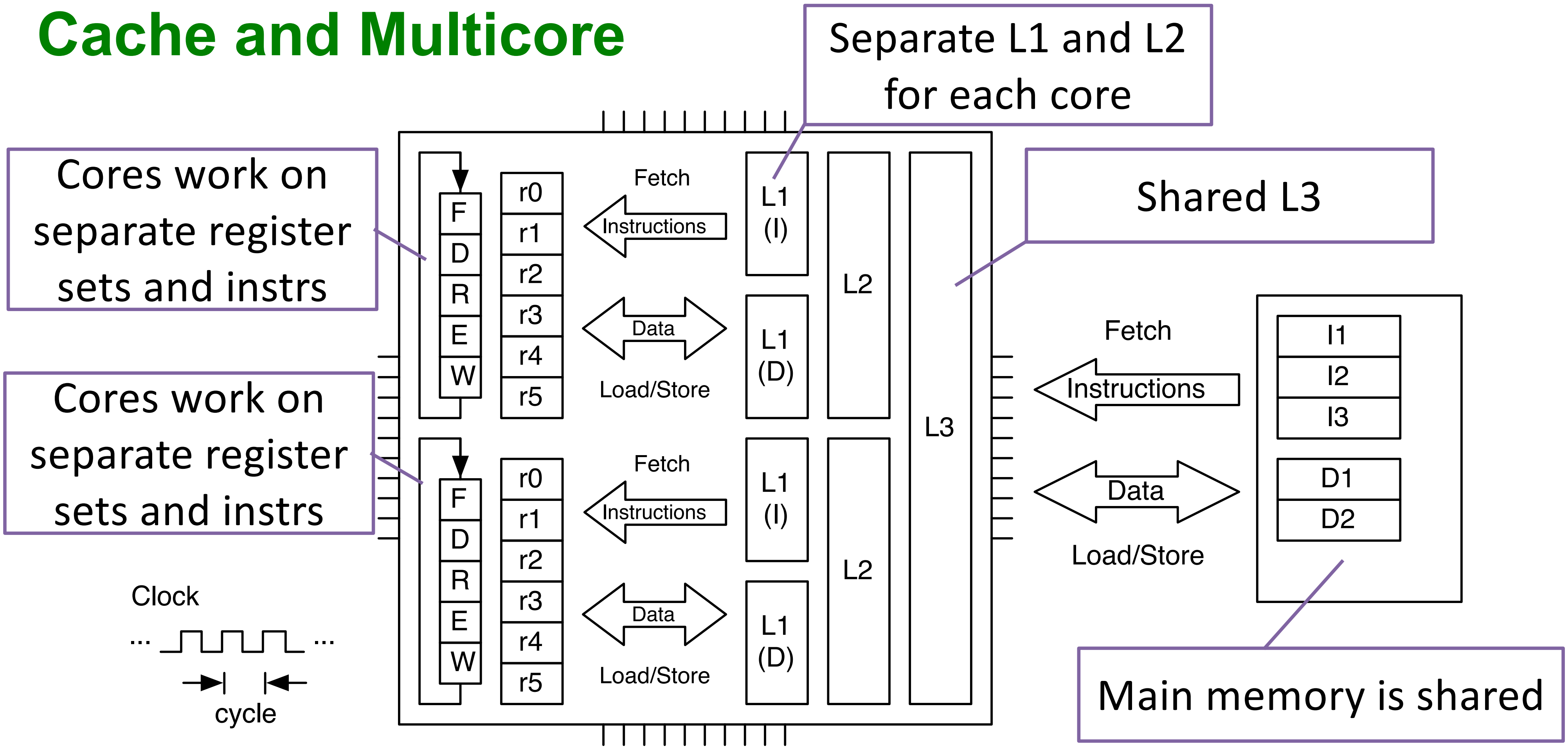
FDE works with data in registers

Data goes from L1 to registers

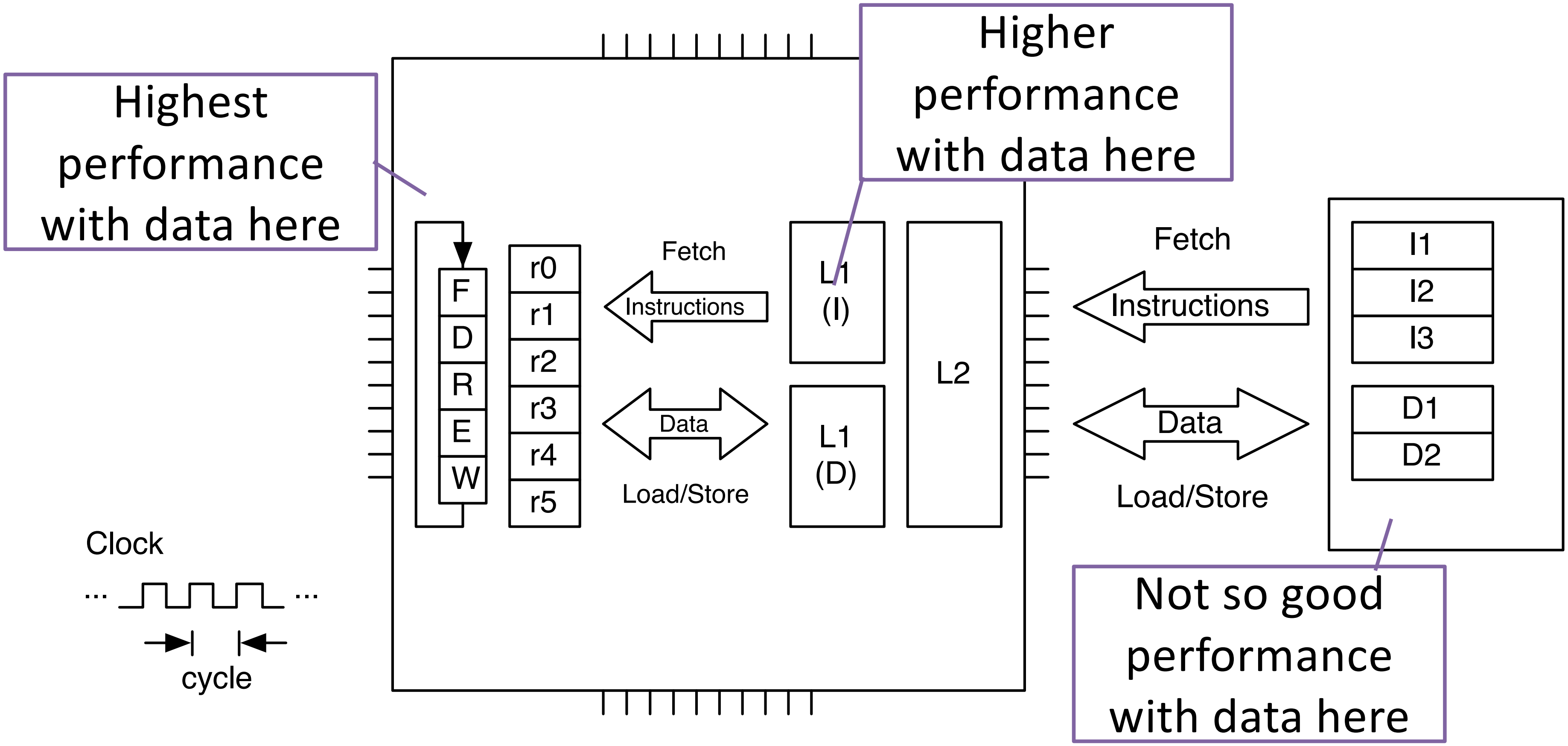
Data goes from main memory to L2



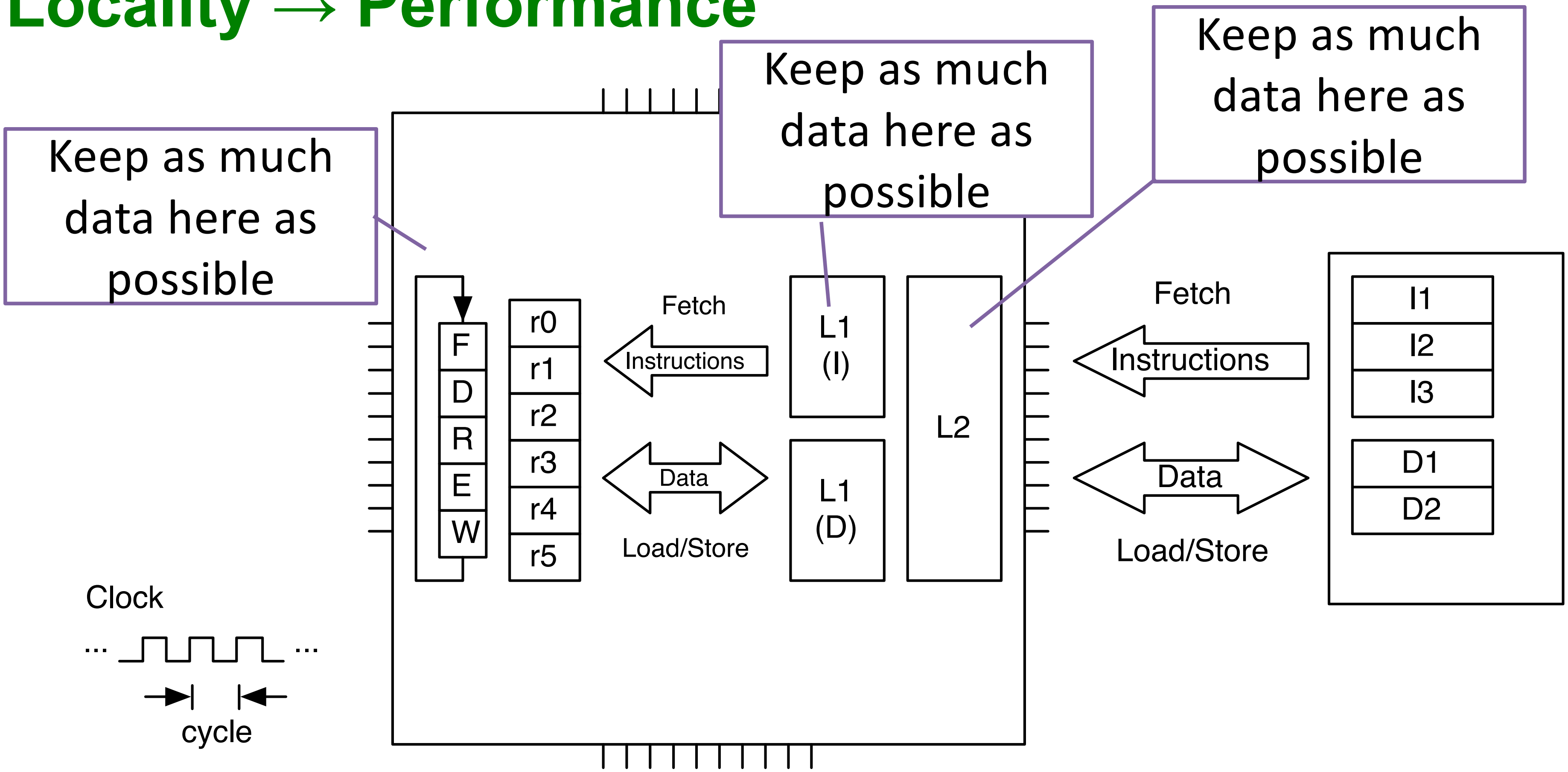
