NORTHWEST INSTITUTE for ADVANCED COMPUTING

# AMATH 483/583
# High Performance Scientific Computing

# Lecture 13:
# Case Studies: TwoNorm, PageRank, Lambda

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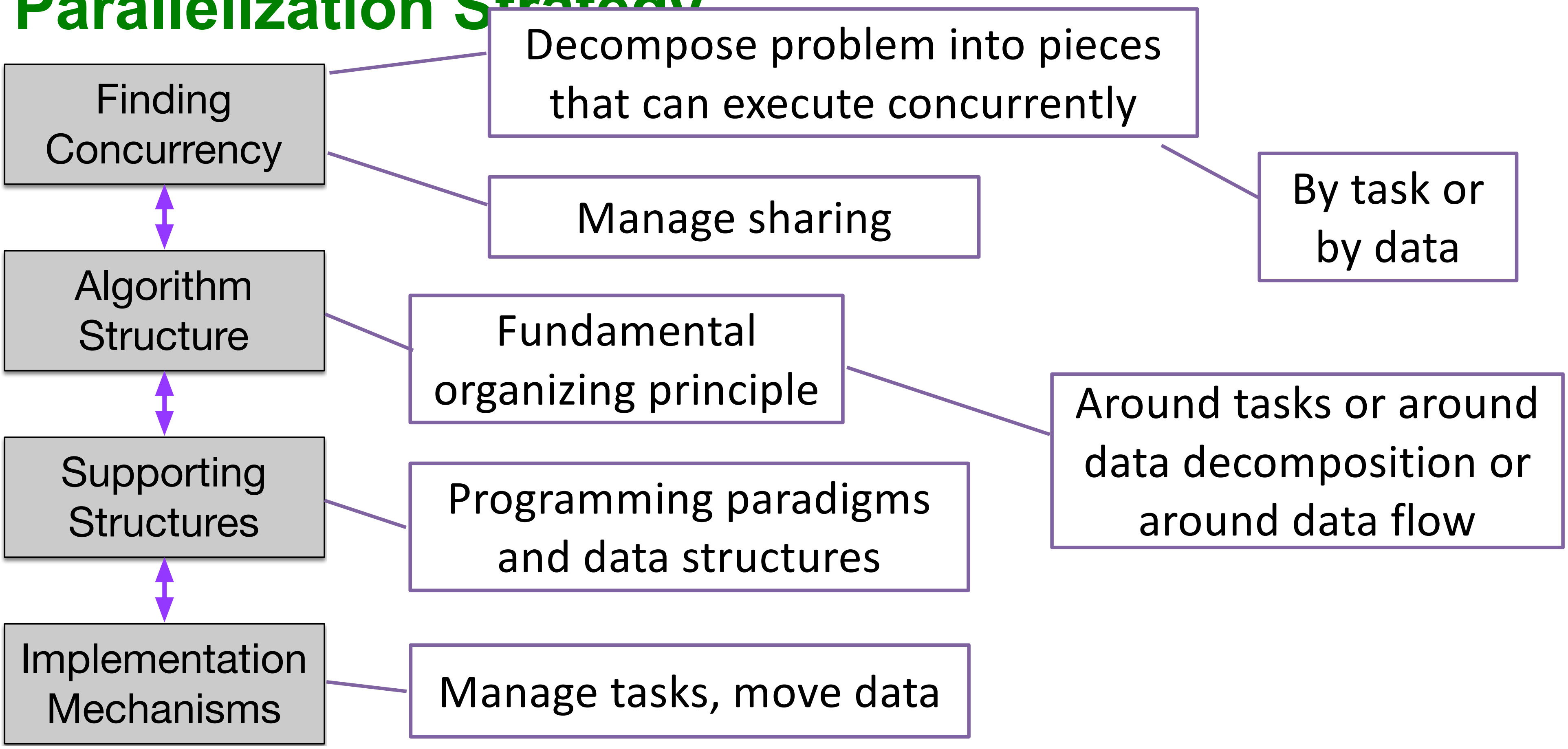
# Questions from Last Time?

# Parallelization Strategy






Timothy Mattson, Beverly Sanders, and Berna Massingill. 2004. *Patterns for Parallel Programming* (First ed.). Addison-Wesley Professional.

# Two Norm Function (Sequential)

```
double two_norm(const Vector& x) {
  double sum = 0.0;
  for (size_t i = 0; i < x.num_rows(); ++i) {
    sum += x(i) * x(i);
  }
  return std::sqrt(sum);
}
```

# Partitioned Vector

```
class PartitionedVector {
public:
  PartitionedVector(size_t M) : num_rows_(M), storage_(num_rows_) {}

        double& operator()(size_t i)       { return storage_[i]; }
  const double& operator()(size_t i) const { return storage_[i]; }

  size_t num_rows() const { return num_rows_; }

  void partition_by_rows(size_t parts) {
    size_t xsize = num_rows_ / parts;
    partitions_.resize(parts+1);
    std::fill(partitions_.begin()+1, partitions_.end(), xsize);
    std::partial_sum(partitions_.begin(), partitions_.end(), partitions_.begin());
  }


private:
  size_t              num_rows_;
  std::vector<double> storage_;
public:
  std::vector<size_t> partitions_;
};
```

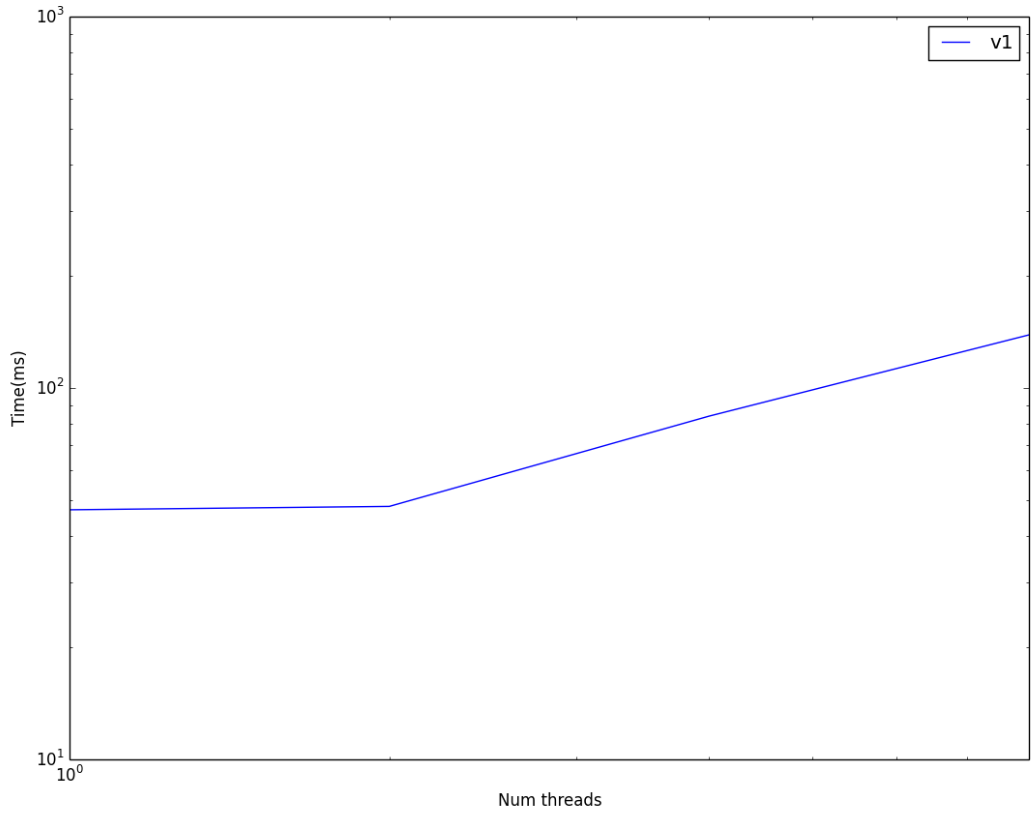
# Two Norm v.1

```
double two_norm_part(const PartitionedVector& x, size_t p) {
  double sum = 0.0;
  for (size_t i = x.partitions_[p]; i < x.partitions_[p+1]; ++i) {
    sum += x(i) * x(i);
  }
  return sum;
}



double two_norm_px(const PartitionedVector& x) {
  std::vector<std::future<double>> futures_;
  for (size_t p = 0; p < x.partitions_.size()-1; ++p) {
    futures_.push_back(std::async(std::launch::async, two_norm_part, x, p));
  }

  double sum = 0.0;
  for (size_t p = 0; p < x.partitions_.size()-1; ++p) {
    sum += futures_[p].get();
  }
  return std::sqrt(sum);
}
```

