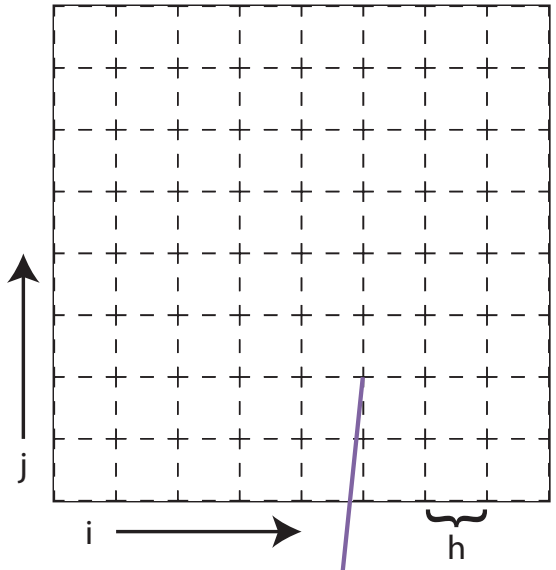
# Discretized




Each grid point (I, j) in the mesh

- Del operator

$$\nabla\phi = \frac{\partial\phi}{\partial x} + \frac{\partial\phi}{\partial y}$$
$$\nabla^2\phi = \frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2}$$

- Finite difference approximation to derivative

First derivative

$$\frac{\mathrm{d}x}{\mathrm{d}t}(t_0) \approx \frac{x(t_0+h)-x(t_0)}{h}$$

Derivative of first derivative

$$\frac{\mathrm{d}^2x}{\mathrm{d}t^2}(t_0) \approx \frac{\frac{\mathrm{d}x}{\mathrm{d}t}(t_0+h)-\frac{\mathrm{d}x}{\mathrm{d}t}(t_0)}{h}$$
$$= \frac{x(t_0+h+h)-x(t_0+h)-x(t_0+h)+x(t_0)}{h^2}$$
$$= \frac{x(t_0+2h)-2x(t_0+h)+x(t_0)}{h^2}$$
$$= \frac{x(t_0+h)-2x(t_0)+x(t_0-h)}{h^2}$$

Approximate second derivative

- Finite difference approximation to del

$$\frac{\phi_{i+1,j} + \phi_{i-1,j} + \phi_{i,j-1} + \phi_{i,j+1} - 4\phi_{i,j}}{h^2} = 0$$

Gives us a linear equation

The value of each point is the avearage of its neighbors