# Cache and Multicore



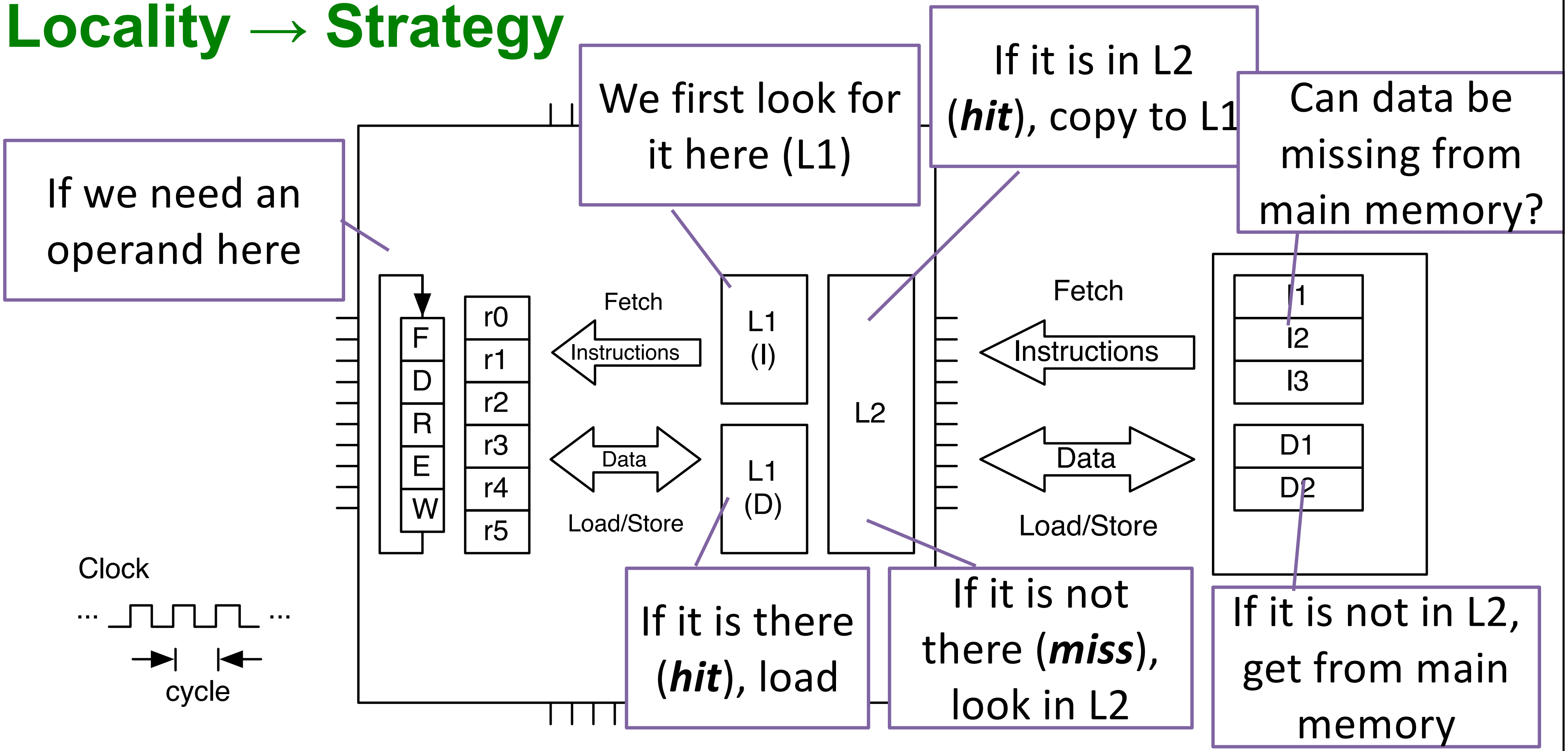
# Performance



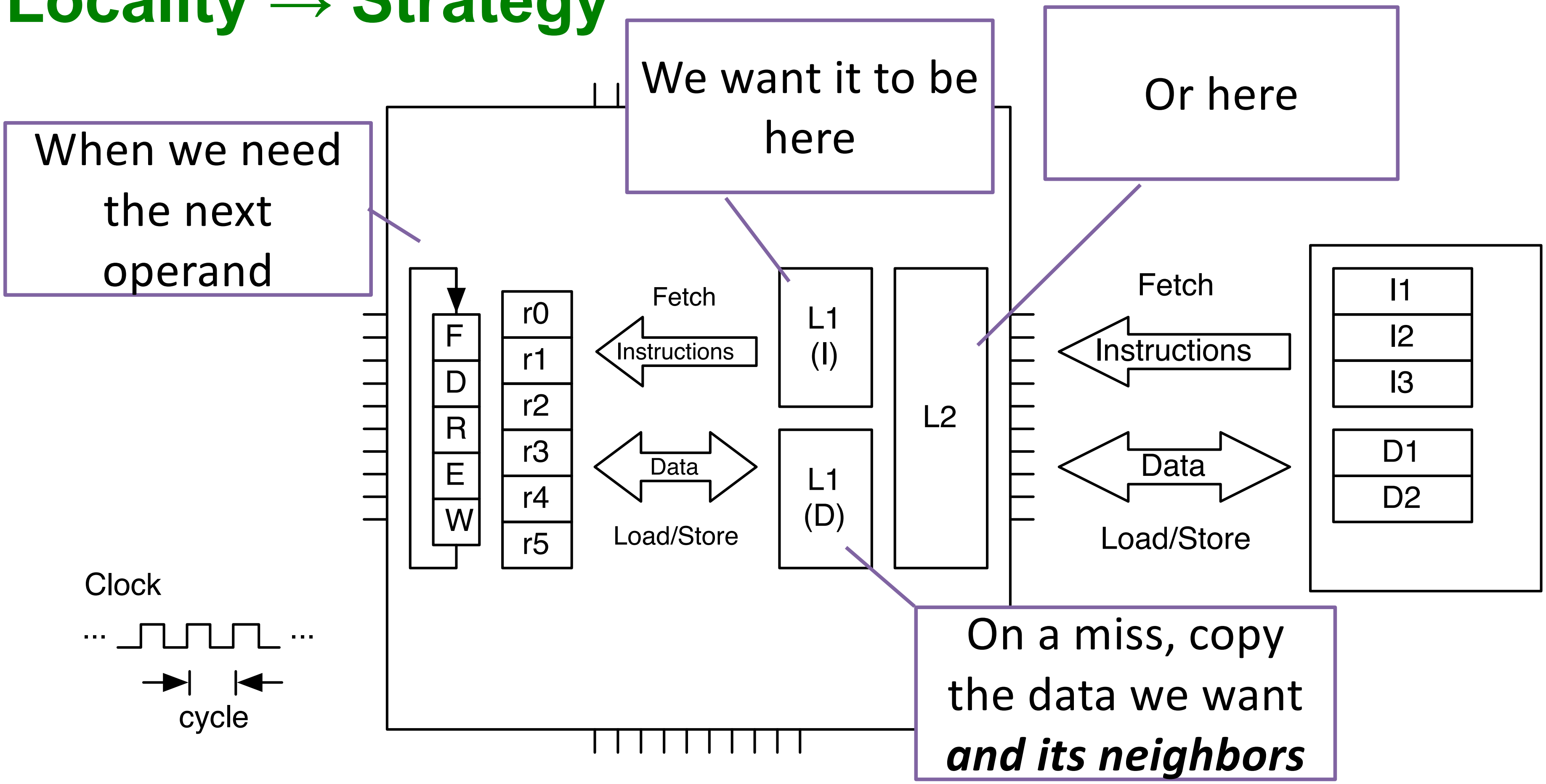
# Locality → Performance



# Locality → Strategy



# Locality → Strategy



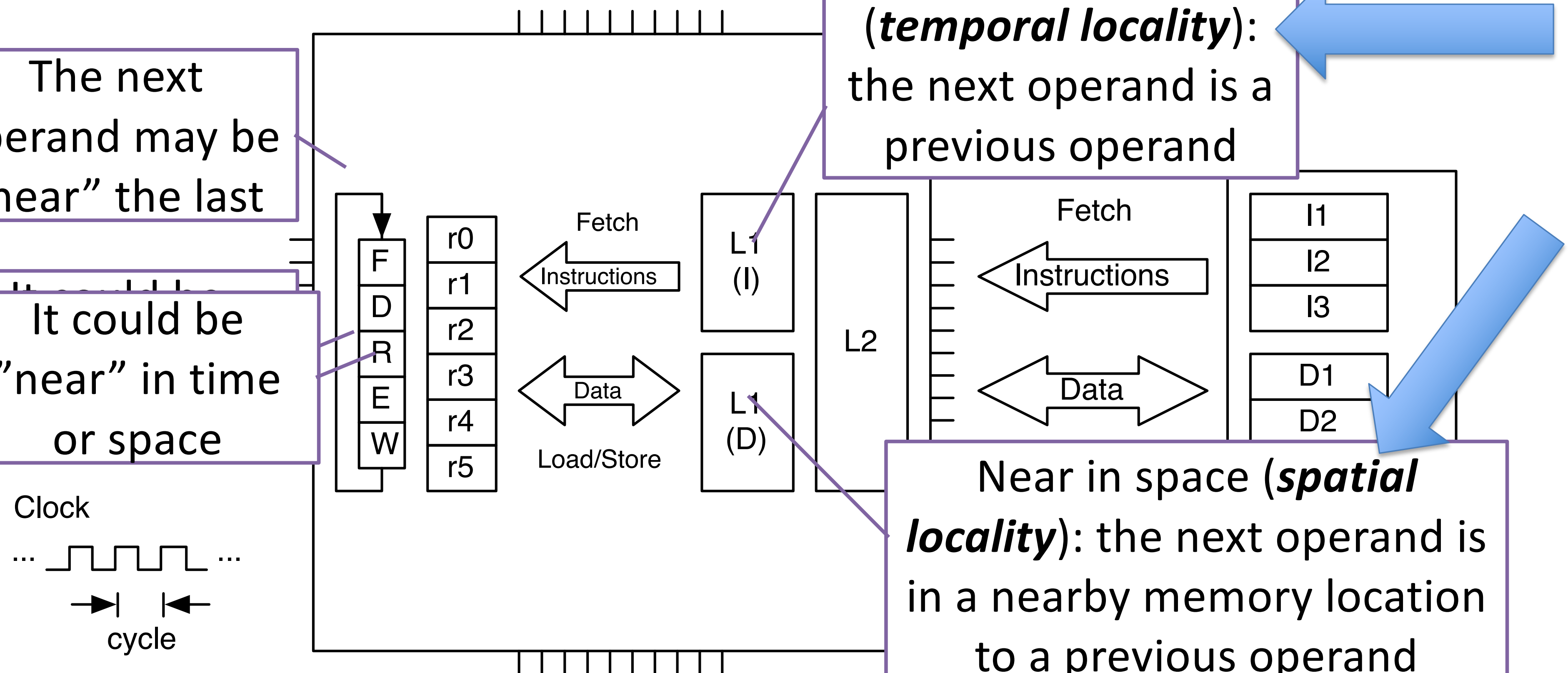
# Locality → Strategy

The next operand may be "near" the last

It could be "near" in time or space

Near in time (***temporal locality***): the next operand is a previous operand

Near in space (***spatial locality***): the next operand is in a nearby memory location to a previous operand



# Locality → Performance

- Caches are much smaller than main memory. How do we decide what data to keep in cache to effect higher performance (more accesses)?
- **Temporal Locality**: if a program accesses a memory location, there is a much higher than random probability that the same location will be accessed again
  - Cache replacement policies attempt to keep cached elements in the cache for as long as possible
- **Spatial Locality**: if a program accesses a memory location, there is a much higher than random probability that nearby locations will also be accessed (soon)
  - Cache policies read contiguous chunks of data – a referenced element and its neighbors – not just single elements

# Thank you!