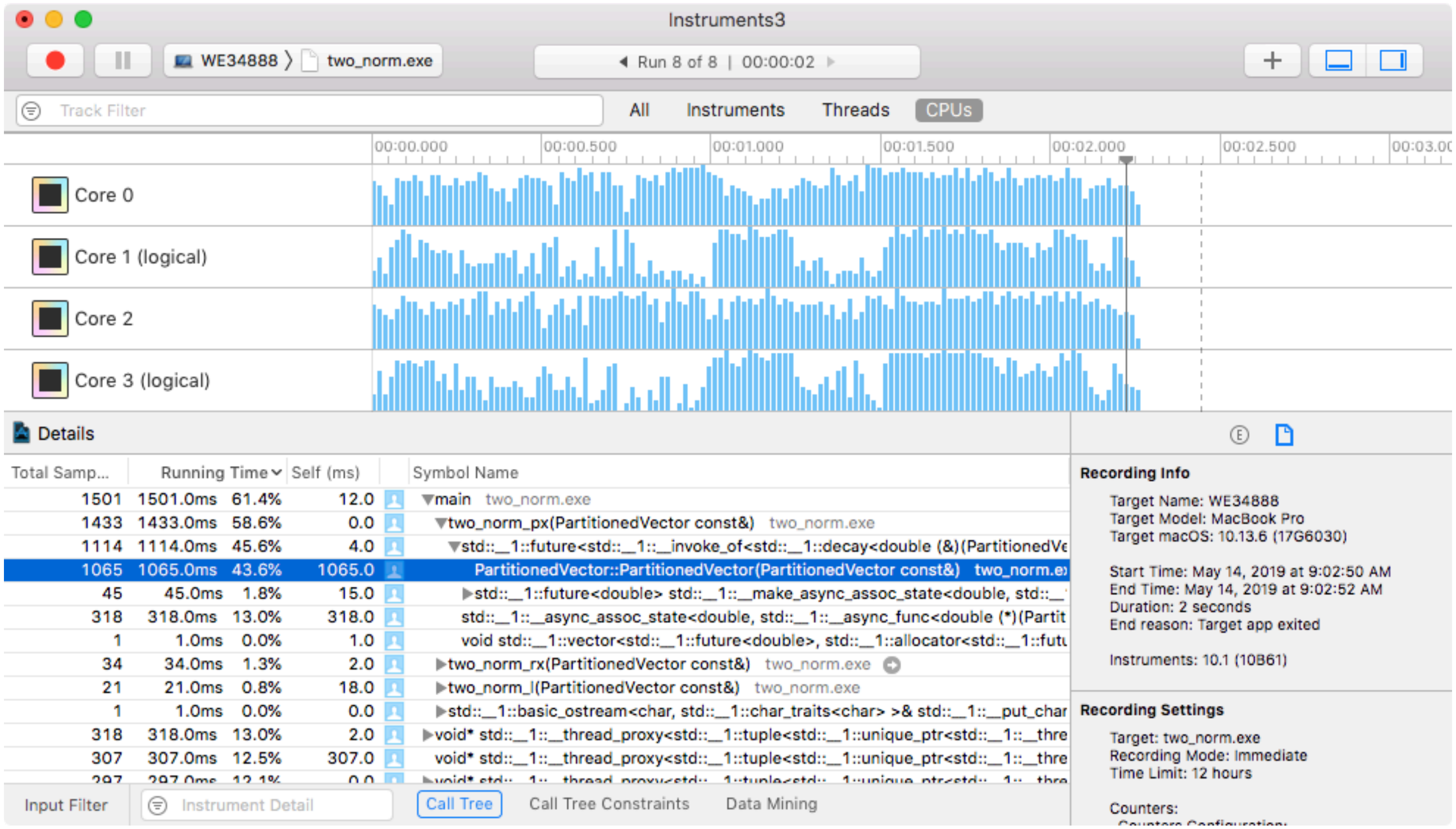
## Timing

```
for (size_t num_threads = 1; num_threads <= 8; num_threads*=2) {
    x.partition_by_rows(num_threads);

    DEF_TIMER(two_norm_rx);
    START_TIMER(two_norm_rx);
    for (size_t i = 0; i < trips; ++i) {
      b += two_norm_rx(x);
    }
    STOP_TIMER(two_norm_rx);
```

# Results



# What Happened?



# What Happened?

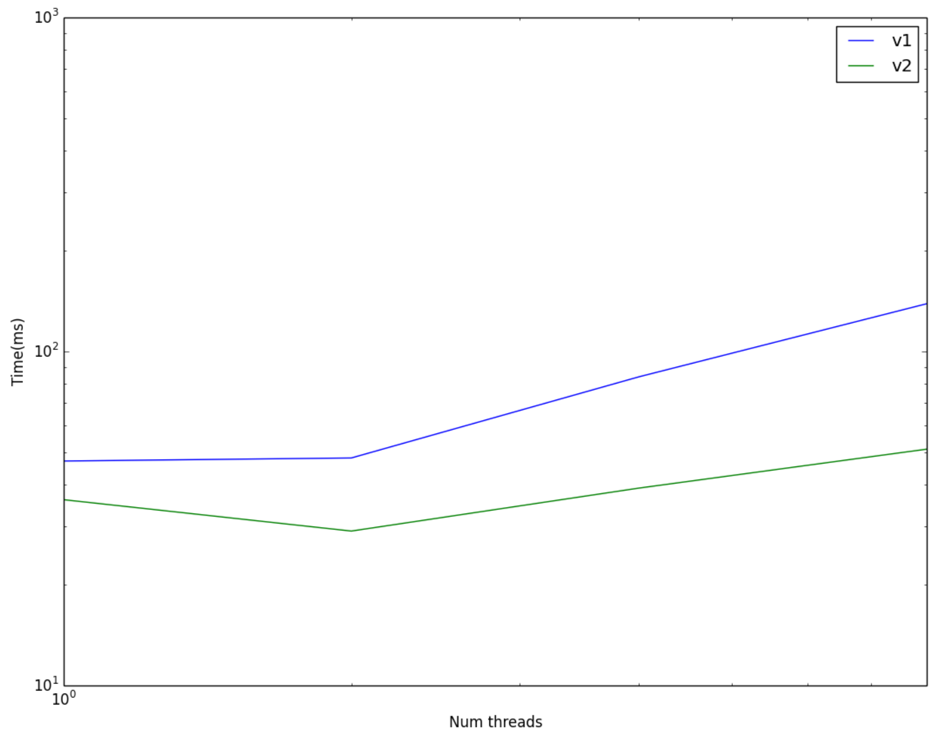
| Total Samp... | Running Time | | Self (ms) | | Symbol Name |
|---|---|---|---|---|---|
| 1501 | 1501.0ms | 61.4% | 12.0 | | ▼main two_norm.exe |
| 1433 | 1433.0ms | 58.6% | 0.0 | | ▼two_norm_px(PartitionedVector const&) two_norm.exe |
| 1114 | 1114.0ms | 45.6% | 4.0 | | ▼std::__1::future<std::__1::__invoke_of<std::__1::decay<double (&)(PartitionedVe |
| 1065 | 1065.0ms | 43.6% | 1065.0 | | PartitionedVector::PartitionedVector(PartitionedVector const&) two_norm.ex |
| 45 | 45.0ms | 1.8% | 15.0 | | ▶std::__1::future<double> std::__1::__make_async_assoc_state<double, std::__ |
| 318 | 318.0ms | 13.0% | 318.0 | | std::__1::__async_assoc_state<double, std::__1::__async_func<double (*)(Partit |
| 1 | 1.0ms | 0.0% | 1.0 | | void std::__1::vector<std::__1::future<double>, std::__1::allocator<std::__1::futu |
| 34 | 34.0ms | 1.3% | 2.0 | | ▶two_norm_rx(PartitionedVector const&) two_norm.exe |
| 21 | 21.0ms | 0.8% | 18.0 | | ▶two_norm_l(PartitionedVector const&) two_norm.exe |
| 1 | 1.0ms | 0.0% | 0.0 | | ▶std::__1::basic_ostream<char, std::__1::char_traits<char> >& std::__1::__put_char |
| 318 | 318.0ms | 13.0% | 2.0 | | ▶void* std::__1::__thread_proxy<std::__1::tuple<std::__1::unique_ptr<std::__1::__thre |
| 307 | 307.0ms | 12.5% | 307.0 | | void* std::__1::__thread_proxy<std::__1::tuple<std::__1::unique_ptr<std::__1::__thre |
| 297 | 297.0ms | 12.1% | 0.0 | | ▶void* std::__1::__thread_proxy<std::__1::tuple<std::__1::unique_ptr<std::__1::__thre |

Input Filter | Instrument Detail | Call Tree | Call Tree Constraints | Data Mining

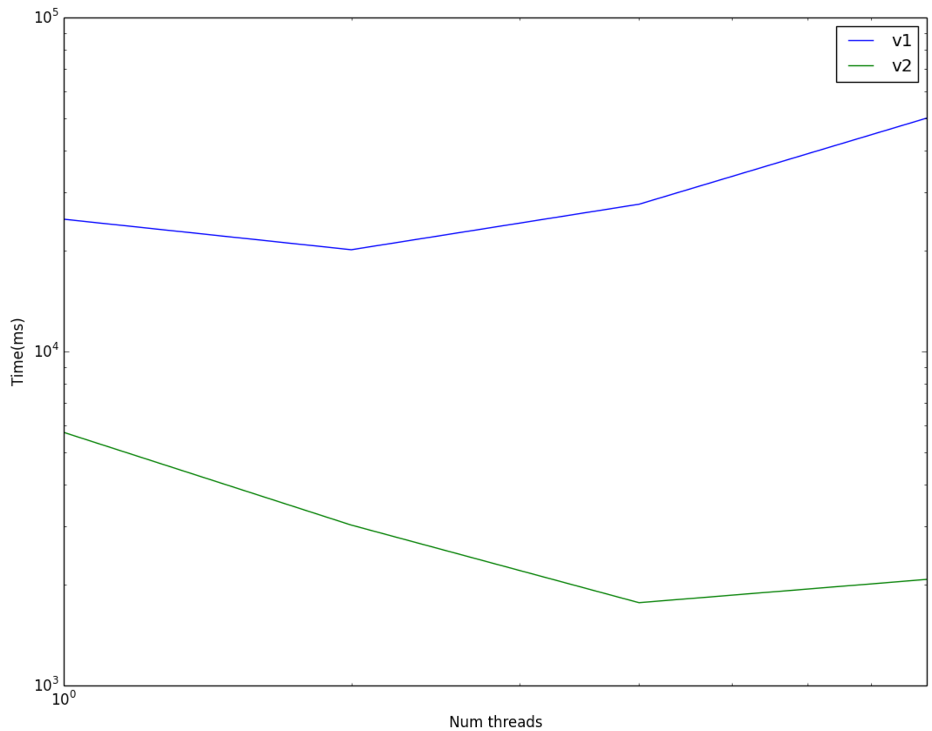
# Two Norm v.2

```
double two_norm_part(const PartitionedVector& x, size_t p) {
  double sum = 0.0;
  for (size_t i = x.partitions_[p]; i < x.partitions_[p+1]; ++i) {
    sum += x(i) * x(i);
  }
  return sum;
}

double two_norm_rx(const PartitionedVector& x) {
  std::vector<std::future<double>> futures_;
  for (size_t p = 0; p < x.partitions_.size()-1; ++p) {
    futures_.push_back(std::async(std::launch::async, two_norm_part, std::cref(x), p));
  }

  double sum = 0.0;
  for (size_t p = 0; p < x.partitions_.size()-1; ++p) {
    sum += futures_[p].get();
  }
  return std::sqrt(sum);
}
```

# Results v.2



# Results v.2



# Walkthrough

# Timing all Three Norms

```
for (size_t num_threads = 1; num_threads <= 8; num_threads *= 2) {

  x.partition_by_rows(num_threads);

  DEF_TIMER(two_norm_px);
  START_TIMER(two_norm_px);
  for (size_t i = 0; i < trips; ++i) {
    a += two_norm_px(x);
  }
  STOP_TIMER(two_norm_px);


for (size_t num_threads = 1; num_threads <= 8; num_threads*=2) {
  x.partition_by_rows(num_threads);

  DEF_TIMER(two_norm_rx);
  START_TIMER(two_norm_rx);
  for (size_t i = 0; i < trips; ++i) {
    b += two_norm_rx(x);
  }
  STOP_TIMER(two_norm_rx);

for (size_t num_threads = 1; num_threads <= 8; num_threads*=2) {
  x.partition_by_rows(num_threads);

  DEF_TIMER(two_norm_l);
  START_TIMER(two_norm_l);
  for (size_t i = 0; i < trips; ++i) {
    c += two_norm_l(x);
  }
  STOP_TIMER(two_norm_l);
```

These are all the same

# Functions as Values

```
void benchmark(const PartitionedVector& x) {
  for (size_t num_threads = 1; num_threads <= 8;  2) {

    x.partition_by_rows(num_threads);

    DEF_TIMER(two_norm_px);
    START_TIMER(two_norm_px);
    for (size_t i = 0; i < trips; ++i) {
      a += <something>(x);
    }
    STOP_TIMER(two_norm_px)
}
```

We want to pass in something

That we call like a function

Double bonus: It just needs an operator()()

Let's not get carried away

# Functions as Values

```
void bench(std::function<double (PartitionedVector&)> two_norm_f,
           PartitionedVector& x) {

  double a = 0;
  for (size_t num_threads = 1; num_threads <= 8; n {

    x.partition_by_rows(num_threads);

    DEF_TIMER(two_norm_px);
    START_TIMER(two_norm_px);
    for (size_t i = 0; i < trips; ++i) {
      a += two_norm_f(std::ref(x));
    }
    STOP_TIMER(two_norm_px);
  }
}
```

Is a function

Parameter f

That returns void

# Two Norm v.2

```
double two_norm_part(const PartitionedVector& x, size_t p) {
  double sum = 0.0;
  for (size_t i = x.partitions_[p]; i < x.partitions_[p+1]; ++i) {
    sum += x(i) * x(i);
  }
  return sum;
}

double two_norm_rx(const PartitionedVector& x) {
  std::vector<std::future<double>> futures_;
  for (size_t p = 0; p < x.partitions_.size()-1; ++p) {
    futures_.push_back(std::async(std::launch::async, two_norm_part, std::cref(x), p));
  }

  double sum = 0.0;
  for (size_t p = 0; p < x.partitions_.size()-1; ++p) {
    sum += futures_[p].get();
  }
  return std::sqrt(sum);
}
```

# Launching async()

```
int main(int argc, char* argv[]) {
  unsigned long intervals   = 1024 * 1024;
  unsigned long num_blocks  = 1;
  double        h           = 1.0 / (double)intervals;
  unsigned long blocksize   = intervals / num_blocks;

  std::vector<std::future<double>> partial_sums;

  for (unsigned long k = 0; k < num_blocks; ++k)
    partial_sums.push_back(
      std::async(std::launch::async,
        partial_pi, k * blocksize, (k + 1) * blocksize, h));

  for (unsigned long k = 0; k < num_blocks; ++k)
    pi += h * partial_sums[k].get();

  std::cout << "pi is approximately " << pi << std::endl;

  return 0;
}
```

"Helper function" (where is it?)

Run right away

Results will be here

# Named function

Return type | Function name | Parameter list | Return value

```
double partial_pi(unsigned long begin, unsigned long end, double h) {
  double partial_pi = 0.0;
  for (unsigned long i = begin; i < end; ++i) {
    partial_pi += 4.0
  }
  return partial_pi;
}
```

Return value | Function name | Parameters

```
double my_pi = partial_pi(0, 100, .001);
```

# Named functions

```
double partial_pi(unsigned long begin, unsigned long end, double h) {
  double partial_pi = 0.0;
  for (unsigned long i = begin; i < end; ++i) {
    partial_pi += 4.0 / (1.0 + (i*h*i*h));
  }
  return partial_pi;
}
```

But what is this really?

Function name

Parameters

```
partial_sums.push_back(
  std::async(std::launch::async,
    partial_pi, k * blocksize, (k + 1) * blocksize, h));
```

# Named variables

```
double pi = 3.14;

double sqrtpi_1 = sqrt583(pi);

double sqrtpi_2 = sqrt583(3.14);
```

Variable name

Variable value

Value will be looked up

Call with variable name

And then sqrt583 will be called

Call with value

Function will be called with same thing as before

# Named functions

```
double partial_pi(unsigned long begin, unsigned long end
  double partial_pi = 0.0;
  for (unsigned long i = begin; i < end; ++i) {
    partial_pi += 4.0 / (1.0 + (i*h*i*h));
  }
  return partial_pi;
}
```

```
partial_sums.push_back(
  std::async(std::launch::async,
    partial_pi, k * blocksize, (k + 1) * blocksize, h));
```

Function name

Can I call std::async directly with the value of partial_pi

(yes)

Value will be looked up

Call with function name

And then std::async will be called

# Name this famous person

Alonzo Church (June 14, 1903 – August 11, 1995) was an American mathematician and logician who made major contributions to mathematical logic and the foundations of theoretical computer science. He is best known for the ***lambda calculus***, Church–Turing thesis, proving the undecidability of the Entscheidungsproblem, Frege–Church ontology, and the Church–Rosser theorem.

Various formalisms for computing

Gottlog Frege

Alan Turing

John Barkley Rosser

# Lambda: Anonymous functions

```
int main(int argc, char* argv[]) {
  unsigned long intervals   = 1024 * 1024;
  unsigned long num_blocks  = 1;
  double        h           = 1.0 / (double)intervals;
  unsigned long blocksize   = intervals / num_blocks;

  std::vector<std::future<double>> partial_sums;

  for (unsigned long k = 0; k < num_blocks; ++k) {
    partial_sums.push_back(std::async(std::launch::async, [&]() -> double {
      double partial_pi = 0.0;
      for (unsigned long i = k * blocksize; i < (k + 1) * blocksize; ++i) {
        partial_pi += 4.0 / (1.0 + (i * h * i * h));
      }
      return partial_pi;
    }));
  }

  double pi = 0.0;
  for (unsigned long k = 0; k < num_blocks; ++k) {
    pi += h * partial_sums[k].get();
  }
  std::cout << "pi is approximately " << std::setprecision(15) << pi << std::endl;

  return 0;
}
```

# Lambda: Anonymous functions

```
for (size_t k = 0; k < num_blocks; ++k) {
  partial_sums.push_back
    (std::async(std::launch::async,
                 [](size_t begin, size_t end, double h) -> double
                 {
                   double partial_pi = 0.0;
                   for (size_t i = begin; i < end; ++i) {
                     partial_pi += 4.0 / (1.0 + (i*h*i*h));
                   }
                   return partial_pi;
                 }
                 ));
}
```

Value of partial_pi

# Two Norm v.3

```
double two_norm_l(const PartitionedVector& x) {
  std::vector<std::future<double>> futures_;
  for (size_t p = 0; p < x.partitions_.size()-1; ++p) {
    futures_.emplace_back(std::async(std::launch::async, [&](size_t p) {
    double sum = 0.0;
    for (size_t i = x.partitions_[p]; i < x.partitions_[p+1]; ++i) {
      sum += x(i) * x(i);
    }
    return sum;
  }, p));

  }

  double sum = 0.0;
  for (size_t p = 0; p < x.partitions_.size()-1; ++p) {
    sum += futures_[p].get();
  }
  return std::sqrt(sum);
}
```

Used to be two_norm_part

lambda

# Before

```
double partial_pi(size_t begin, size_t end, double h)
{
  double partial_pi = 0.0;
  for (size_t i = begin; i < end; ++i) {
    partial_pi += 4.0 / (1.0 + (i*h*i*h));
  }
  return partial_pi;
}
```

# After

```
auto partial_pi(size_t begin, size_t end, double h) -> double
{
  double partial_pi = 0.0;
  for (size_t i = begin; i < end; ++i) {
    partial_pi += 4.0 / (1.0 + (i*h*i*h));
  }
  return partial_pi;
}
```

# Before

```
auto partial_pi(size_t begin, size_t end, double h) -> double
{
  double partial_pi = 0.0;
  for (size_t i = begin; i < end; ++i) {
    partial_pi += 4.0 / (1.0 + (i*h*i*h));
  }
  return partial_pi;
}
```

# After

```
auto partial_pi = [](size_t begin, size_t end, double h) -> double
{
  double partial_pi = 0.0;
  for (size_t i = begin; i < end; ++i) {
    partial_pi += 4.0 / (1.0 + (i*h*i*h));
  }
  return partial_pi;
};
```

# Function values

“Lambda” (this is a function value)

Function parameters

```
auto partial_pi = [](size_t begin, size_t end, double h) -> double
{
  double partial_pi = 0.0;
  for (size_t i = begin; i < end; ++i) {
    partial_pi += 4.0 / (1.0 + (i*h*i*h));
  }
  return partial_pi;
};
```

Return type

Return value

What is the value of partial_pi?

# Before

```
(std::async(std::launch::async,
            partial_pi,




               k * blocksize, (k + 1) * blocksize, h
            ));
```

# After

```
(std::async(std::launch::async,
            [](size_t begin, size_t end, double h) -> double
            {
              double partial_pi = 0.0;
              for (size_t i = begin; i < end; ++i) {
                partial_pi += 4.0 / (1.0 + (i*h*i*h));
              }
              return partial_pi;
            }, k * blocksize, (k + 1) * blocksize, h
            ));
```

# Before

```
(std::async(std::launch::async,
            partial_pi,



               k * blocksize, (k + 1) * blocksize, h
            ));
```

Function name

# After

Function value

```
(std::async(std::launch::async,
            [](size_t begin, size_t end, double h) -> double
            {
              double partial_pi = 0.0;
              for (size_t i = begin; i < end; ++i) {
                partial_pi += 4.0 / (1.0 + (i*h*i*h));
              }
              return partial_pi;
            }, k * blocksize, (k + 1) * blocksize, h
            ));
```

async "sees" the same thing

# All together

```cpp
int main(int argc, char* argv[]) {
  size_t intervals   = 1024 * 1024;
  size_t num_blocks  = 1;
  double       h           = 1.0 / (double)intervals;
  size_t blocksize   = intervals / num_blocks;

  std::vector<std::future<double>> partial_sums;

  for (size_t k = 0; k < num_blocks; ++k) {
    partial_sums.push_back
      (std::async(std::launch::async,
                  [](size_t begin, size_t end, double h) -> double
                  {
                    double partial_pi = 0.0;
                    for (size_t i = begin; i < end; ++i) {
                      partial_pi += 4.0 / (1.0 + (i*h*i*h));
                    }
                    return partial_pi;
                   , k * blocksize, (k + 1) * blocksize, h
                  ));
  }

  double pi = 0.0;
  for (size_t k = 0; k < num_blocks; ++k) {
    pi += h * partial_sums[k].get();
  }
  std::cout << "pi is approximately " << std::setprecision(15) <<
↪  pi << std::endl;

  return 0;
}
```

# All together zoomed

```
size_t intervals   = 1024 * 1024;
size_t num_blocks  = 1;
double       h         = 1.0 / (double)intervals;
size_t blocksize   = intervals / num_blocks;

std::vector<std::future<double>> partial_sums;

for (size_t k = 0; k < num_blocks; ++k) {
  partial_sums.push_back
    (std::async(std::launch::async,
                [](size_t begin, size_t end, double h) -> double
                {
                  double partial_pi = 0.0;
                  r (size_t i = begin; i < end; ++i) {
                  artial_pi += 4.0 / (1.0 + (i*h*i*h));

                  turn partial_pi;
                 , k * blocksize, (k + 1) * blocksize, h
                ));
```

Function parameters

Why can’t we use k, blocksize, and h directly?

Passed parameters

# Capture

```
size_t intervals   = 1024 * 1024;
size_t num_blocks  = 1;
double       h          = 1.0 / (doub
size_t blocksize   = intervals / num_bl

std::vector<std::future<double>> partia

for (size_t k = 0; k < num_blocks; ++k)
  partial_sums.push_back
    (std::async(std::launch::async,
                []() -> double
                {
                  double partial_pi = 0
                  for (size_t i = k*blo
                    partial_pi += 4.0 /
                  }
                  return partial_pi;
                }
                ));
```

```
$ c++ -std=c++11 capture.cpp
capture.cpp:31:23: error: variable 'k' cannot be implicitly captured in a lambda with no capture-default specified
                    for (size_t i = k*blocksize; i < (k+1)*blocksize; ++i) {
                                    ^
capture.cpp:25:15: note: 'k' declared here
  for (size_t k = 0; k < num_blocks; ++k) {
              ^
capture.cpp:28:5: note: lambda expression begins here
                  []() -> double
                  ^
capture.cpp:31:25: error: variable 'blocksize' cannot be implicitly captured in a lambda with no capture-default specified
                    for (size_t i = k*blocksize; i < (k+1)*blocksize; ++i) {
                                      ^
capture.cpp:21:10: note: 'blocksize' declared here
  size_t blocksize   = intervals / num_blocks;
         ^
capture.cpp:28:5: note: lambda expression begins here
                  []() -> double
                  ^
capture.cpp:31:41: error: variable 'k' cannot be implicitly captured in a lambda with no capture-default specified
                    for (size_t i = k*blocksize; i < (k+1)*blocksize; ++i) {
                                                      ^
capture.cpp:25:15: note: 'k' declared here
  for (size_t k = 0; k < num_blocks; ++k) {
              ^
capture.cpp:28:5: note: lambda expression begins here
                  []() -> double
                  ^
capture.cpp:31:46: error: variable 'blocksize' cannot be implicitly captured in a lambda with no capture-default specified
                    for (size_t i = k*blocksize; i < (k+1)*blocksize; ++i) {
                                                           ^
capture.cpp:21:10: note: 'blocksize' declared here
  size_t blocksize   = intervals / num_blocks;
         ^
capture.cpp:28:5: note: lambda expression begins here
                  []() -> double
                  ^
capture.cpp:32:39: error: variable 'h' cannot be implicitly captured in a lambda with no capture-default specified
                      partial_pi += 4.0 / (1.0 + (i*h*i*h));
                                                    ^
capture.cpp:20:17: note: 'h' declared here
  double       h          = 1.0 / (double)intervals;
               ^
capture.cpp:28:5: note: lambda expression begins here
                  []() -> double
                  ^
capture.cpp:32:43: error: variable 'h' cannot be implicitly captured in a lambda with no capture-default specified
                      partial_pi += 4.0 / (1.0 + (i*h*i*h));
                                                        ^
capture.cpp:20:17: note: 'h' declared here
  double       h          = 1.0 / (double)intervals;
               ^
capture.cpp:28:5: note: lambda expression begins here
                  []() -> double
                  ^
6 errors generated.
```

# Before

```
size_t intervals   = 1024 * 1024;
size_t num_blocks  = 1;
double       h            = 1.0 / (double)intervals;
size_t blocksize   = intervals / num_blocks;

std::vector<std::future<double>> partial_sums;

for (size_t k = 0; k < num_blocks; ++k) {
  partial_sums.push_back
    (std::async(std::launch::async,
                []() -> double
                {
                  double partial_pi = 0.0;
                  for (size_t i = k*blocksize; i < (k+1)*blocksize; ++i) {
                    partial_pi += 4.0 / (1.0 + (i*h*i*h));
                  }
                  return partial_pi;
                }
                ));
```

# After

```
size_t intervals   = 1024 * 1024;
size_t num_blocks  = 1;
double       h            = 1.0 / (double)intervals;
size_t blocksize   = intervals / num_blocks;

std::vector<std::future<double>> partial_sums;

for (size_t k = 0; k < num_blocks; ++k) {
  partial_sums.push_back
    (std::async(std::launch::async,
                [&]() -> double
                {
                  double partial_pi = 0.0;
                  for (size_t i = k*blocksize; i < (k+1)*blocksize; ++i) {
                    partial_pi += 4.0 / (1.0 + (i*h*i*h));
                  }
                  return partial_pi;
                }
                ));
```

# After after

```
size_t intervals   = 1024 * 1024;
size_t num_blocks  = 1;
double        h           = 1.0 / (double)intervals;
size_t blocksize   = intervals / num_blocks;

std::vector<std::future<double>> partial_sums;

for (size_t k = 0; k < num_blocks; ++k) {
  partial_sums.push_back
    (std::async(std::launch::async,
                [=]() -> double
                {
                  double partial_pi = 0.0;
                  for (size_t i = k*blocksize; i < (k+1)*blocksize; ++i) {
                    partial_pi += 4.0 / (1.0 + (i*h*i*h));
                  }
                  return partial_pi;
                }
                ));
```

# After after after

```
size_t intervals   = 1024 * 1024;
size_t num_blocks  = 1;
double       h          = 1.0 / (double)intervals;
size_t blocksize   = intervals / num_blocks;

std::vector<std::future<double>> partial_sums;

for (size_t k = 0; k < num_blocks; ++k) {
  partial_sums.push_back
    (std::async(std::launch::async,
                [k, blocksize, &h]() -> double
                {
                  double partial_pi = 0.0;
                  for (size_t i = k*blocksize; i < (k+1)*blocksize; ++i) {
                    partial_pi += 4.0 / (1.0 + (i*h*i*h));
                  }
                  return partial_pi;
                }
                ));
```

# Capture all by reference

```
size_t intervals   = 1024 * 1024;
size_t num_blocks  = 1;
double        h          = 1.0 / (double)intervals;
size_t blocksize   = intervals / num_blocks;

std::vector<std::future<doub

for (size_t k = 0; k < num_b
  partial_sums.push_back
    (std::async(std::launch:
                   [&]() -> double
                   {
                     double partial_pi = 0.0;
                     for (size_t i = k*blocksize; i < (k+1)*blocksize; ++i) {
                       partial_pi += 4.0 / (1.0 + (i*h*i*h));
                     }
                     return partial_pi;
                   }
                   ));
```

Capture all by reference

# Capture all by value

```
size_t intervals   = 1024 * 1024;
size_t num_blocks  = 1;
double        h           = 1.0 / (double)intervals;
size_t blocksize   = intervals / num_blocks;

std::vector<std::future<doub

for (size_t k = 0; k < num_b
  partial_sums.push_back
    (std::async(std::launch:
                    [=]() -> double
                    {
                      double partial_pi = 0.0;
                      for (size_t i = k*blocksize; i < (k+1)*blocksize; ++i) {
                        partial_pi += 4.0 / (1.0 + (i*h*i*h));
                      }
                      return partial_pi;
                    }
                    ));
```

Capture all by value

# Capture some by value, some by reference

```
size_t intervals   = 1024 * 1024;
size_t num_blocks  = 1;
double        h            = 1.0 / (double)intervals;
size_t blocksize   = intervals / num_blocks;

std::vector<std::future<doub

for (size_t k = 0; k < num_b
  partial_sums.push_back
    (std::async(std::launch:
                  [k, blocksize, &h]() -> double
                  {
                    double partial_pi = 0.0;
                    for (size_t i = k*blocksize; i < (k+1)*blocksize; ++i) {
                      partial_pi += 4.0 / (1.0 + (i*h*i*h));
                    }
                    return partial_pi;
                  }
                  ));
```

Pick and choose

# Who Wants to be a Billionaire?

US006285999B1

(12) **United States Patent**
**Page**

(10) **Patent No.:** **US 6,285,999 B1**
(45) **Date of Patent:** **Sep. 4, 2001**

(54) **METHOD FOR NODE RANKING IN A LINKED DATABASE**

(75) Inventor: **Lawrence Page**, Stanford, CA (US)

(73) Assignee: **The Board of Trustees of the Leland Stanford Junior University**, Stanford, CA (US)

( * ) Notice: Subject to any disclaimer, the term of this patent is extended or adjusted under 35 U.S.C. 154(b) by 0 days.

(21) Appl. No.: **09/004,827**

(22) Filed: **Jan. 9, 1998**

**Related U.S. Application Data**

(60) Provisional application No. 60/035,205, filed on Jan. 10, 1997.

(51) **Int. Cl.**[7] ........................................ **G06F 17/30**
(52) **U.S. Cl.** ............................ **707/5**; 707/7; 707/501
(58) **Field of Search** ............................ 707/100, 5, 7, 707/513, 1–3, 10, 104, 501; 345/440; 382/226, 229, 230, 231

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(List continued on next page.)

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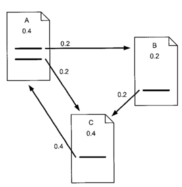
(74) *Attorney, Agent, or Firm*—Harrity & Snyder L.L.P.

(57) **ABSTRACT**


A method assigns importance ranks to nodes in a linked database, such as any database of documents containing citations, the world wide web or any other hypermedia database. The rank assigned to a document is calculated from the ranks of documents citing it. In addition, the rank of a document is calculated from a constant representing the probability that a browser through the database will randomly jump to the document. The method is particularly useful in enhancing the performance of search engine results for hypermedia databases, such as the world wide web, whose documents have a large variation in quality.


**29 Claims, 3 Drawing Sheets**





US006285999B1

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(58) **Field of Search** ............................ 707/100, 5, 7,

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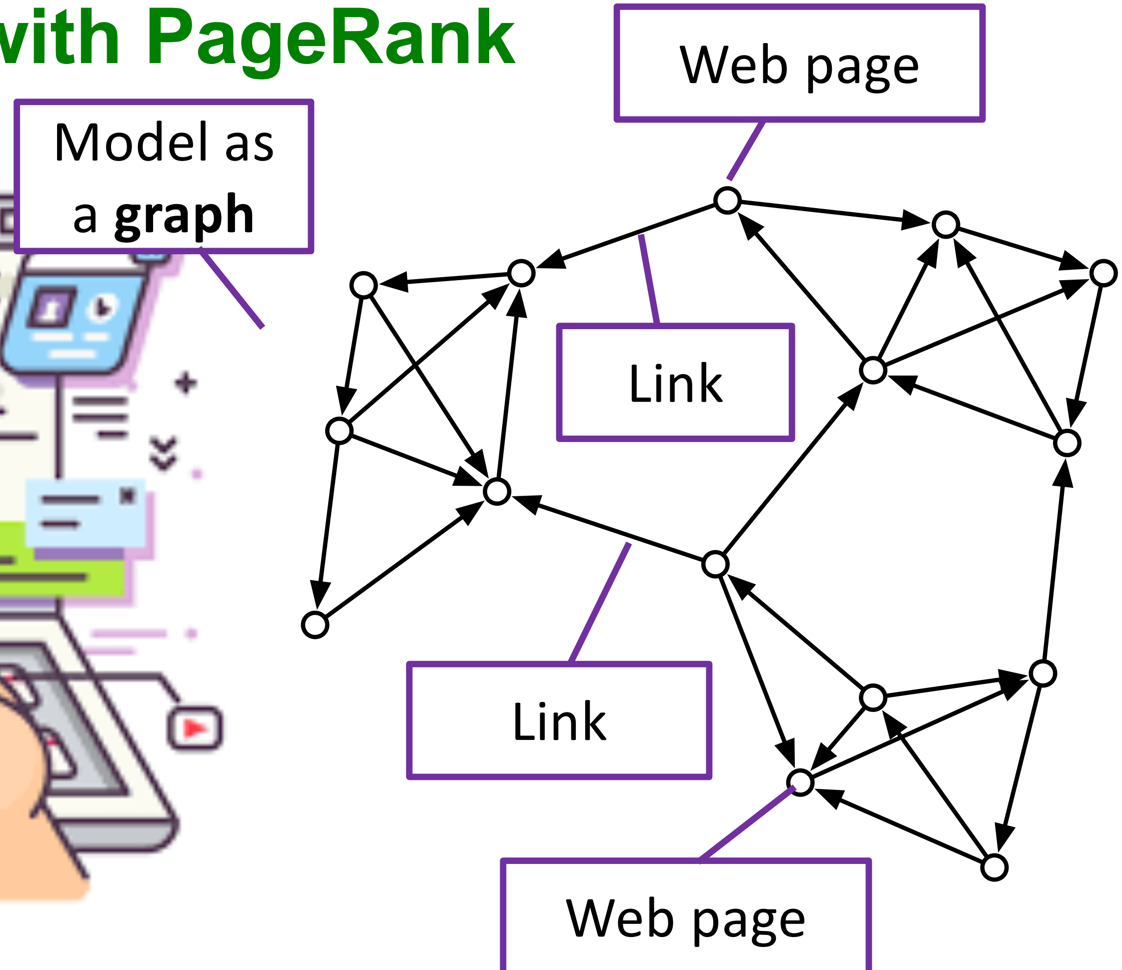
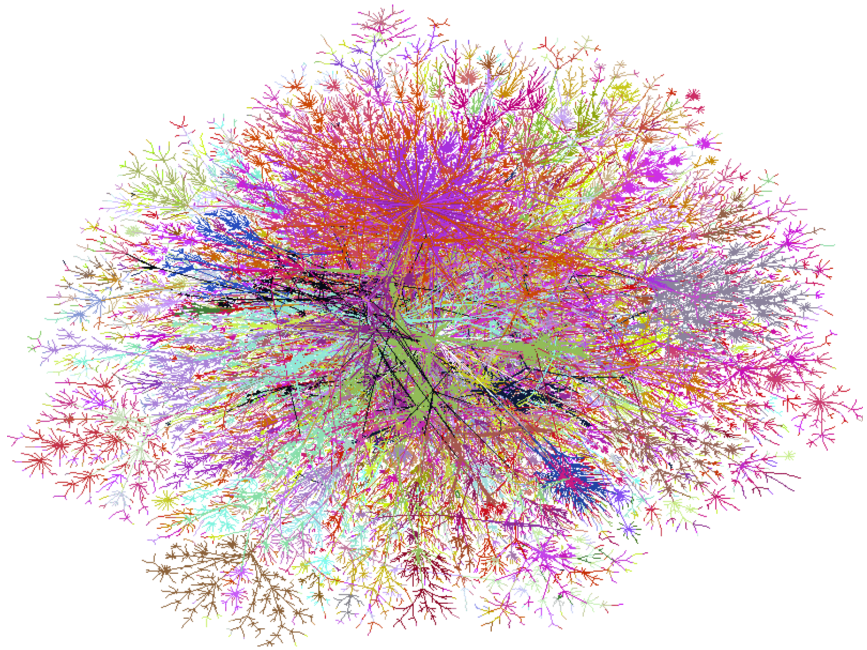
P. Doreian, "Measuring the relative standing of disciplinary journals," 1988, Inf. Proc. And Management, vol. 24, pp. 45–56.

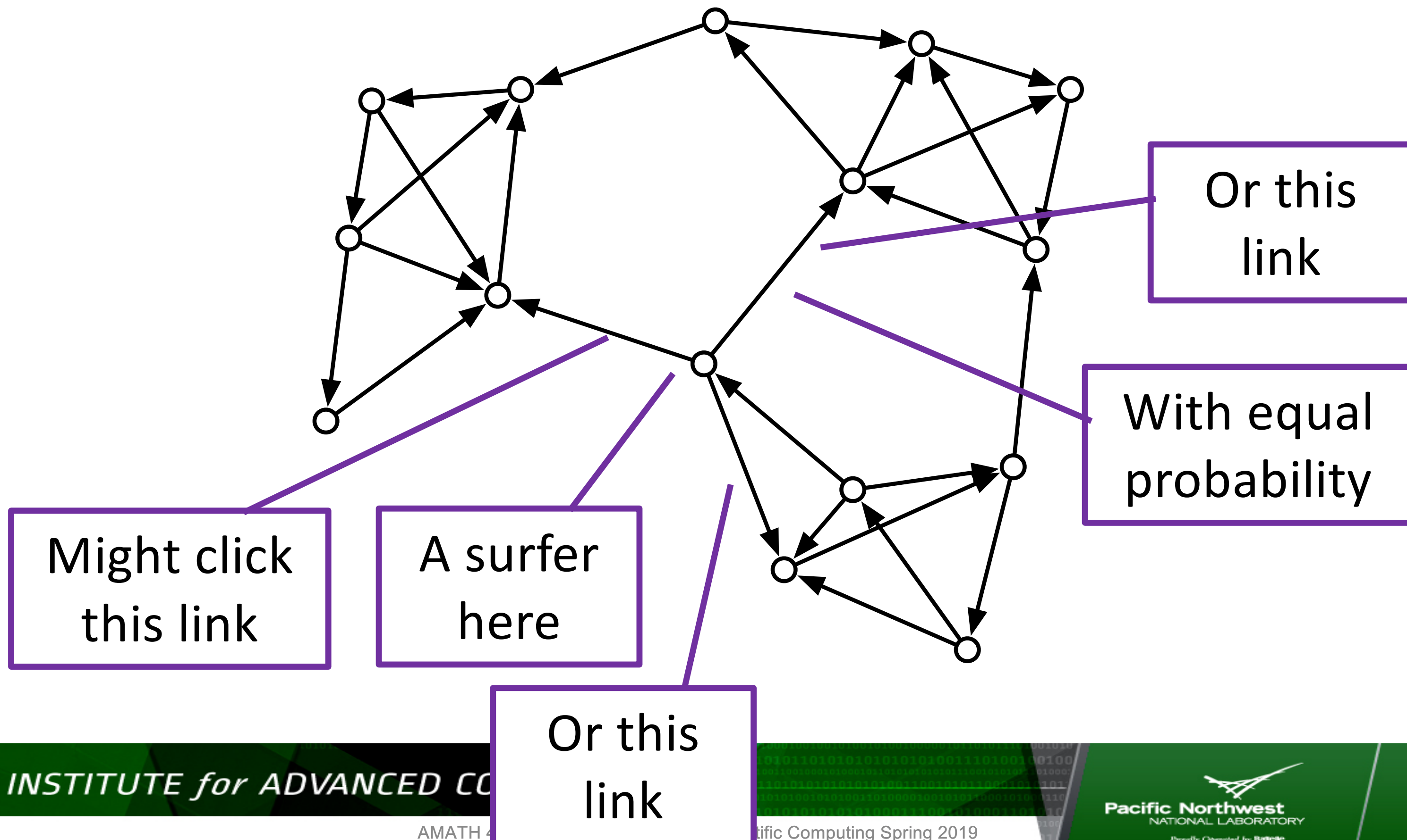
# Ranking Web Pages with PageRank



# Ranking Web Pages with PageRank



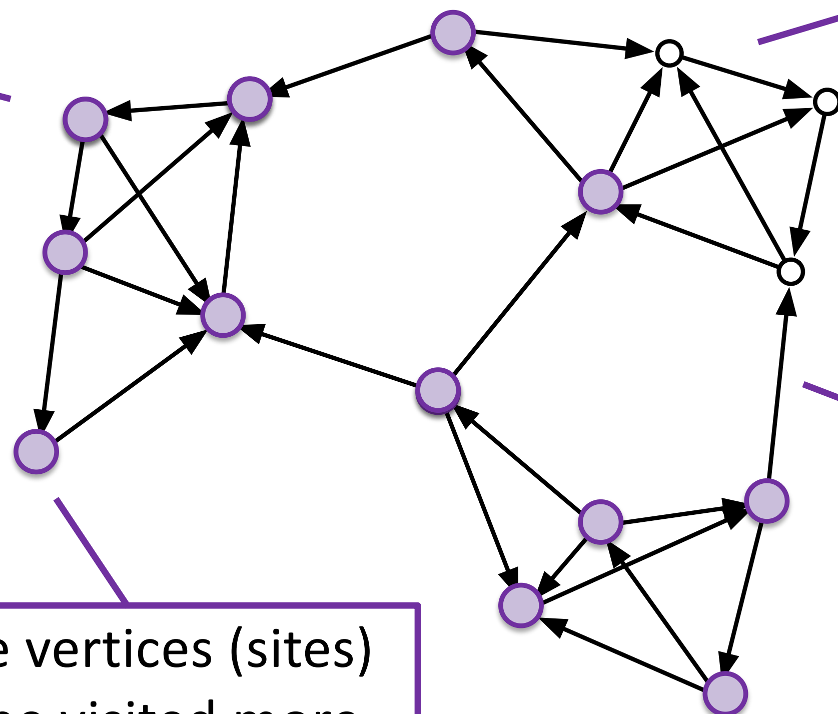
# Surfing: Random Walk on the Web Graph



# Surfing: Random Walk on the Web Graph

If we do this for a long time

"Important" vertex (site)



Modified random walk includes "teleportation"

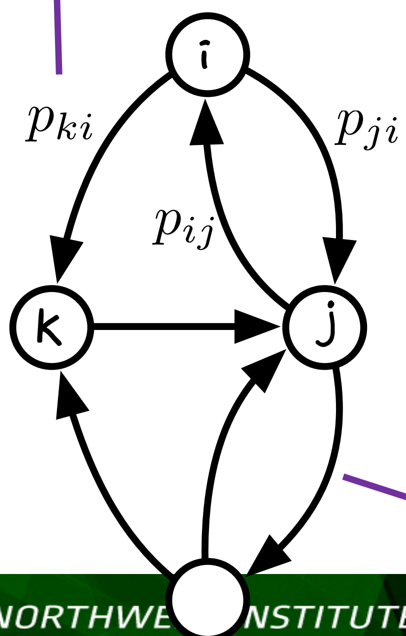
PageRank: Order vertices by importance

Some vertices (sites) will be visited more often than others

# Vector Representation

Probability that user will follow link from i to k

Probability that user will follow link from i to k




$$p_{ji} + p_{ki} = 1$$

Stochasticity

Graph of links

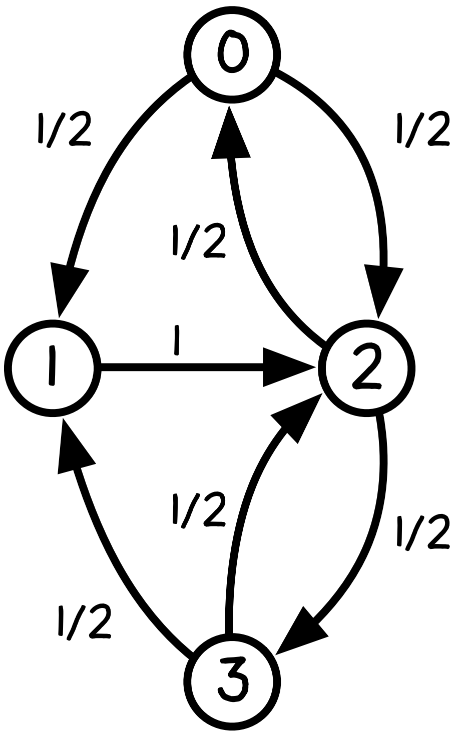
$$\begin{bmatrix} 0 \\ \vdots \\ 0 \\ p_{ji} \\ 0 \\ \vdots \\ 0 \\ p_{ki} \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

Stochastic (column) vector for node i

Entry at row j for edge from i

Entry at row k for edge from i

# Matrix Vector




Node 0

$$\begin{bmatrix} 0 \\ \frac{1}{2} \\ \frac{1}{2} \\ 0 \end{bmatrix}$$

Node 1

$$\begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

Node 2

$$\begin{bmatrix} \frac{1}{2} \\ 0 \\ 0 \\ \frac{1}{2} \end{bmatrix}$$

Node 3

$$\begin{bmatrix} 0 \\ \frac{1}{2} \\ \frac{1}{2} \\ 0 \end{bmatrix}$$

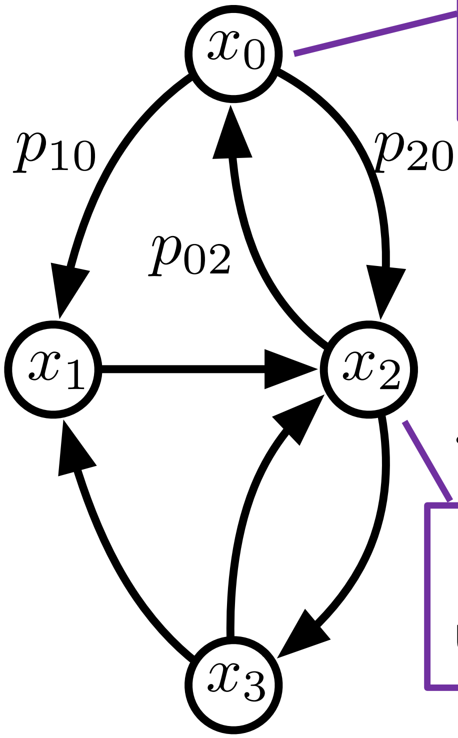
Probability that user will follow link from i to k

$$\begin{bmatrix} 0 & 0 & \frac{1}{2} & 0 \\ \frac{1}{2} & 0 & 0 & \frac{1}{2} \\ \frac{1}{2} & 1 & 0 & \frac{1}{2} \\ 0 & 0 & \frac{1}{2} & 0 \end{bmatrix}$$

Put vectors together into a matrix

$$\sum_i p_{ij} = 1 \quad \forall j$$

# Random Walk / Markov Process




Probability user is at 0

Probability user moves from 0 to 2

Probability user is at 2

What is the eigenvalue?

x is an eigenvector of P

$$x_2 = p_{20}x_0 + p_{21}x_1 + p_{23}x_3$$

$$x_i = \sum_j p_{ij}x_j$$

$$x = Px$$

$$\sum_i p_{ij} = 1 \quad \forall j$$

$$\sum_j x_j = 1$$

## Some Facts

- Exploit $\sum_i p_{ij} = 1 \quad \forall j$ and consider left eigenvalues (which are same as right eigenvalues
- By Gershgorin, all (left) eigenvalues are in or on a circle of radius 1
- That is, spectral radius is equal to unity
- By Perron-Frobenius, there is a unique eigenvalue at the spectral radius (there is unique eigenvalue equal to unity)
- Conclusion, there is an x that satisfies $x = Px$

# Computing Solution

- Let $\tilde{x} = P\tilde{x}$
- Claim

$$\lim_{k\to\infty} P^k y = \tilde{x} \text{ for any y}$$

So: $\tilde{x} = z$

Let

$$z = \lim_{k\to\infty} P^k y$$

Then

$$z = \lim_{k\to\infty} P^k y$$

$$= \lim_{k\to\infty} PP^k y$$

$$= P \lim_{k\to\infty} P^k y$$

$$= Pz \Rightarrow z = Pz$$

But $\tilde{x}$ is unique

# Computing Solution

Matrix-matrix product (k of them)

Matrix-vector product (k of them)

$$\lim_{k\to\infty} P^k y = \tilde{x}$$ for any y

$$(P^k)x = P(P(P\ldots(Px)))$$

Expensive!

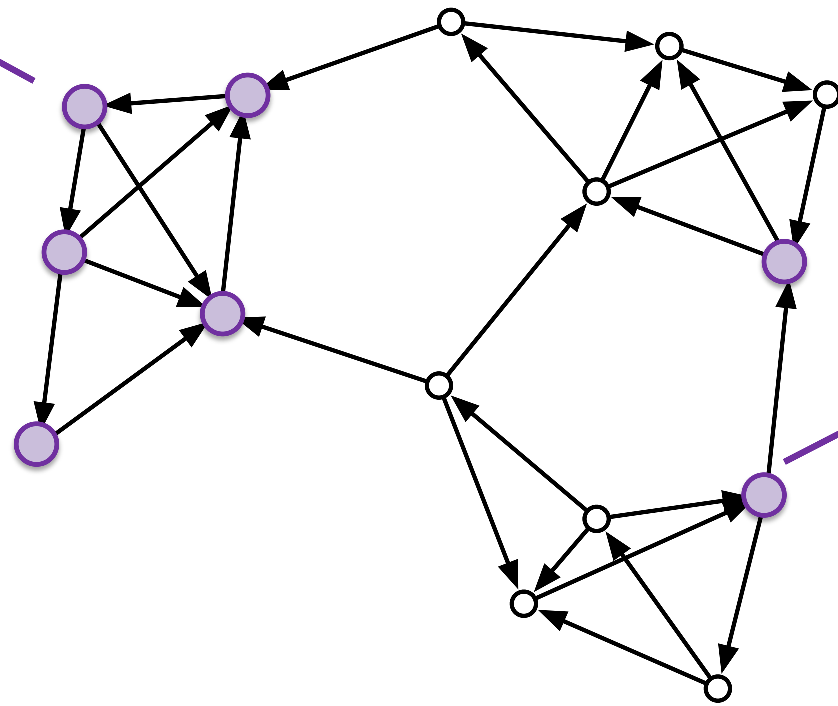
Much cheaper!

```
Vector x(N);
randomize(x);
x = (1.0 / one_norm(x)) * x;

for (size_t i = 0; i < max_iters; ++i) {
  Vector y = P * x;
  if (two_norm(x - y) < tol) {
    return y;
  }
  x = y;
}
```

# Teleportation

Once we get into this cycle we can't get out



PageRank includes "teleportation"

# Teleportation

Include teleportation computationally

Small probability that user might go from a site to any other site

$$Q = \frac{\alpha}{N_p} \begin{bmatrix} 1 & 1 & \dots & 1 \\ 1 & 1 & \dots & 1 \\ \vdots & \vdots & & \vdots \\ 1 & 1 & \dots & 1 \end{bmatrix} + (1-\alpha)P$$

Scale to maintain Markov chain properties

Sum of all elements in column is equal to unity

# Simplifying Teleportation

$$\frac{1}{N_p}\begin{bmatrix} 1 & 1 & \ldots & 1 \\ 1 & 1 & \ldots & 1 \\ \vdots & \vdots & & \vdots \\ 1 & 1 & \ldots & 1 \end{bmatrix} x = \frac{1}{N_p}\begin{bmatrix} |x|_1 \\ |x|_1 \\ \vdots \\ |x|_1 \end{bmatrix} = \frac{1}{N_p}\begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix}$$

$$x \leftarrow (1-\alpha)Px + \frac{\alpha}{N}$$

Small bias

# Algorithm with Teleportation

```
Vector x(N);
randomize(x);
x = (1.0 / one_norm(x)) * x;

for (size_t i = 0; i < max_iters; ++i) {
  Vector y = (1.0 - alpha) * P * x + alpha / x.num_rows();
  if (two_norm(x - y) < tol) {
    return y;
  }
  x = y;
}
```

Teleportation bias

# Parallelization Strategy

- **Finding Concurrency**
  - Decompose problem into pieces that can execute concurrently
    - By task or by data
  - Manage sharing
- **Algorithm Structure**
  - Fundamental organizing principle
    - Around tasks or around data decomposition or around data flow
- **Supporting Structures**
  - Programming paradigms and data structures
- **Implementation Mechanisms**
  - Manage tasks, move data

Timothy Mattson, Beverly Sanders, and Berna Massingill. 2004. *Patterns for Parallel Programming* (First ed.). Addison-Wesley Professional.

# Walkthrough

# Thank you!

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